$$D_A \equiv \frac{S}{\theta}$$

$$\theta(t_e) = \frac{S}{\tilde{\omega} a(t_e)}$$

$$\theta(z) = \frac{S(1+z)}{\tilde{\omega}(z)}$$

$$\frac{a(t_e)}{a(\mathrm{now})} = \frac{1}{1+z}$$

$$\tilde{\omega}(z) = \frac{2c}{H_0}\left(\frac{1}{1+z}\right)\left(\frac{1}{\Omega_M^2}\right)\left[\Omega_M z - (2-\Omega_M)\left(\sqrt{1+\Omega_M z}-1\right)\right]$$

$$\theta(z) = \frac{S(1+z)}{\frac{2c}{H_0}\left(\frac{1}{1+z}\right)\left(\frac{1}{\Omega_M^2}\right)\left[\Omega_M z - (2-\Omega_M)\left(\sqrt{1+\Omega_M z}-1\right)\right]}$$

$$= \frac{(SH_0/2c)(1+z)^2\,\Omega_M^2}{\left[\Omega_M z - (2-\Omega_M)\left(\sqrt{1+\Omega_M z}-1\right)\right]}$$