

Lecture 5. Interstellar Dust and Extinction

1. Introduction
2. Extinction
3. Mie Scattering
4. Dust to Gas Ratio

References

Spitzer Ch. 7

Tielens, Ch. 5

Draine, ARAA, 41, 241, 2003

1. History of Dust

- Presence of nebular gas was long accepted but existence of absorbing interstellar dust remained controversial into the 20th cy.
- Star counts (W. Herschel, 1738-1822) found few stars in some directions, later demonstrated by E. E. Barnard's photos of dark clouds.
- Lick Observatory's Trumpler (PASP 42 214 1930) conclusively demonstrated interstellar absorption by comparing luminosity distances & angular diameter distances for open clusters:
 - Angular diameter distances are systematically smaller
 - Discrepancy grows with distance
 - Distant clusters are redder
 - Estimated ~ 2 mag/kpc absorption
 - Attributed it to Rayleigh scattering by tiny grains (25A)

Some More History

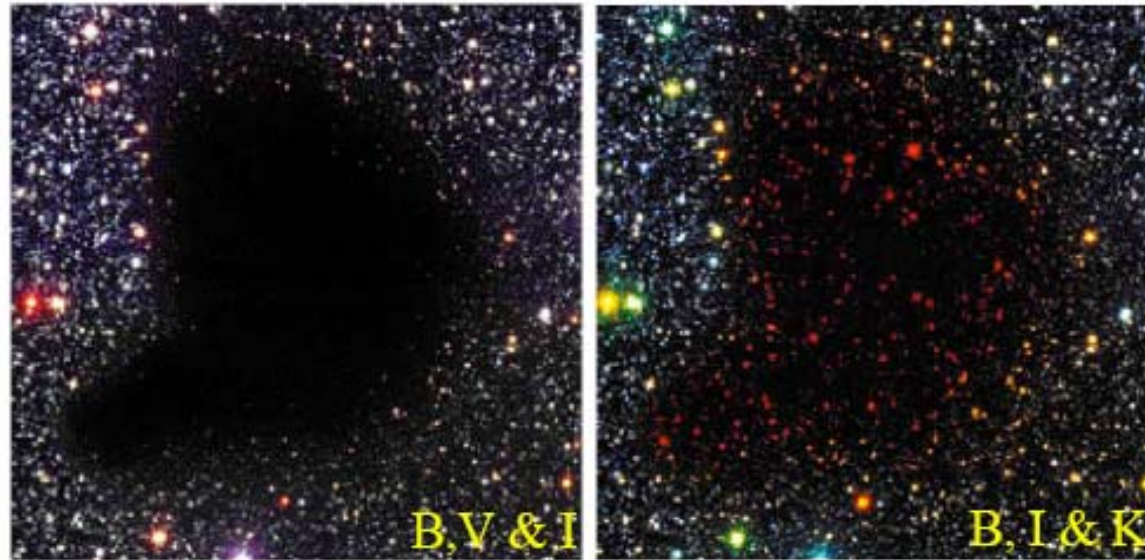
- The advent of photoelectric photometry (1940-1954) led to the discovery and quantification of **interstellar reddening** by precise comparison of stars of the same type
- Interstellar reddening is ascribed to the continuous (with wavelength) absorption of small macroscopic particles, quantified by the **extinction** A_λ , i.e., by the difference in magnitudes:
 - A_λ , varies roughly as $\lambda^{-1.5}$ from 0.3-2.3 μm
 - the condition, $2\pi\lambda/a \sim 1$, suggests particle sizes $a \sim 0.1\mu\text{m}$
- Starlight is polarized in regions of high extinction with one polarization state selectively removed by scattering from small elongated conducting or dielectric particles aligned in the galactic magnetic field

Summary of the Evidence for Interstellar Dust

- Extinction, reddening, polarization of starlight
 - dark clouds
- Scattered light
 - reflection nebulae
 - diffuse galactic light
- Continuum IR emission
 - diffuse galactic emission correlated with HI & CO
 - young and old stars with large infrared excesses
- Depletion of refractory elements from the interstellar gas (e.g., Ca, Al, Fe, Si)

Examples of the Effects of Dust

B68 dark cloud
extincted stars
appear red



Pleiades starlight
scattered by dust



2. Extinction

For simplicity, we consider homogeneous spherical dust particles of radius a and introduce the ***cross section for extinction***

$$\sigma(\lambda) = \pi a^2 Q_{ext}(\lambda),$$

where $Q_{ext}(\lambda)$ is the ***efficiency factor for extinction***. The optical depth along a line sight with volumetric dust density n_d is then

$$\tau_{\lambda}^{ext} = \int n_{dust} \sigma_{\lambda}^{ext} ds = \sigma_{\lambda}^{ext} \int n_{dust} = \pi a^2 Q_{ext}(\lambda) N_{dust}$$

Extinction in magnitudes A_{λ} is defined in terms of the reduction in the intensity cause by the presence of the dust :

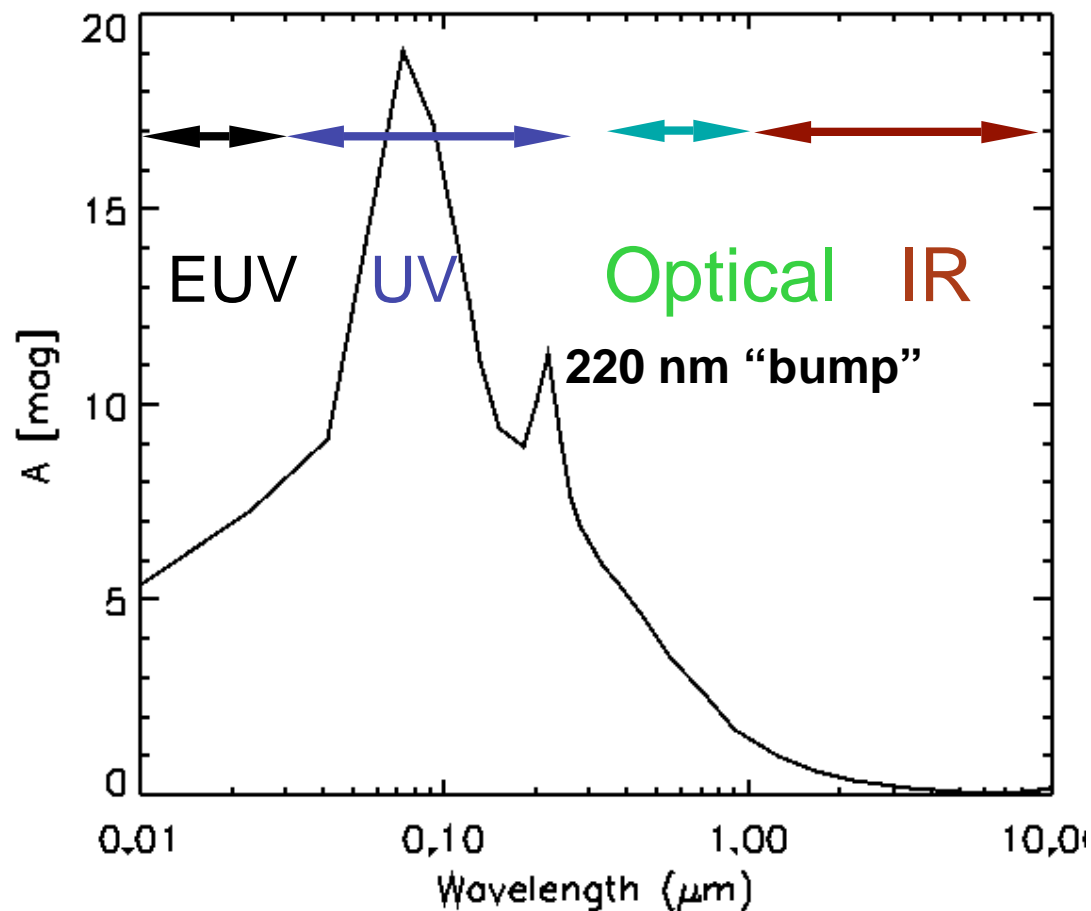
$$I(\lambda) = I_0(\lambda) \exp[-\tau_{\lambda}^{ext}],$$

$$A_{\lambda} = -2.5 \log_{10}[I(\lambda)/I_0(\lambda)] = 2.5 \log_{10}(e) \tau_{\lambda}^{ext} = 1.086 \tau_{\lambda}^{ext}$$

Schematic Plot of Extinction

General Properties

1. $\lambda^{-1.5}$ trend in the optical/NIR
3. steep UV rise with peak at $\sim 800 \text{ \AA}$
4. strong features:
 - $\lambda = 220 \text{ nm}$, $\Delta \lambda = 47 \text{ nm}$
 - $\lambda = 9.7 \text{ \mu m}$, $\Delta \lambda \sim 2\text{-}3 \text{ \mu m}$(not visible on this plot)



Magnitudes of extinction vs, wavelength (schematic)

Scattering and Absorption

The extinction efficiency is the sum of **scattering and absorption**

$$Q_{ext} = Q_{sca} + Q_{abs}$$

For spherical grains,

$$\sigma_{abs} = Q_{abs} \pi a^2 \quad \sigma_{sca} = Q_{sca} \pi a^2$$

$$\sigma_{ext} = Q_{ext} \pi a^2 = (Q_{abs} + Q_{sca}) \pi a^2$$

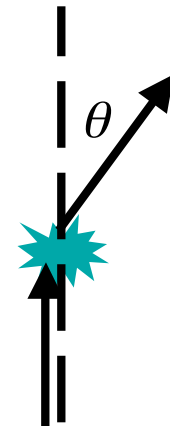
The amount and character of the scattering is expressed by the

$$\text{albedo} = \frac{\sigma_{sca}}{\sigma_{ext}} = \frac{Q_{sca}}{Q_{ext}} \leq 1$$

All of the Q 's and the albedo are functions of λ and a . The scattering phase function is

$$g = \langle \cos \theta \rangle = \frac{\int_0^\pi I(\theta) \cos \theta d\Omega}{\int_0^\pi I(\theta) d\Omega}$$

$\langle \cos \theta \rangle = 1, 0, -1$ correspond to forward, isotropic, and back scattering



Measures of Extinction

The (“general”) extinction A_λ can also be written as terms of the brightness in magnitudes of a source, but this requires knowing its distance and luminosity

$$m_\lambda = M_\lambda + 5 \log d - 5 + A_\lambda.$$

Instead we use the distance-independent “***selective extinction***”, which is the additional *color excess* due to extinction

$$E(B-V) = A(B) - A(V) = (B-V) - (B-V)_0$$

The “***normalized selective extinction***” at any wavelength is also a common measure of extinction:

$$E(\lambda, V) / E(B-V)$$

The “***normalized extinction***”: $1/R_V$, measures the steepness of the extinction curve

$$R_V = A(V) / [A(B) - A(V)] = A(V) / E(B-V)$$

It is steep in the diffuse ISM: $R_V = 3.1 \pm 0.2$, shallower in dark clouds:

$$R_V \approx 5$$

Copernicus Observations of Two Similar Stars

Raw (uncorrected, unnormalized) count rates from the UV satellite *Copernicus* for two O7 stars, S Mon (top, “un-reddened”) ξ Per (bottom, “reddened”).

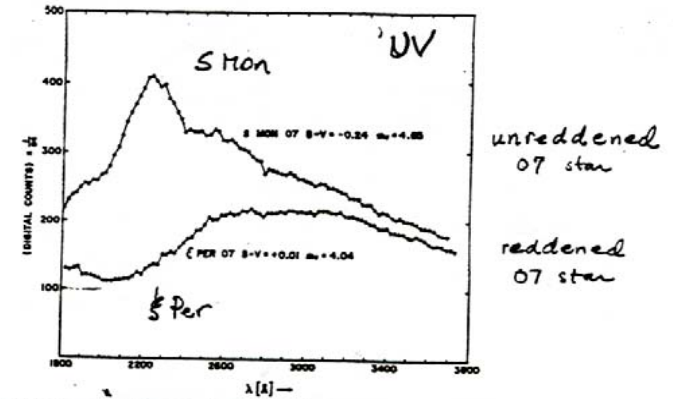


FIG. 1.—Spectral scans of S Mon and ξ Per over the region $\lambda\lambda 1800-3700$, made by Spectrometer 1 on OAO-2. Note the large extinction effects, especially near $\lambda 2200$. These scans are neither normalized nor corrected for the instrumental sensitivity function.

Upper panel: 1800 - 2000 Å
Bottom panel :1000 – 2000 Å

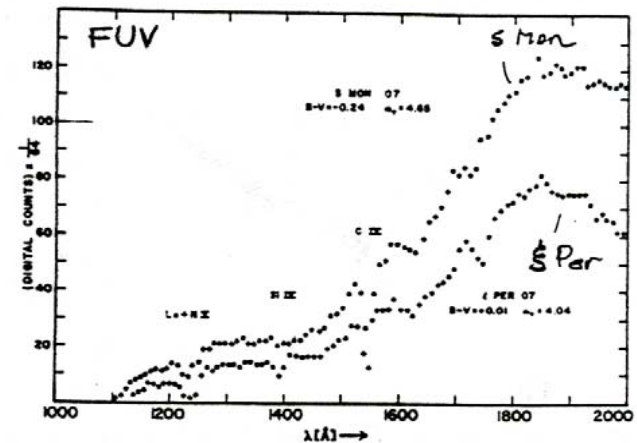
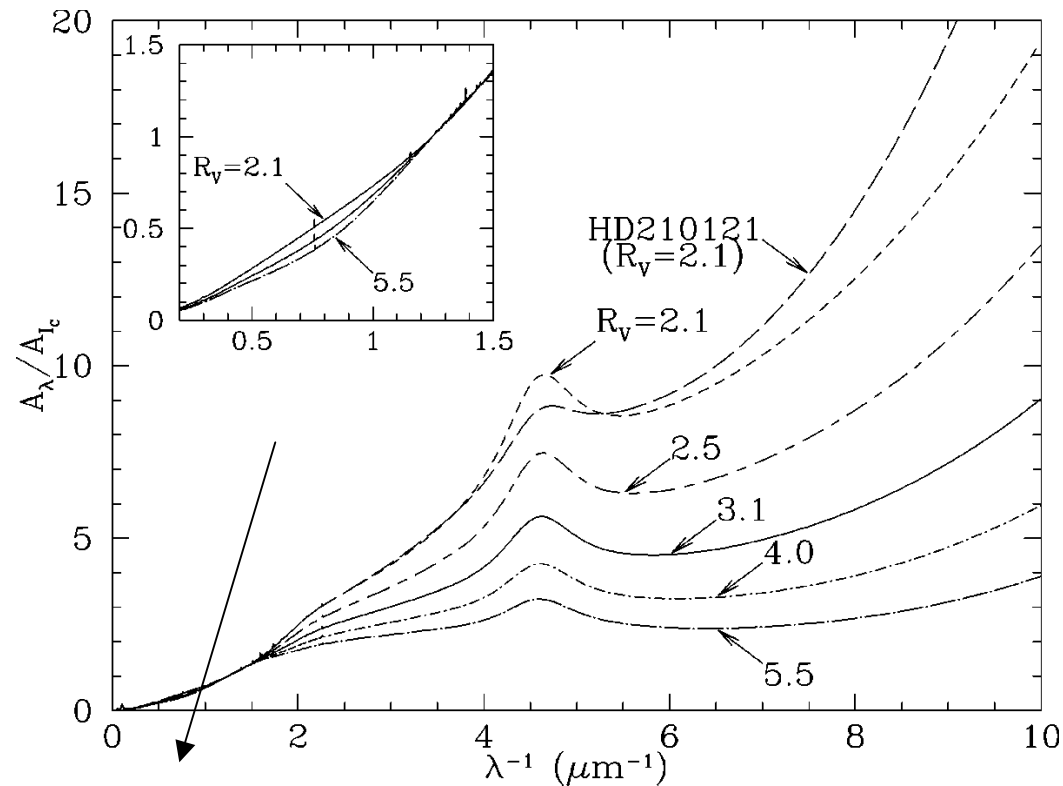


FIG. 2.—Spectral scans of S Mon and ξ Per over the region $\lambda\lambda 1100-2000$ made by Spectrometer 2. As with the curves in Fig. 1, these scans are raw data with only a small, constant background correction made.

Empirical Interstellar Extinction Curves



Based on extinction measurements from the UV to NIR, c.f. Fitzpatrick, PASP 111 63 1999 & Draine, ARAA 41 241 2003

Illustrating the effect of varying the “normalized extinction $1/R_V$ which measures the slope of the extinction curve

Correlation between Gas and Dust

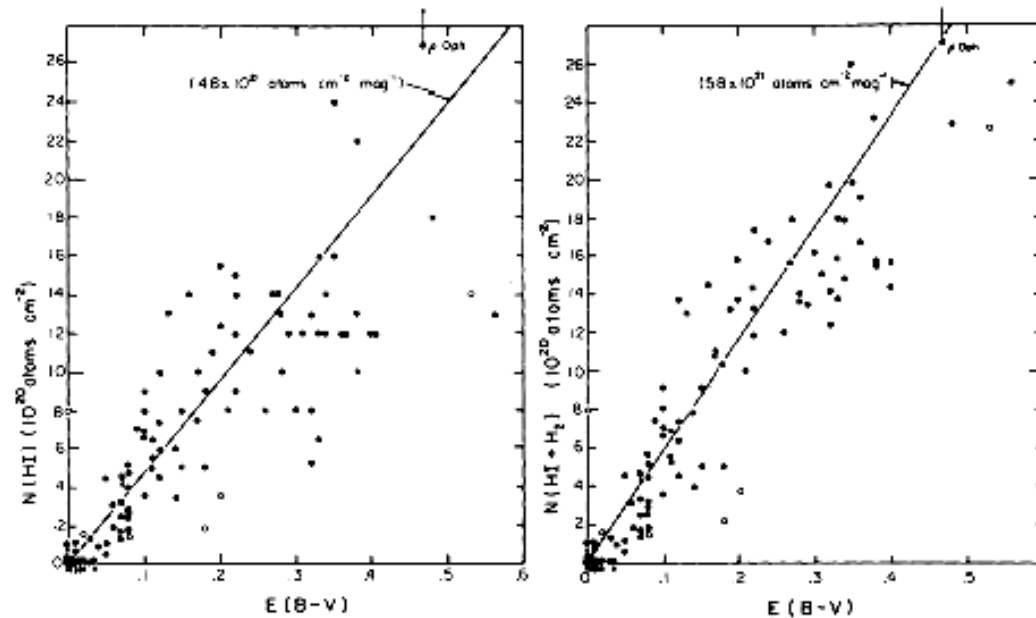


Figure 2 Correlations between gas column densities and interstellar reddening for 100 stars from the Copernicus atomic and molecular hydrogen survey (Savage et al. 1977, Bohlin, Savage & Drake 1978): (a) shows the atomic hydrogen column density, $N(\text{HI})$, versus $E(B-V)$, (b) shows the total hydrogen column density, $N(\text{HI} + \text{H}_2) = N(\text{HI}) + 2N(\text{H}_2)$, versus $E(B-V)$. Be stars are denoted with the open symbols. The solid line in (a) gives the average atomic hydrogen to $E(B-V)$ ratio $4.8 \times 10^{21} \text{ atoms cm}^{-2} \text{ mag}^{-1}$. In (b) the solid line gives the average total hydrogen to $E(B-V)$ ratio of $5.8 \times 10^{21} \text{ atoms cm}^{-2} \text{ mag}^{-1}$. The point for $\rho \text{ Oph}$ in (a) and (b) should be moved upward by about a factor of 2.7.

Plots of HI and H₂ vs. selective extinction $E(B-V)$ towards 100 stars observed by Copernicus (Savage & Mathis ARAA 17 33 1979).

The correlation on the right is

$$N_{\text{H}} = 5.8 \times 10^{20} \text{ cm}^{-2} \text{ mag}^{-1}$$

and measures of the dust to gas ratio for diffuse interstellar clouds.

The Dust to Gas Ratio

Extinction measurements of local diffuse clouds yield

$$E(B, V) \approx N_H / 1.58 \times 10^{20} \text{ mag cm}^{-2} \quad R_V \approx 3.1$$

or , with $R_V = A(V) / E(B-V)$,

$$A(V) \approx N_H / 1.8 \times 10^{21} \text{ mag cm}^{-2}$$

But $A_\lambda = 1.086 \tau_\lambda^{ext}$ and

$$\begin{aligned} \tau_\lambda^{ext} &= \int ds n_{dust} \sigma_\lambda^{ext} = \int ds (\rho_{dust} / m_{dust}) \sigma_\lambda^{ext} \\ &= \int ds n_H (\rho_{dust} / \rho_{gas}) (m_{gas} / m_{dust}) \sigma_\lambda^{ext} \end{aligned}$$

where ρ_{dust} / ρ_{gas} = the **dust to gas ratio**, m_{dust} is the mass of a dust grain, and m_{gas} is the mean gas particle mass per H nucleus $\sim 1.4 m_H$.
Extracting mean values from the integral gives

$$\tau_\lambda^{ext} = \langle \rho_{dust} / \rho_{gas} \rangle \langle m_{gas} / m_{dust} \rangle Q_\lambda^{ext} \pi a^2 N_H$$

Gas to Dust Ratio

Introducing the internal grain mass density $\rho_{\text{gr}} \sim 2.5 \text{ gr cm}^{-3}$

$$A(V) = 1.086 \frac{3m_{\text{gas}}Q_{\lambda}^{\text{ext}}}{4\rho_d a} \langle \rho_{\text{dust}} / \rho_{\text{gas}} \rangle N_{\text{H}}$$

Numerically comparing this with the empirical value on the previous slide gives an estimate of the dust to gas ratio in terms of $a = 0.1 \mu\text{m } a_{-5}$

$$\langle \rho_{\text{dust}} / \rho_{\text{gas}} \rangle = 0.0073 \frac{a_{-5}}{Q_{\lambda}^{\text{ext}}} \frac{\rho_{\text{gr}}}{2.5 \text{ gr cm}^{-3}} \approx 0.01$$

Another order of magnitude result is to rewrite $A_{\lambda} = 1.086 \tau_{\lambda}^{\text{ext}}$ as

$$A(V) = 1.086 N_d \sigma_V^{\text{ext}}$$

and to divide by the gas column N_{H} to obtain the **mean extinction cross section per H nucleus**

$$N_d / N_{\text{H}} \sigma_V^{\text{ext}} \sim 5 \times 10^{-22} \text{ cm}^2$$

3. Electromagnetic Scattering by Small Particles

A. N. Mie (Ann Phys 25 377 1908) solved Maxwell's Equations for scattering by a uniform sphere of radius a and a general index of refraction

$$\mathbf{m} = n - i k \quad \text{with} \quad m = m(\lambda),$$

The mathematical basis is a *multipole expansion* of the scattered wave in terms of (vector) spherical harmonics times, each multiplied by a radial Bessel function, plus the application of appropriate boundary conditions at the surface of the sphere. The classic reference is: H.C. van de Hulst, "Light Scattering from Small Particles"

The basic parameter, measuring radius in terms of wavelength is: $x = 2\pi a/\lambda$

$x \ll 1$: long wavelength *diffraction limit* - need only a few terms

$x \gg 1$: short wavelengths *geometrical optics limit* - need many terms

Asymptotic Mie Formulae For $x = 2\pi a/\lambda \ll 1$

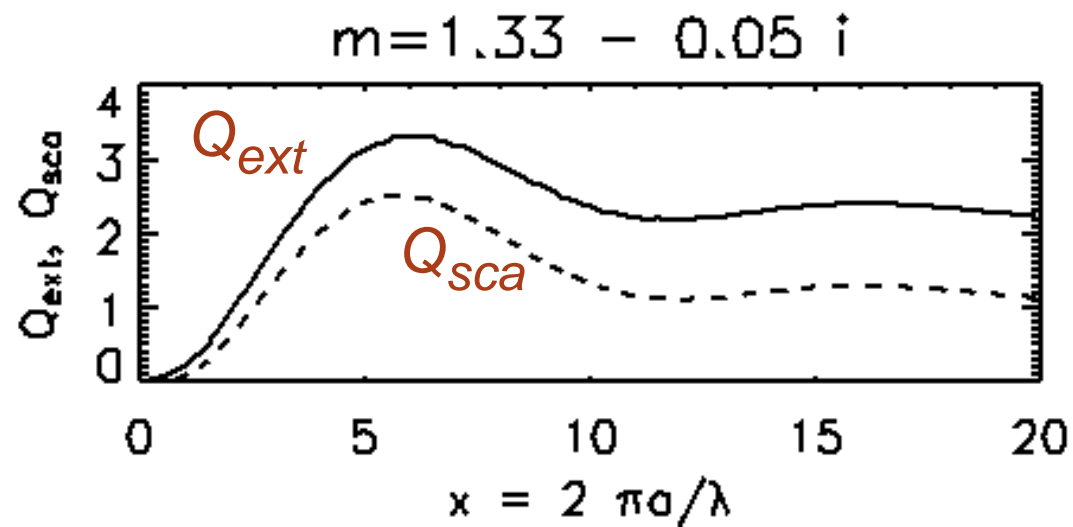
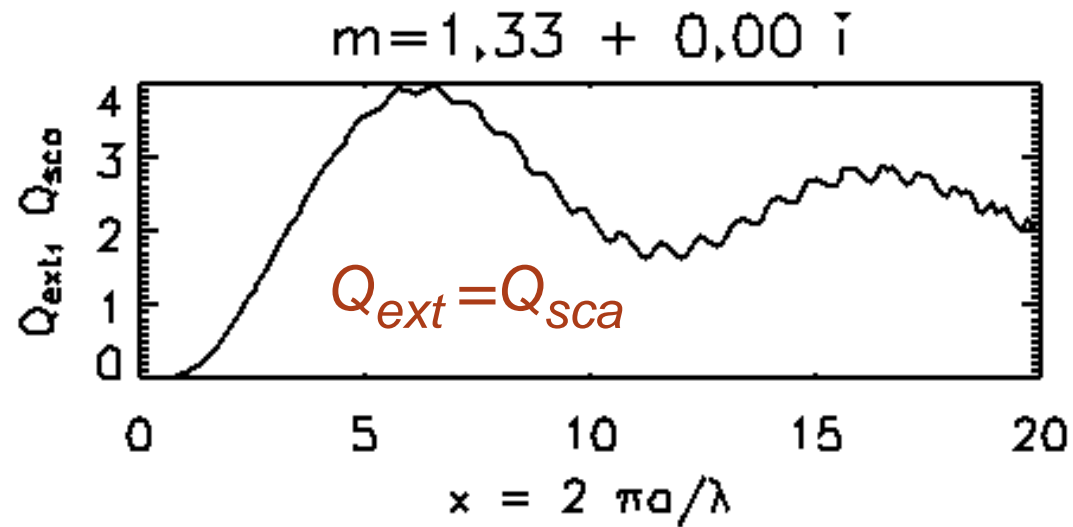
$$Q_{abs} = -4x \operatorname{Im}\left(\frac{m^2 - 1}{m^2 + 2}\right) \propto \lambda^{-1}$$

$$Q_{sca} = \frac{8}{3}x^4 \operatorname{Re}\left\{\left(\frac{m^2 - 1}{m^2 + 2}\right)^2\right\} \propto \lambda^{-4}$$

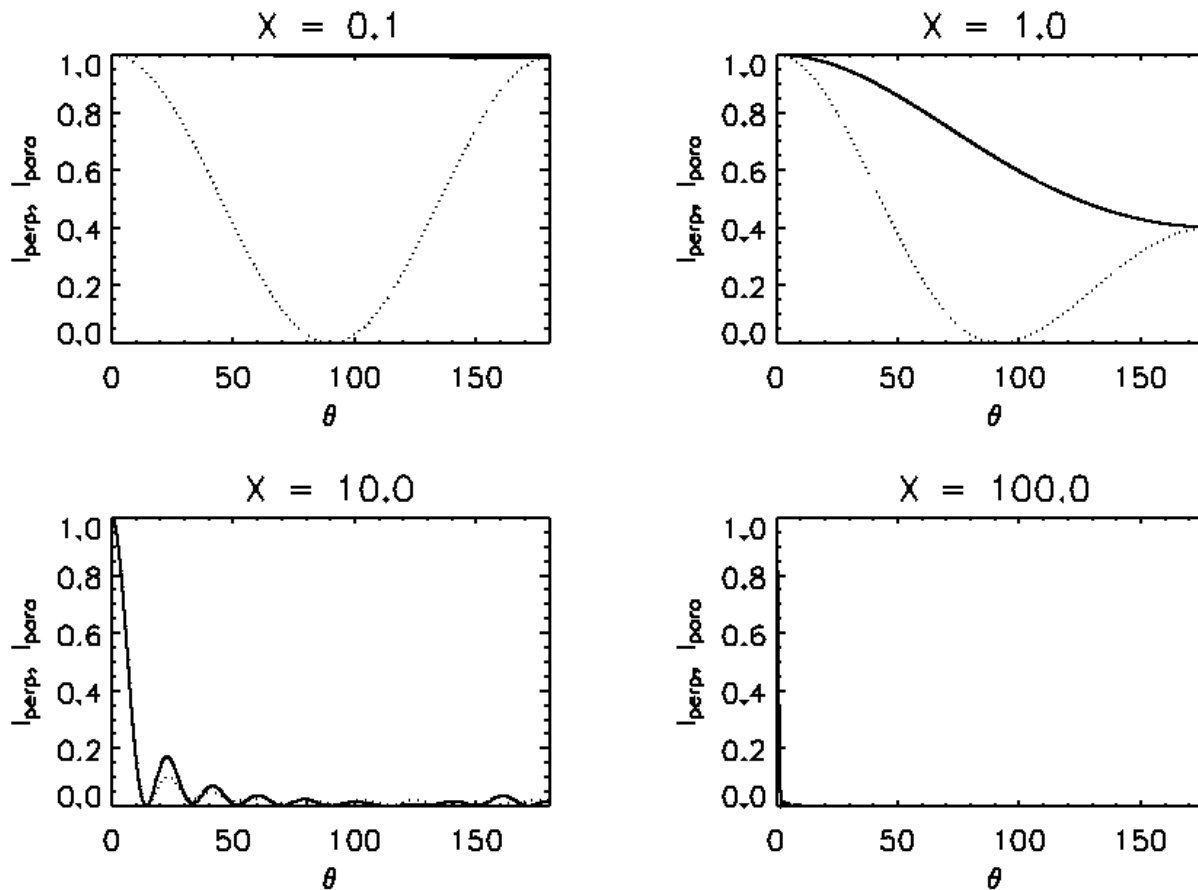
In this long wavelength or Rayleigh limit, the absorption cross section depends only on the mass of the grain:

$$\sigma_{abs} = Q_{abs} \pi a^2 \propto a^3 \propto m_{dust}$$

Pure vs. Dirty Dielectrics



Scattering Angular Distributions ($m = 1.33$)



- The angular distribution becomes highly peaked in the forward direction for $x \gg 1$.
- Note the difference between the \mathbf{E} perpendicular (dotted curve) and parallel (solid curve) to the scattering plane.

Dust to Gas Ratio Following Purcell (1969)

Purcell (ApJ 158, 433m 1969) used the Kramers-Kronig relations to relate the integrated extinction coefficient to the (constant) index m and the dust column N_d :

$$\int_0^\infty Q_{ext} d\lambda = 4\pi^2 a \left(\frac{m^2 - 1}{m^2 + 2} \right)$$

$$\tau_{ext} = Q_{ext} \pi a^2 N_d, \quad N_d = \rho_d L / m_d$$

$$\int_0^\infty \tau_{ext} d\lambda = 4\pi^3 a^3 N_d \left(\frac{m^2 - 1}{m^2 + 2} \right)$$

NB: This provides only a lower limit on grain volume for other shapes; spherical grains have been assumed here.

Reintroduce the extinction A_λ

$$A_\lambda = 1.086 \tau_\lambda = 1.086 Q_{ext} \pi a^2 N_d$$

to derive a relation between it and the dust column.

Gas to Dust Ratio Following Purcell

This Krarners-Kronig result involves integrated quantities

$$\frac{\int_0^\infty A_\lambda d\lambda}{L} = 3\pi^2 1.086 \left(\frac{m^2 - 1}{m^2 + 2} \right) n_d V_{gr}, \quad V_{gr} = \frac{4\pi}{3} a^3$$
$$n_d V_{gr} = \frac{n_d m_{gr}}{m_{gr} / V_{gr}} = \frac{\rho_d}{\rho_{gr}} = \frac{\text{spatial average dust density}}{\text{density of a grain}}$$

where m_g is the mass of an individual grain and L is the distance along the line of sight.

- The integrated extinction A_λ measures the mean dust density.
- Dividing the integral by $\rho = mn_H = mN_H/L$, gives the mean dust to gas mass ratio ($m = 1.35m_H =$ gas particle mass)

Estimate of the Gas to Dust Ratio

$$\frac{1}{L} \int_0^{\infty} A_{\lambda} d\lambda = 3\pi^2 1.086 \left(\frac{m^2 - 1}{m^2 + 2} \right) \frac{\rho_{dust}}{\rho_{gas}} \frac{(N_H/L)m}{\rho_{gr}}$$

$$\int_0^{\infty} A_{\lambda} d\lambda = 3\pi^2 1.086 \left(\frac{m^2 - 1}{m^2 + 2} \right) \frac{m}{\rho_{gr}} N_H$$

or

$$\frac{\rho_{dust}}{\rho_{gas}} = \frac{\int_0^{\infty} A_{\lambda} d\lambda}{N_H} \left[3\pi^2 1.086 \left(\frac{m^2 - 1}{m^2 + 2} \right) \frac{m}{\rho_{gr}} \right]^{-1}$$

We assume silicate grains with a real index = 1.5 and internal density $\rho_{gr} = 2.5 \text{ g cm}^{-3}$ and use the empirical extinction law, $A(V) \approx N_H / 1.8 \times 10^{21} \text{ mag cm}^{-2}$ and obtain

$$\frac{\rho_{dust}}{\rho_{gas}} = 19.2 \frac{\int_0^{\infty} A_{\lambda} d\lambda}{N_H}$$

If we use the approximation $A \sim \lambda^{-1.5}$ for $\lambda > 0.25 \mu\text{m}$, the dust to gas ratio is ~ 0.004 , and if we guess that the UV contributes another 50-100%, we would get a ratio of $\sim 0.06-0.08$. Tielens would calculate for this internal dust density 0.006.

Implications of the Dust to Gas ratio

$$\frac{\rho_{dust}}{\rho_{gas}} \approx 0.006$$

- A significant fraction of O heavy elements are in interstellar dust grains.
- Since the mass fraction of heavy elements is $Z = 0.016$ (solar), $\sim 40\%$ are in grains, and an even larger fraction when abundant volatile elements are excluded
- Typical grain models include
 - Silicates: Mg, Si, Fe, O (20%) in $(\text{Mg,Fe})_2\text{SiO}_4$ etc.
 - Carbonaceous material (graphite & organics): C (60%)
 - Some SiC

The presence of C and Si are supported by dust spectral features, as discussed in the next lecture

Sample Interstellar Dust Composition

Element	Abundance	A	M/M_H
C	0.6 \times 2.5×10^{-4}	12	0.0018
Mg	3.4×10^{-5}	24	0.0008
Fe	2.8×10^{-5}	56	0.0016
Si	3.2×10^{-5}	28	0.0009
O	0.2 \times 4.6×10^{-4}	16	0.0014
Total			0.0066

Summary of Grain Properties

- The shape of the interstellar extinction curve contains information about the size and chemical composition of interstellar dust grains.
- The relatively smooth variation of the extinction with wavelength from 0.1 to 3 μm indicates that a wide distribution of grain sizes are involved.
- A substantial fraction of the heavy elements are bound up in dust.
- The 220 nm bump and the 9.7 & 18 μm features indicate that dust contains C and Si, respectively,