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ORIGINAL ARTICLE

Proposals for the masses of the three largest asteroids, the Moon-Earth mass ratio and the Astronomical Unit

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Abstract We propose to the NSFA (the IAU Working Group on Numerical Standards for Fundamental Astronomy) the following representative values and realistic uncertainties for the masses of the three largest asteroids (Ceres, Pallas, Vesta), to be used as the current best estimates:

$$M_{\text{Ceres}}/M_{\odot} = 4.72(3) \cdot 10^{-10},$$
$$M_{\text{Pallas}}/M_{\odot} = 1.03(3) \cdot 10^{-10},$$
$$M_{\text{Vesta}}/M_{\odot} = 1.35(3) \cdot 10^{-10}.$$

Unlike the values previously adopted in the Astronomical Almanac, these are consistent with nearly all of the twenty or so modern accurate determinations from various authors. We also have proposed the following values for the Moon-Earth mass ratio and the astronomical unit in meters obtained from the ephemeris improvement processes at JPL in Pasadena and at IAA RAS in St.Petersburg: $M_{\text{Moon}}/M_{\text{Earth}} = 0.0123000371(4)$ and $AU = 149597870700(3)$ m. The numerical value of the AU in meters is identical in both the TDB-based and the TCB-based systems of units if one uses the conversion proposed by Irwin and Fukushima, Brumberg and Groten, Brumberg and Simon.

Keywords Ceres mass · DE ephemerides · EPM ephemerides · Pallas mass · Vesta mass · Moon-Earth mass ratio · Astronomical Unit

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1 Introduction

The largest asteroids are massive enough to significantly affect the orbits of other bodies in the Solar System. Direct dynamical determinations of their masses come from their perturbations upon other Solar System bodies.

In the classical example, the perturbed body is some other asteroid, and the accuracy of the mass determination depends upon the geometry of a close encounter and upon the accuracy and extent of the observational data obtained before and after the encounter. Since the first asteroid mass estimation by Hertz (1966) from Vesta's encounters with Arete, several tens of asteroid masses have been determined using asteroid-asteroid perturbations. Unfortunately, this classical method is known to have significant problems. As was shown, for example, by Krasinsky et al. (2001); Hilton (2002) the classical method of determining asteroid masses is limited by uncertainties in the masses of the largest asteroids, unmodeled or poorly modeled perturbations by other asteroids, and the quality of the observations themselves. Thus, the method may lead to inaccurate or poorly-determined estimations. Accurate determinations may be obtained for only cases when very close encounters are provided with useful data before and after the encounter. For example, recently, a new determination of (15) Eunomia has been obtained by Vitagliano and Stoss (2006) and confirmed by two independent groups (Kochetova 2006; Baer and Chesley 2008): $(1.64 \pm 0.06) \cdot 10^{-11} M_{\odot}$. In particular, in the last paper, Baer and Chesley obtained mass values for 21 asteroids by examining over 2500 candidate events of close encounters.

The masses of Ida (243), Gaspra (951), Eros (433), and Mathilda (253) have been derived from their perturbations upon spacecraft during flybys.

Some asteroids are now known to be double or to have satellites, so their masses are also determined with reasonably good accuracy.

The masses of Ceres, Pallas, and Vesta, as well as a few others, may also be estimated from analysis of the highly accurate ranging data of the spacecraft orbiting Mars or landed upon its surface using high precision ephemerides.

Recent years have also yielded significant improvements to a whole set of constants for planet ephemerides. Some constants, namely, the masses of Ceres, Pallas, Vesta, the Earth–Moon mass ratio, and the value of the Astronomical Unit are hereby proposed to the NSFA WG as current best estimates.

The values of the masses of Ceres, Pallas, Vesta, the Earth–Moon mass ratio, and the value of the Astronomical Unit proposed to the NSFA WG are based on original investigations carried out by the authors by using planet ephemerides of Jet Propulsion Laboratory—DE and of Institute of Applied Astronomy of Russian Academy of Sciences—EPM and comparison of them for demonstration of their reliability with the best values of other researchers.

2 DE and EPM ephemerides, evaluation of their parameters

To ensure space flights in the late 1960's numerical planetary ephemerides were coming into construction by several groups in the USA and Russia. The ephemerides of the two independent groups—JPL and IAA RAS, having about the same accuracy, continue to be improved since that time.

Common to these and other modern planet ephemerides (the recent French ephemerides INPOP06 of IMCCE, Fienga et al. 2008) is the simultaneous numerical integration of the equations of motion of the nine major planets, the Sun, 300 or more biggest asteroids, the Moon, and the lunar physical libration performed in the Parameterized Post-Newtonian metric for General Relativity taking into account perturbations due to the solar oblateness, the gravity fields of the Moon and Earth, and a massive ring of small asteroids. The ephemerides have been oriented to the International Celestial Reference Frame (ICRF) with an accuracy better than 1 mas by including into the total solution the ICRF-base VLBI measurements of spacecraft near the planets.

The dynamical models of different versions for these ephemerides differ slightly by:

- the modeling of the lunar libration, but this difference doesn't influence significantly the planet ephemerides;
- the modeling of the perturbations from asteroids (the dynamical models includes perturbations from 300–343 largest asteroids and the massive ring of small asteroids);
- sets of observations to which ephemerides are adjusted;
- sets of solution parameters.

Observations to which ephemerides have been fitted include many thousands of measurements (about 550000 ones in 2008) of different types. They are classical and modern optical observations of the outer planets and their satellites (since 1911), ranging to planets, the martian landers and spacecraft, including the data of Mariner-9, Viking, Pathfinder, MGS, Odyssey, MRO, Venus Express (1961–2008), differenced range (1976–1987), VLBI spacecraft observations (1990–2007).

Several hundreds of parameters are determined simultaneously while improving the planetary part of the ephemerides. In addition to the orbital elements of all the planets and the main satellites of the outer planets, this set includes masses of celestial bodies, parameters of surface topography of planets and rotation of Mars, the coefficients of the solar corona and the solar oblateness, parameters of the orientation of planet ephemerides to the ICRF, etc. The value of Astronomical Unit, masses of Ceres, Pallas, Vesta and the Earth–Moon mass ratio are significant parameters of the DE and EPM ephemerides.

The reduction of observations includes all the relevant corrections. The main reductions for the optical observations of planets are correction for the additional phase effect (the main phase corrections were made by observers themselves) and corrections for referencing observations obtained in different catalogues to the ICRF reference frame. The reduction for ranging observations includes the relativistic corrections—the time delay (the Shapiro effect) and path-bending of the propagation of radio-signals in the gravitational fields of the Sun, Jupiter, and Saturn, and the reduction of observations from the coordinate time of the ephemerides to the proper time of the observer; the delay from the solar corona; the delay from the Earth's troposphere; and the correction of planetary radar observations for topography.

The basic flow of the ephemeris creation and parameter determination process is that of a least-squares iteration which can be reduced to the following:

- Numerical integration of the equations of motion for the major planets, Moon, and Sun and variational equations for producing the partial derivatives.
- Computing the model observations "C" (e.g. time delays) from the produced ephemerides for the time of each observation "O", calculating the residuals (O-C) and the partial derivatives.
- Obtaining the values of the parameters being determined, and deriving the residuals of the observations after the improvement by the least-squares adjustment; for that, observations are weighted in accordance with their a priori accuracy.

The concrete versions of JPL ephemerides (DE200, DE405, DE410, DE414, DE421) and IAA RAS ones (EPM87, EPM2000, EPM2006, EPM2008) mentioned in this paper are described in Standish 1990, 1998, 2003, 2005, 2006; Folkner et al. 2008; Krasinsky et al. 1993; Pitjeva 2001, 2007, 2009.

As experience shows, the formal accuracy of determining the parameters by LS is overly optimistic. The actual accuracy could be an order of magnitude lower due to the deviation of the distribution of observations from the Gaussian law and due to the systematic errors in the observations, often of an unknown nature. The actual accuracies of the parameters can be estimated by comparing the values obtained in dozens of different test LS solutions that differed by the sets of observations, their weights, and the sets of parameters included in the solution, as well as by comparing parameter values and ephemerides produced by independent groups. In order to make sure of reliability of DE and EPM ephemerides a comparison of all stages of the ephemeris creation (calculation of different types of residuals, their partial derivatives, evaluation of ephemeris parameters by the least-squares method and coordinates themselves of produced ephemerides) was made. This comparison has shown good agreement, thus demonstrating the reliability of the numerical values obtained for the ephemeris parameters. The comparison of DE and EPM ephemerides is given in Standish 2000; Pitjeva 2001, 2005, 2007.

3 The masses of Ceres, Pallas, Vesta

Masses of several asteroids which affect Mars and the Earth most strongly may be estimated from the observations of the martian landers and the spacecraft orbiting Mars. Standish and Hellings (1989) determined the masses of Ceres, Pallas and Vesta from Viking measurements. This paper inspired further investigations on the determination of the masses of the largest asteroids from data of the martian landers and spacecraft. The perturbation of the largest asteroids are considered in this paper, and it was shown that the masses of Ceres, Pallas, Vesta may be estimated by this method, i.e. while fitting planet ephemerides to observations with estimating masses of these asteroids simultaneously with all other parameters of planet ephemerides. At a later time with the addition of the exceedingly precise ranging to the Mars Global Surveyor, Odyssey, Mars Express and Mars Reconnaissance Orbiter, the masses of these asteroids obtained by this method have been improved significantly (Standish 1998, 2000, 2006; Pitjeva 2001, 2005, 2009; Fienga et al. 2008; Folkner et al. 2008. For comparison the values of the masses of Ceres, Pallas and Vesta, obtained by different authors from close encounters with other asteroids and from their perturbations upon the orbit of Mars (the authors whose results were obtained by this last method are marked by *), are presented in Table 1.

In selecting the representative values of Ceres, Pallas, Vesta masses we have preferred the estimations obtained from their perturbations upon the orbit of Mars, as the classical method based on their perturbations upon other asteroids often gets inexact results, as noted above; and it is seen from Table 1 that the scattering of such estimations is significantly greater. Moreover, the proposing values have been obtained by the averaging of the recent author's estimations.

Although modern ephemerides (JPL's DE405–DE421, IAA's EPM2000–EPM2008), and observations from which asteroid masses are taken are extremely accurate, some unknown, unmodeled, and systematic effects can possibly remain which correlate with the estimated parameters. Thus, comparison of different solutions is desirable in order to get a more realistic idea of the uncertainties involved. Our values are in the very good agreement with recent results obtained by Fienga et al. (2008) and Folkner et al. (2008). The real uncertainties of these values have been obtained from the different authors' solutions produced by varying data sets, their weights and solution parameters as well as from the comparison with values of other authors. From a consideration of the many different determinations, we have suggested

Table 1 Masses of Ceres, Pallas and Vesta in $10^{-10} M_\odot$

Ceres	Pallas	Vesta	Year	Authors
5.0 ± 0.2	1.4 ± 0.2	1.5 ± 0.3	1989	Standish and Hellings*
4.796 ± 0.085			1992	Sitarski and Todororovic
4.62 ± 0.07		1.396 ± 0.043	1995	Sitarski and Todororovic
4.64	1.05	1.34	1995	Standish et al.*
4.35 ± 0.05	1.60 ± 0.04	1.52 ± 0.09	1997	Hilton
4.759 ± 0.023			1998	Viateau and Rapaport
4.70	1.00	1.30	1998	Standish*
4.39 ± 0.04	1.59 ± 0.05	1.69 ± 0.11	1999	Hilton
4.70 ± 0.04	1.21 ± 0.26	1.36 ± 0.05	2000	Michalak
	1.17 ± 0.03		2001	Goffin
		1.306 ± 0.016	2001	Viateau and Rapaport
4.76 ± 0.02	1.08 ± 0.04	1.35 ± 0.02	2000	Standish*
4.81 ± 0.01	1.00 ± 0.01	1.36 ± 0.01	2001	Pitjeva*
4.69 ± 0.01	1.05 ± 0.01	1.36 ± 0.01	2003	Standish*
4.753 ± 0.007	1.027 ± 0.003	1.344 ± 0.001	2005	Pitjeva*
4.699 ± 0.028	1.026 ± 0.028	1.358 ± 0.016	2006	Konopliv et al.*
4.736 ± 0.026			2007	Kovacevic and Kuzmanovski
4.75 ± 0.03	1.06 ± 0.13	1.34 ± 0.01	2008	Baer and Chesley
4.746 ± 0.006	0.995 ± 0.003	1.338 ± 0.002	2008	Fienga et al.*
4.685	1.010	1.328	2008	Folkner et al.*
4.712 ± 0.006	1.027 ± 0.007	1.344 ± 0.003	2009	Pitjeva*

Table 2 Masses of Ceres, Pallas and Vesta proposed to NSFA

Object	Previous adopted values	Proposed new values
$M_{\text{Ceres}}/M_\odot$	$4.39(4)\cdot 10^{-10}$	$4.72(3)\cdot 10^{-10}$
$M_{\text{Pallas}}/M_\odot$	$1.59(5)\cdot 10^{-10}$	$1.03(3)\cdot 10^{-10}$
$M_{\text{Vesta}}/M_\odot$	$1.69(11)\cdot 10^{-10}$	$1.35(3)\cdot 10^{-10}$

to the NSFA WG as the current best estimates, the following mean representative values of Ceres, Pallas, and Vesta masses and their realistic uncertainties set out in Table 2. From Tables 1 and 2, it is evident that the values previously adopted in 2006 in the Astronomical Almanac are not consistent with any of the many other determinations.

4 The Earth–Moon mass ratio

The ratio of the masses of the Earth and the Moon has been obtained by Standish while fitting the DE414 to all the data (Standish 2006); $M_{\text{Earth}}/M_{\text{Moon}} = 81.300568$. This value is nearly identical to the newer value obtained recently by Folkner et al. 2008 for the DE421 ephemeris: $M_{\text{Earth}}/M_{\text{Moon}} = 81.3005691 \pm 0.0000005$; and the value for the EPM2008 ephemeris (Pitjeva 2009): $M_{\text{Earth}}/M_{\text{Moon}} = 81.3005676 \pm 0.0000001$, where for the two last values the *formal* uncertainties are indicated. A realistic uncertainty may be more than what estimations obtained in different solutions show. Therefore we have proposed the following value of the Earth–Moon mass ratio

$$M_{\rm Earth}/M_{\rm Moon} = 81.300568 \pm 0.000003$$

or the Moon–Earth mass ratio, as it is customary in the NSFA

$$M_{\rm Moon}/M_{\rm Earth} = 0.0123000371(4).$$

5 The value of the Astronomical Unit

Since the beginning Venus radar echoes in 1961 the value of the Astronomical Unit, which fixes the scale distance in the Solar System, has been determined exclusively from ranging data of planets and spacecraft. The AU value is one of basic parameters of JPL (DExxx) and IAA RAS (EPMxxxx) ephemerides. Some values for these ephemerides are given in Table 3.

The AU values for DE405 (Standish 1998) and EPM2000 (Pitjeva 2001) coincide due to the similarity of the dynamical models and sets of observations used in these ephemerides. At present (DE414, DE421 and EPM2006, EPM2008/9) the AU values differ by 4–5 m, giving, therefore, a rough estimate of the uncertainty involved with this parameter. Some tests show that this difference is caused by the specific choices of the parameter sets (asteroid masses, data bias parameters, etc.). We have proposed to the NSFA WG the value AU = 149597870700(3) m which is consistent with the value $GM_{\rm Sun} = 1.32712442099(10) \cdot 10^{20}[{\rm m}^3{\rm s}^{-2}]$ proposed to the NSFA WG by W. Folkner.

The value $GM_{\rm Sun}$ in the physical system of units $[{\rm m}^3{\rm s}^{-2}]$ may be estimated from fitting ephemerides to observations. However, up to the present, this value has been calculated from an entirely equivalent process—that of adjusting the AU value, obtained in meters while fitting planet ephemerides, by using the relation,

$$GM_{\rm Sun}[{\rm m}^3\,{\rm s}^{-2}] = k^2\; AU[{\rm m}]^3/86400[{\rm s}]^2, \tag{1}$$

where $k = 0.01720209895$ is Gaussian gravitational constant.

Equation 1 effectively defines the Astronomical Unit, and is used for the transition between the physical system of units—SI and the astronomical system of units.

All of the main ephemerides of JPL and IAA RAS have been constructed in the TDB time scale. It is noted here that the AU in meters has the same numerical value for ephemerides built in the TDB or the TCB time scale, if one uses the conversion between TDB and TCB proposed by Irwin and Fukushima (1999); Brumberg and Groten (2001); Brumberg and Simon (2004).

As a test, two versions of the IAA RAS ephemerides were constructed in 2002, one in the TDB time system and the other in the TCB system, in accordance with this conversion

Table 3 The value of the Astronomical Unit in km

Ephemerides	AU-149597870	Ephemerides	AU-149597870	Real uncertainty
DE200	0.66 ± 0.03	EPM87	0.62 ± 0.18	0.2
DE405	0.6916 ± 0.0005	EPM2000	0.6912 ± 0.0002	0.01
DE414	0.7008 ± 0.00015	EPM2006	0.6953 ± 0.0001	0.005
DE421	0.6996 ± 0.00015	EPM2009	0.6966 ± 0.0001	0.003

(Pitjeva 2003). These ephemerides were fitted to data (1913–2002); all the rms residuals were identical, and the same AU value was obtained for both of these ephemerides, as expected.

6 Conclusions

The values for the masses of Ceres, Pallas, and Vesta adopted in Astronomical Almanac in 2006 have been seen to disagree with virtually all other determinations. We have therefore proposed to the WG NSFA the adoption of values which are representative of most of the modern, accurate determinations. We have also proposed consistent values of the Astronomical Unit and of the Earth–Moon mass ratio. The proposed values are based mainly on original investigations carried out by the authors by using planet ephemerides—DE (JPL) and EPM (IAA RAS).

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