Equiv width of Na I 5890 Å line

$$W = 0.76 \text{ Å}$$

$$\rightarrow \log\left(\frac{W}{5890\text{ Å}}\right) = -3.89$$

From curve of growth,

$$\log\left(Nf\left(\frac{\lambda}{5000\text{ Å}}\right)\right) = 14.82$$

We know

$$\lambda = 5890 \text{ Å}$$
$$f = 0.645$$

Solve for N_a, # atoms in absorbing state per cm²

$$N_a = \frac{10^{14.82}}{(0.645)(5890/5000)} = 8.74 \times 10^{14} \frac{\text{atoms}}{\text{cm}^2}$$

Now, what fraction of neutral sodium atoms are in the ground state? Use Boltzmann equation

Let a = ground state
b = excited state

We know the wavelength corresponding to this transition:

$$\lambda = 5890 \text{ Å}$$
$$\rightarrow \Delta E = hc/\lambda = 3.37 \times 10^{-19} \text{ J} = 2.105 \text{ eV}$$

$$\frac{N_b}{N_a} = \left(\frac{g_b}{g_a}\right) e^{-(\Delta E)/kT} \rightarrow$$

p2

We don't know g_a or g_b (yet), but the exponential term is

$$e^{-(3.37\times10^{-19}\,J)/(1.38\times10^{-23}\,J/K)(6000\,K)}$$

$$= 0.017$$

So most atoms are probably in the ground state. After a little digging, one can find for sodium

ground state: outer electron in $3s^1$

$\rightarrow l=0,\ m=0,\ s=\pm\frac{1}{2}$ $\quad g_a=2$

first excited state: outer electron in $3p^1$

$\rightarrow l=1,\ m=-1,0,1\ \ s=\pm 1/2$ $\quad g_b=6$

So

$$\frac{N_b}{N_a} = \left(\frac{6}{2}\right)(0.017) \approx 0.05$$

Yes, most sodium atoms are probably in the ground state; to an order of magnitude, we'll estimate

$>$ ground state includes 80% of all neutral sodium

Now, what about the ratio of ionized to neutral? The Saha eqn states

$$\frac{N_{II}}{N_I} = \frac{2kT}{P_e}\frac{Z_{II}}{Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi/kT}$$

where $Z_I = 2.4$
$Z_{II} = 1.0$
$\chi = 5.1\ eV = 8.17 \times 10^{-19}\ J$
$P_e \cong 200\ dyne/cm^2 = 20\ N/m^2$
$T \cong 6000\ K$

Putting this all together,

$$\frac{N_{II}}{N_I} = \left(8.28 \times 10^{-23}\right)\left(\frac{1.0}{2.4}\right)\left(\frac{4.739\times10^{-49}}{4.390\times10^{-67}}\right)^{3/2} e^{-\left(\frac{8.17\times10^{-19}J}{8.28\times10^{-20}}\right)}$$

$$\left(8.28\times10^{-21}\right)\left(\frac{1}{2.4}\right)\left(1.12\times10^{27}\right)\left(5.19\times10^{-5}\right)$$

$$\cong 200 \quad \text{ionized for each neutral}$$

Note that your text chooses $P = 10\ dyne/cm^2$, $T = 5800\ K$, which yields a very different ratio of 2430 ionized per neutral

Continuing with my values for P_e and T, we can figure out the total number of sodium atoms:

$$N(tot) = N_a * \frac{N(\text{excited})}{N(\text{ground})} * \frac{N(\text{ionized + neutral})}{N(\text{neutral})}$$

$$= \left(8.74\times10^{14}\ \frac{\text{atoms}}{cm^2}\right)\left(\frac{1}{0.80}\right)\left(\frac{201}{1}\right)$$

$$\cong \underline{\underline{2.2\times10^{17}\ \text{atoms}/cm^2}}$$

(compare to 2.4×10^{18} atom/cm^2 in book, due mostly to different value of P_e)

Now, we are told the column density of hydrogen is

$$N_H = 6.6 \times 10^{23} \text{ atoms}/\text{cm}^2$$

So the abundance of sodium relative to hydrogen is

$$\frac{N(\text{sodium})}{N(H)} = \frac{2.2 \times 10^{17} \text{ atoms}/\text{cm}^2}{6.6 \times 10^{23} \text{ atoms}/\text{cm}^2}$$

$$= 3.3 \times 10^{-7}$$

Astronomers often quote abundances like so:

$$12 + \log_{10}\left(\frac{\text{mass density of X}}{\text{mass density of H}}\right)$$

which for us would give

$$\log(\text{sodium}) \sim 12 + \log_{10}\left(\frac{2.2 \times 10^{17} \text{ atoms}/\text{cm}^2 \cdot 23 \text{ amu}/\text{atom}}{6.6 \times 10^{23} \text{ atoms}/\text{cm}^2 \cdot 1 \text{ amu}/\text{atom}}\right)$$

$$\sim 6.9$$

The actual solar photospheric abundance is $\simeq 6.4$, so we didn't do a terrible job.