EASTER VACATION WORK AT PORT ERIN.

THE Easter vacation party at the Port Erin Biological Station has this year been larger than ever before, and has carried out a longer programme of work—both in the laboratory and on the seashore. During the last few weeks (March and April) the number of researchers and senior students enrolled in the books of the station has reached the record total of eighty-five, including half a dozen professors and a dozen university lecturers and demonstrators, while nearly half of the total number were post-graduate researchers. Altogether twelve universities or university colleges have been represented. Practically all the senior students and post-graduate workers of the botanical and zoological departments of the University of Liverpool, under Profs. Harvey Gibson and Herdman, migrated to the Port Erin laboratory for the vacation. Prof. Cole brought a considerable contingent from the University College of Reading, and Dr. Stuart Thomson a number from Manchester; Mr. Holden came with some students from University College, Nottingham, and smaller groups came from Birmingham, Cambridge, Oxford, Bristol, Bangor, Cardiff, London, and Melbourne. In addition to the laboratory work of the students and their collecting expeditions on the seashore, the activities of the biological station at this time of year are threefold: first, the flat-fish hatching (seen at its best during March and April); secondly, the plankton investigation going on at sea from the s.y. *Runa*; and thirdly, the special investigations of the post-graduate researchers.

The spawning of the mature plaice in the open-air fish-ponds started at the beginning of February this year, at least a fortnight earlier than usual, and it is by no means finished yet. Already more than eight millions of eggs have been skimmed from the ponds, and about seven millions of young fish have been set free in the sea round the south end of the Isle of Man.

Work at sea was much hampered by bad weather during the earlier part of the time, and it was sometimes difficult to get the periodic plankton hauls taken. This is now the eighth year of Prof. Herdman's scheme of intensive study of the nature and distribution of the plankton, of which it is hoped to complete ten years' statistics before winding up the investigation. Up to the present the phytoplankton this spring has been characterised by the prevalence of Coscinodiscus.

In addition to the collecting and recording of rare species, both of animals and sea-weeds, which has gone on very much as in former years, there has been a large amount of special investigation both at sea and in the laboratory on the part of those who are engaged in the preparation of L.M.B.C. Memoirs, and also of others who are at various researches. For example, Mr. R. D. Laurie has been making observations on the movements of Amphidinium in the sand, Mr. S. T. Burfield has been working at Sagitta, Miss Gleave at Archidoris, Mr. H. G. Jackson on Decapod larvæ in the plankton, and Prof. B. Moore and Mr. E. Whitley on the nutrition of marine animals and the variations in the alkalinity of the sea-water. The memoir on Echinoderm larvæ which Mr. Chadwick has been engaged on for some years is now in the printer's hands, and will be published at an early date. The pressure on the laboratory accommodation has been very great during this vacation, and the need of further extension of the building is urgent.

W. A. H.

RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS. *

I.

Historical.

INVESTIGATIONS into the nature of the stars must necessarily be very largely based upon the average characteristics of groups of stars selected in various ways—as by brightness, proper motion, and the like. The publication within the last few years of a great wealth of accumulated observational material makes the compilation of such data an easy process; but some methods of grouping appear to bring out much more definite and interesting relations than others, and, of all the principles of division, that which separates the stars according to their spectral types has revealed the most remarkable differences, and those which most stimulate attempts at a theoretical explanation.

In the present discussion, I shall attempt to review very rapidly the principal results reached by other investigators, and shall then ask your indulgence for an account of certain researches in which I have been engaged during the past few years.

Thanks to the possibility of obtaining with the objective prism photographs of the spectra of hundreds of stars on a single plate, the number of stars the spectra of which have been observed and classified now exceeds one hundred thousand, and probably as many more are within the reach of existing instruments. The vast majority of these spectra show only dark lines, indicating that absorption in the outer and least dense layers of the stellar atmospheres is the main cause of their production. Even if we could not identify a single line as arising from some known constituent of these atmospheres, we could nevertheless draw from a study of the spectra, considered merely as line-patterns, a conclusion of fundamental importance.

The spectra of the stars show remarkably few radical differences in type. More than 99 per cent. of them fall into one or other of the six great groups which, during the classic work of the Harvard College Observatory, were recognised as of fundamental importance, and received as designations, by the process of "survival of the fittest," the rather arbitrary series of letters B, A, F, G, K, and M. That there should be so few types is noteworthy; but much more remarkable is the fact that they form a continuous series. Every degree of gradation, for example, between the typical spectra denoted by B and A may be found in different stars, and the same is true to the end of the series, a fact recognised in the familiar decimal classification, in which B5, for example, denotes a spectrum half-way between the typical examples of B and A. This series is not merely continuous; it is *linear*. There exist indeed slight differences between the spectra of different stars of the same spectral class, such as AO; but these relate to minor details, which usually require a trained eye for their detection, while the difference between successive classes, such as A and F, are conspicuous to the novice. Almost all the stars of the small outstanding minority fall into three other classes, denoted by the letters O, N, and R. Of these O undoubtedly precedes B at the head of the series, while R and N, which grade into one another, come probably at its other end, though in this case the transition stages, if they exist, are not yet clearly worked out.

From these facts it may be concluded that the prin-

* An address delivered before a joint meeting of the Astronomical and Astrophysical Society of America and Section A of the American Association for the Advancement of Science, at Atlanta, Georgia, December 30, 1913, with a few additions, by Prof. H. N. Russell.

cipal differences in stellar spectra, however they may originate, arise in the main from variations in a single physical condition in the stellar atmospheres. This follows at once from the linearity of the series. If the spectra depended, to a comparable degree, on two independently variable conditions, we should expect that we would be obliged to represent their relations, not by points on a line, but by points scattered over an area. The minor differences which are usually described as "peculiarities" may well represent the effects of other physical conditions than the controlling one.

The first great problem of stellar spectroscopy is the identification of this predominant cause of the spectral differences. The hypothesis which suggested itself immediately upon the first studies of stellar spectra was that the differences arose from variations in the chemical composition of the stars. Our knowledge of this composition is now very extensive. Almost every line in the spectra of all the principal classes can be produced in the laboratory, and the evidence so secured regarding the uniformity of nature is probably the most impressive in existence. The lines of certain elements are indeed characteristic of particular spectral classes; those of helium, for instance, appear only in Class B, and form its most distinctive characteristic. But negative conclusions are proverbially unsafe. The integrated spectrum of the sun shows no evidence whatever of helium, but in that of the chromosphere it is exceedingly conspicuous. Were it not for the fact that we are near this one star of Class G, and can study it in detail, we might have erroneously concluded that helium was confined to the "helium stars." There are other cogent arguments against this hypothesis. For example, the members of a star-cluster, which are all moving together, and presumably have a common origin, and even the physically connected components of many double stars, may have spectra of very different types, and it is very hard to see how, in such a case, all the helium and most of the hydrogen could have collected in one star, and practically all the metals in the other. A further argument—and to the speaker a very convincing one—is that it is almost unbelievable that differences of chemical composition should reduce to a function of a single variable, and give rise to the observed linear series of spectral types.

I need not detain you with the recital of the steps by which astrophysicists have become generally convinced that the main cause of the differences of the spectral classes is difference of temperature of the stellar atmospheres. There is time only to review some of the most important evidence which, converging from several quarters, affords apparently a secure basis for this belief.

The first argument is based upon the behaviour of the spectral lines themselves. To appreciate its full force, one must familiarise himself with a multitude of details. A typica[1] instance is that of the heavy bands in the region of longer wave-length, which are the most characteristic feature of spectra of Class M, appear faintly in Class K5, and are absent in Class K and all those higher in the series. Fowler has shown[1] that these bands are perfectly reproduced in the spectrum of the outer flame of an electric arc charged with some compound of titanium, while the spectrum of the core of the arc, though showing conspicuously the bright lines of titanium, does not contain the bands. Here we are evidently dealing with some compound—perhaps titanium oxide—the vapour of which is present in the relatively cool flame of the arc, and emits a spectrum of the banded type, characteristic of compounds, while in the hotter core it is dissociated, and only the lines of the metal are seen. There seems then to be no escape from the conclusion that the atmospheres of stars of Class M are cool enough to permit the existence of this compound, and hence cooler than the core of the arc, and that the temperature of its dissociation is approached in Class K5, and surpassed in Class K. In general, those metallic lines which are relatively strong in the spectra produced in the oxyhydrogen flame or the electric furnace are also strong in spectra of Classes M and K; the lines most prominent in Class G are the typical arc lines; and the relatively few metallic lines which persist into Classes A and B are those which appear exclusively, or with greatly enhanced intensity, in the spark spectra of the laboratory.

The second line of evidence is afforded by the distribution of intensity in the continuous background of the spectra, the differences of which from type to type are obvious to the eye as differences in the colour of the stars. This characteristic is fortunately capable of accurate measurement. For the brighter stars, spectro-photometric comparisons may be made with a terrestrial light-source the energy curve of which is known, as has been done visually by Wilsing and Scheiner,[2] and photographically by Rosenberg.[3] Much fainter stars may be reached by the comparison of their brightness as measured visually (or on isochromatic plates with a suitable colour-screen), and photographically on ordinary plates. The "colour-index" so obtained, which expresses, in stellar magnitudes, the relative photographic brightness of stars of equal visual brightness, is found to be very intimately related to the spectral type, the differences within each spectral class being scarcely greater than the errors of observation. The results of King,[4] Parkhurst,[5] and Schwarzschild,[6] working with different instruments and on stars of very different brightness, are in excellent agreement, as is shown in Table I. The near approach to equality among the differences in colour-index from class to class is very remarkable, when it is considered that these types were picked out somewhat arbitrarily according to the general appearance of the photographic spectra. The judgment of the Harvard observers in selecting the really important points of difference was evidently very good.

TABLE I.

Spectrum	Colour-index King	Parkhurst	Schwarzschild	Temperature
Bo	−0·32			20,000
B5	−0·17	−0·21	−0·20	14,000
Ao	0·00	0·00	0·00	11,000
A5	0·19	0·23	0·20	9,000
Fo	0·30	0·43	0·40	7,500
F5	0·42	0·65	0·60	6,000
Go	0·72	0·86	0·84	5,000
G5	0·98	1·07	1·10	4,500
Ko	1·10	1·30	1·35	4,200
K5	1·62	1·51	1·80	3,200
M	1·62	1·68		3,100
N		2·5		2,300

If the spectral sensitiveness of the plates used in such investigations has been determined (as Parkhurst has done) it is possible to calculate the temperature at which a black-body would emit light of the same colour as that observed; and similar calculations can be made, with greater accuracy, from the spectrophotometric data. The last column of Table I. gives the effective temperatures thus derived (based mainly on the work of Wilsing and Scheiner). The absolute

[1] Proc. Roy. Soc., vol. lxxii., pp. 219–225, 1904.

[2] Potsdam Publications, vol. xix., part 1.

[3] *A.N.*, 4628, 1913.

[4] Harvard Annals, vol. lix., p. 179.

[5] *Astrophys. Jour.*, vol. xxxvi., p. 218, 1912.

[6] Göttingen Aktinometrie, Teil B, p. 19.

values of the temperatures here given may be considerably in error, especially at the top of the scale (in fact, Rosenberg's work indicates a much greater range), but there can be no doubt about the relative order.

Of a third independent confirmation of the temperature hypothesis, based on the determination of the surface brightness of the stars, I shall have occasion to speak later.

It should be expressly stated that the "temperatures" here spoken of are the effective "black-body" temperatures corresponding to the spectral distribution of the radiation. Unless the surfaces of the stars possess decided selective emissivity for certain wave-lengths, these effective temperatures should also indicate with tolerable accuracy the energy-density of the flux of radiation which escapes from them. This tells us little about the temperature of the deeper regions; but it must be the main, if not the only, factor in determining the temperature of those outer and nearly transparent layers of the atmospheres in which the characteristic line absorption takes place. If we further assume, in accordance with Abbot's studies of the solar atmosphere,[7] that the absorption is nearly complete in so small a thickness of the atmosphere that wide variations in its depth and density would modify its total absorption but little, it becomes easy to see how the influence of its temperature (which presumably determines the relative strength of absorption in different lines) may predominate so greatly over all that of all other factors in determining the spectral type.

We may now review rapidly some of the relations which have been brought to light between other characteristics of the stars and their spectral types. First, as regards the relative numbers of stars of the different classes, we have in Table II. some results of counts made at Harvard.[8]

TABLE II.

Spectrum	O	B	A	F	G	K	M	N
No. above 3.25m.	3	52	32	16	20	35	21	0
" 6.25m.	20	696	1885	720	609	1719	457	8
Percentage in Galactic region	100	82	66	57	58	56	54	87

Classes A and K make up more than half of all the stars brighter than 6·25 m—that is, of the stars visible to the naked eye. The remaining stars are divided fairly evenly among the other four principal classes, while only one star in 300 is of Class O, and only one in 800 of Class N. The relative proportions of the different classes are, however, different in different parts of the heavens, as is indicated by the last line of the table, which give the percentage of stars of each class which lie in a belt covering one-half of the celestial sphere, and extending for 30° each side of the Milky Way. All the stars of Class O are close to the central line of the Galaxy (except for a few in the Magellanic Clouds). The stars of Class B are very strongly concentrated in the galactic region; those of Class A are considerably so; those of the following classes very little, except in the case of Class N (for which the tabular percentage is derived, not from the eight brightest stars of this class alone, but from a much larger number of fainter ones).[9]

The relative proportions of the different classes vary also with the apparent brightness of the stars. Among the stars brighter than 3·25m., as the table shows, Class B has more representatives than any other; but the percentage of this type steadily diminishes as we pass to fainter stars. The percentage of stars of Class A at first increases with diminishing visual brightness; but there is good reason to believe that, at least in regions remote from the Galaxy, the relative proportion of these too falls off rapidly in the neighbourhood of the ninth magnitude[10]; and Fath's work on the integrated spectrum of the Milky Way[11] shows that, even there, the bulk of the very faint stars which form the galactic clouds must be of Secchi's second type (F, G, or K).

Counts of the stars down to any given magnitude may, however, be very misleading unless we bear in mind the enormous preference which this method of observation gives to the stars of great actual luminosity, which can be seen afar off, and hence are being sought in a much greater volume of space than those of small luminosity. A difference of but five magnitudes in the real brightness of two groups of stars gives the brighter kind (if both are uniformly distributed in space) a thousand-fold better chance of getting into our catalogues; and this example understates the actual conditions in some cases. Mere counts of stars need therefore to be supplemented by such knowledge as we can obtain concerning their distances.

Much information can be obtained from the average proper-motions of the stars of the various classes, and still more by deriving their average parallaxes from the mean parallactic drift due to the motion of the solar system in space. Studies of this character have been made by several investigators of the first rank. Their results, which are summarised in Table III., show certain apparent discrepancies, which, however, arise principally from differences in the methods according to which the various workers have selected the groups of stars for investigation.

TABLE III.

Spectrum	Mean centennial proper-motion: Kapteyn	Boss	% rejected	Mean parallax: Kapteyn	Boss	Campbell	% rej.
	"	"		"	"	"	
O		1·6	0		0·004		
B	2·6	2·4	0	0·007	0·007	0·006	0
A	5·8	4·6	3	0·010	0·010	0·016	3
F	14·5	7·7	28		0·012	0·035	3
G	27·0	5·2	20	0·022	0·008	0·022	8
K	13·0	5·7	6		0·010	0·015	9
M	5·9	5·0	6	0·011	0·008	0·011	3
N		3·2		0·0007			

Kapteyn's data[12] represent the mean proper motions and parallaxes of all the stars of the fifth magnitude of each class, except for Class N, in which, to get enough stars, it was necessary to include faint objects, so that the average magnitude is here 8·3. His results show a conspicuous maximum of average proper-motion and parallax for Class G, with a rapid fall on both sides of it. The stars of Class N would have to be brought about five times nearer to appear as bright as the others, but even then they would have the smallest mean parallax of all.

Boss,[13] in his investigation of the solar motion, had at his disposal very accurate proper-motions of all the stars down to 5·7m., and about half as many more between this and the seventh magnitude. The average magnitude of his stars is therefore nearly the same as that of Kapteyn's. But, for very good reasons, he excluded from his main solution all stars with proper-motions exceeding 20″ per century. The percentage of stars thus excluded (which differs greatly from class to class) is given in the fourth column of Table III. It is natural that this often drastic rejection of the large proper-motions, and hence in general

7 Abbot, "The Sun," p. 252, 1911.
8 Harvard Annals, vol. lxiv., p. 134.
9 Harvard Annals, vol. lxvi., p. 213.
10 *Astronomical Journal*, vol. xxvi., p. 153, 1910.
11 *Astrophys. Journal*, vol. xxxvi., pp. 362–367, 1912.
12 *Astrophys. Journal*, vol. xxx., p. 295 : vol. xxxii., p. 91, 1909–10.
13 *Astronomical Journal*, vol. xxvi., pp. 187–201, 1911. The mean proper-motions of the few stars of Classes O and N which appear in Boss's Catalogue have been added by the writer.

of the nearer stars, should greatly diminish his mean values. Among the classes in which the mean proper-motion is small, the percentage of exclusion is also small, and the results are but little modified. But it is noteworthy that the exclusion of 6 per cent. of the stars of Class K has reduced the mean proper-motion in a greater ratio than that of 28 per cent. of those of Class F, and also that the removal of one-fifth of the stars of Class G decreases the mean for the remainder to less than one-fifth of its initial value. It appears from these results that a large majority of the stars of Classes F, G, and K have nearly, if not quite, as small parallaxes and proper-motions as those of Classes A and M, though they are not quite so remote as the stars of Class B. The large mean values obtained for all the stars of these classes are due to the presence of a relatively small proportion of near and apparently rapidly moving stars, of which the percentage decreases, but the mean proper-motion and parallax increase, from F to K.

Campbell's results [14] are derived from a comparison of the radial velocities and proper-motions of nearly 1200 stars, mostly brighter than the fifth magnitude, and averaging about a magnitude brighter than Boss's stars, which would lead us to expect that their mean parallaxes should be 40 or 50 per cent. greater. In his work, "a few stars having proper-motions abnormally large for their classes were omitted in accordance with definitely set limits" (which unfortunately are not described more specifically). The approximate percentage of exclusion is given in the last column of the table. It appears on inspection that the differences between Campbell's and Boss's results for stars of Classes A, K, and M arise mainly from the greater brightness of Campbell's stars; those for Classes F and G are due mainly to the different percentages of exclusion, and that the only significant difference is that Campbell's B stars, though averaging much brighter to the eye than Boss's, have a slightly smaller mean parallax, and therefore must be, on the average, of greater real brightness.

Closely allied with these investigations is the determination of the mean peculiar velocity of the stars of each spectral class. The results of Boss and Campbell, reached almost simultaneously, and from quite independent data—proper-motions in one case and radial velocities in the other—are in extraordinary agreement. The values found for the average component of motion in any arbitrary direction are (in kilometres per second):—

Spectrum	B	A	F	G	K	M
Campbell	6·5	10·5	14·4	15·9	16·8	17·1
Boss	6·3	10·2	16·2	18·6	15·1	17·1

The rapid increase of the mean velocity from B to F is very remarkable. The slow further gain from F to M would attract little attention if it were not in the same direction.

It should here be added that the phenomenon known as preferential motion, or "star-streaming"—the excess of the average peculiar velocity of the stars in a certain direction above those in the perpendicular directions—is almost absent in Class B, very conspicuous in Class A, and somewhat less so in the following classes, being partially concealed by the greater average magnitude of the velocities.

Another notable difference between the various spectral classes may be found in the number of binary stars, both visual and spectroscopic, among them. We may distinguish two classes of visual double stars; binary stars for which orbits have been computed (with periods rarely exceeding two centuries), and physical pairs, the real connection of which is proved by common proper-motion, but the relative motions of which are slow, and periods long—probably often thousands of years. The counts of the two classes here given are from a list prepared in the course of my work, and include all stars for which the necessary data could be obtained, including many stars for which unpublished observations of spectra have been generously furnished me from Harvard. For the spectroscopic binaries, Campbell's counts have been taken from his catalogue of 1910.[15] They include all the systems the periods of which were then known, and are divided into two groups, one including all the periods of which are less than ten days, and also all those the periods of which, though not exactly known, are described as short; the other all the known periods exceeding ten days, and those which, though not precisely determined, are known to be long.

TABLE IV.

Spectrum	Visual binaries	Physical pairs	Spectroscopic binaries: Short period	Spectroscopic binaries: Long period
B	0	52	33	15
A	14	152	15	14
F	33	115	11	9
G	24	74	8	14
K	12	62	0	13
M	0	11	0	2

It appears that, in Campbell's picturesque phrase, visual double stars of relatively short period "abhor" Classes B and M, the greatest number being of Class F, with G a good second. Among the physical pairs, of long period, the most favoured class is A. Class B is abundantly represented, and Class M very sparingly.

The percentage of stars which are found to be spectroscopic binaries is very probably greater among Classes B and A than lower down the list. As time goes on, indeed, more and more of the stars of these "later" types are found to be spectroscopically double, but of long period; but among these classes the detection of such systems, where the range of velocity is small, is much easier than among the stars of the first type, the lines of which are diffuse. In any case it is certain that short periods are almost confined to Classes B, A, and F, and are especially abundant in the first of these. The few short-period stars of Class G which appear in the table are all Cepheid variables, most of which were selected for observation on this account, and would not otherwise have got into the list.

Finally, we may note that, among variable stars, those of the eclipsing type, such as Algol or Beta Lyræ, are for the most part of Classes A and B, though there are a number of Classes F and G, and one at least of Class K; that the Cepheid variables are almost all of classes F and G, with a few A's and K's; and that almost all the irregular variables, and all the variables of long period, are of Classes M or N. Stars of Class M the spectra of which show bright hydrogen lines are without exception variable, and almost all the stars of Class N are also subject to changes in brightness.

(To be continued.)

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

CAMBRIDGE.—Dr. James Ward, professor of mental philosophy and logic, has been nominated to represent the University on the occasion of the celebration at Oxford on June 10 of the seventh centenary of the birth of Roger Bacon, and Dr. Sorley, Knightbridge professor of moral philosophy, to represent the Univer-

[14] Lick Observatory Bulletin, vol. vi., p. 134, 1911.

[15] Lick Observatory Bulletin, vol. vi., p. 38, 1910.

RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS.*

II.

Brightness and Spectral Class.

HAVING thus made a rapid survey of the general field, I shall now ask your attention in greater detail to certain relations which have been the more special objects of my study.

Let us begin with the relations between the spectra and the real brightness of the stars. These have been discussed by many investigators—notably by Kapteyn and Hertzsprung—and many of the facts which will be brought before you are not new; but the observational material here presented is, I believe, much more extensive than has hitherto been assembled. We can only determine the real brightness of a star when we know its distance; but the recent accumulation of direct measures of parallax, and the discovery of several moving clusters of stars the distances of which can be determined, put at our disposal far more extensive data than were available a few years ago.

Fig. 1 shows graphically the results derived from all the direct measures of parallax available in the spring of 1913 (when the diagram was constructed). The spectral class appears as the horizontal coordinate, while the vertical one is the absolute magnitude, according to Kapteyn's definition—that is, the visual magnitude which each star would appear to have if it should be brought up to a standard distance, corresponding to a parallax of 0·1″ (no account being taken of any possible absorption of light in space). The absolute magnitude, −5, at the top of the diagram, corresponds to a luminosity 7500 times that of the sun, the absolute magnitude of which is 4·7. The absolute magnitude 14, at the bottom, corresponds to 1/5000 of the sun's luminosity. The larger dots denote the stars for which the computed probable error of the parallax is less than 42 per cent. of the parallax itself, so that the probable error of the resulting absolute magnitude is less than ±1·0m. This is a fairly tolerant criterion for a "good parallax," and the small dots, representing the results derived from the poorer parallaxes, should scarcely be used as a basis for any argument. The solid black dots represent stars the parallaxes of which depend on the mean of two or more determinations; the open circles, those observed but once. In the latter case, only the results of those observers whose work appears to be nearly free from systematic error have been included, and in all cases the observed parallaxes have been corrected for the probable mean parallax of the comparison stars to which they were referred. The large open circles in the upper part of the diagram represent mean results for numerous bright stars of small proper-motion (about 120 altogether) the observed parallaxes of which scarcely exceed their probable errors. In this case the best thing to do is to take means of the observed parallaxes and magnitudes for suitable groups of stars, and then calculate the absolute magnitudes of the typical stars thus defined. These will not exactly

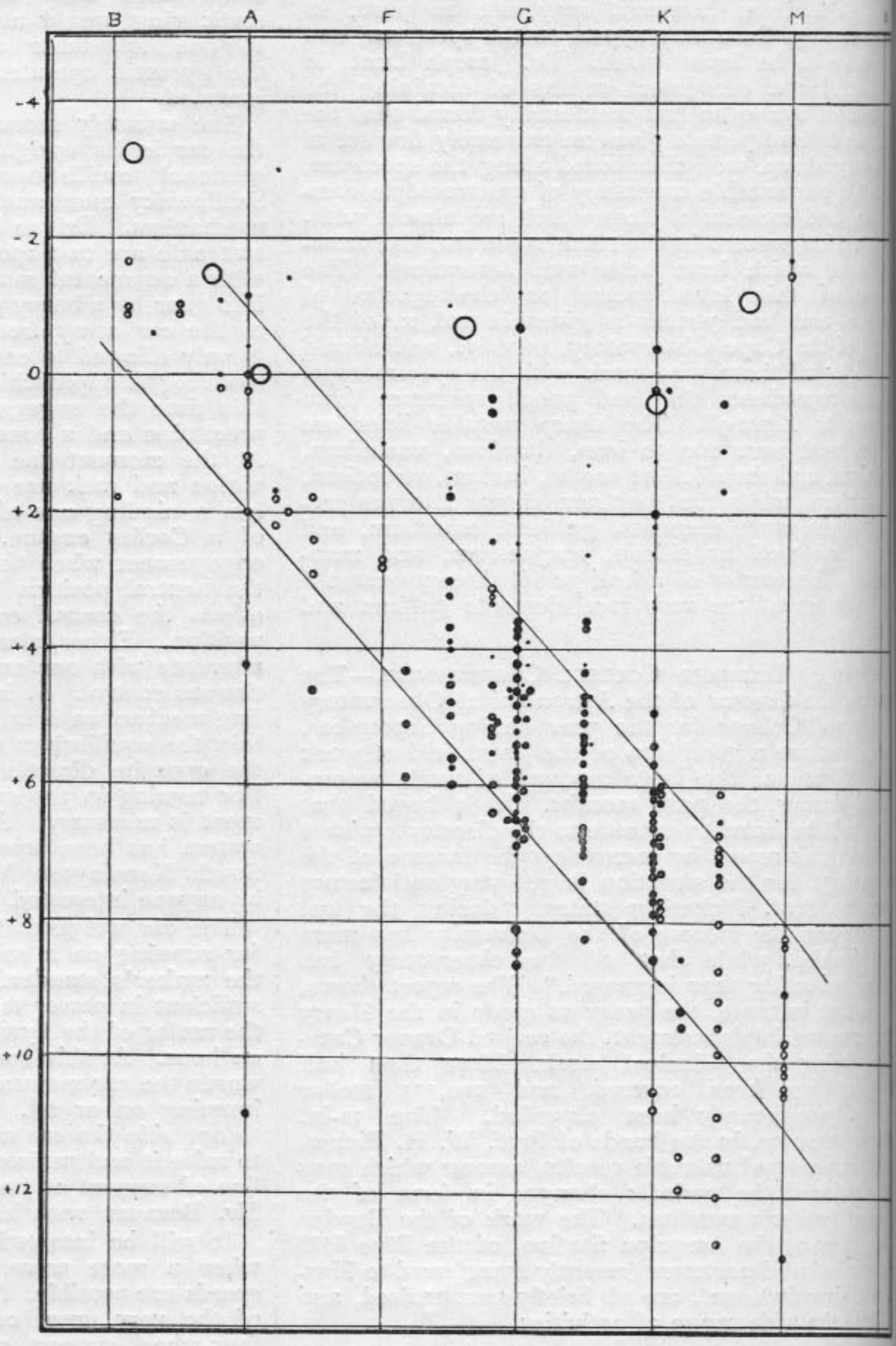


FIG. 1.

* An address delivered before a joint meeting of the Astronomical and Astrophysical Society of America and Section A of the American Association for the Advancement of Science, at Atlanta, Georgia, December 30, 1913, with a few additions, by Prof. H. N. Russell. Continued from p. 230.

correspond to the mean of the individual absolute magnitudes which we could obtain if we knew all the parallaxes exactly, but they are pretty certainly good enough for our purpose.

Upon studying Fig. 1 several things can be observed. (1) All the white stars, of Classes B and A, are bright, far exceeding the sun; and all the very faint stars—for example, those less than 1/50 as bright as the sun—are red, and of Classes K and M. We may make this statement more specific by saying, as Hertzsprung does,[16] that there is a certain limit of brightness for each spectral class, below which stars of this class are very rare, if they occur at all. Our diagram shows that this limit varies by rather more than two magnitudes from class to class. The single apparent exception is the faint double companion to 0, Eridani, concerning the parallax and brightness of which there can be no doubt, but the spectrum of which, though apparently of Class A, is rendered very difficult of observation by the proximity of its far brighter primary.

(2) On the other hand, there are many red stars of great brightness, such as Arcturus, Aldebaran, and Antares, and these are as bright, on the average, as the stars of Class A, though probably fainter than those of Class B. Direct measures of parallax are unsuited to furnish even an estimate of the upper limit of brightness to which these stars attain, but it is clear that some stars of all the principal classes must be very bright. The range of actual brightness among the stars of each spectral class therefore increases steadily with increasing redness.

(3) But it is further noteworthy that all the stars of Classes K5 and M which appear on our diagram are either very bright or very faint; there are none comparable with the sun in brightness. We must be very careful here not to be misled by the results of the methods of selection employed by observers of stellar parallax. They have for the most part observed either the stars which appear brightest to the naked eye, or stars of large proper-motion. In the first case, the method of selection gives an enormous preference to stars of great luminosity, and, in the second, to the nearest and most rapidly moving stars, without much regard to their actual brightness. It is not surprising, therefore, that the stars picked out in the first way (and represented by the large circles in Fig. 1) should be much brighter than those picked out by the second method (and represented by the smaller dots). But if we consider the lower half of the diagram alone, in which all the stars have been picked out for proper-motion, we find that there are no very faint stars of Class G, and no relatively bright ones of Class M. As these stars were selected for observation entirely without consideration of their spectra (most of which were then unknown) it seems clear that this difference at least is real, and that there is a real lack of red stars comparable in brightness with the sun, relatively to the number of those 100 times fainter.

The appearance of Fig. 1 therefore suggests the hypothesis that, if we could put on it some thousands of stars instead of the 300 now available, and plot their absolute magnitudes without uncertainty arising from observational error, we would find the points representing them clustered principally close to two lines, one descending sharply along the diagonal, from B to M, the other starting also at B, but running almost horizontally. The individual points, though thickest near the diagonal lines, would scatter above and below it to a vertical distance corresponding to at least two magnitudes, and similarly would be thickest near the horizontal line, but scatter above and below it to a distance which cannot so far be definitely specified, so that there would be two fairly broad bands in which most of the points lay. For Classes A and F these two zones would overlap, while their outliers would still intermingle in Class G, and probably even in Class K. There would, however, be left a triangular space between the two zones, at the right-hand edge of the diagram, where very few (if any) points appeared, and the lower left-hand corner would be still more nearly vacant.

We may express this hypothesis in another form by saying that there are two great classes of stars, one of great brightness (averaging, perhaps, a hundred times as bright as the sun), and varying very little in brightness from one class of spectrum to another; the other of smaller brightness, which falls off very rapidly with increasing redness. These two classes of stars were first noticed by Hertzsprung,[17] who has applied to them the excellent names of *giant* and *dwarf* stars. The two groups, on account of the considerable internal differences in each, are only distinctly separated among the stars of Class K or redder. In Class F they are partially, and in Class A thoroughly, intermingled, while the stars of Class B may be regarded equally well as belonging to either series.

In addition to the stars of directly measured parallax, represented in Fig. 1, we know with high accuracy the distances and real brightness of about 150 stars which are members of the four moving clusters the convergent points of which are known, namely, the Hyades, the Ursa Major group, the 61 Cygni group, and the large group in Scorpius, discovered independently by Kapteyn, Eddington, and Benjamin Boss, the motion of which appears to be almost entirely parallactic. The data for the stars of these four groups are plotted in Fig. 2, on the same system as in Fig. 1. The solid black dots denote the members of the Hyades; the open circles, those of the group in Scorpius; the crosses, the Ursa Major group; and the triangles, the 61 Cygni group. Our lists of the members of each group are probably very nearly complete down to a certain limiting (visual) magnitude, but fail at this point, owing to lack of knowledge regarding the proper motions of the fainter stars. The apparently abrupt termination of the Hyades near the absolute magnitude 7·0, and of the Scorpius group at 1·5, arises from this observational limitation.

The large circles and crosses in the upper part of Fig. 2 represent the absolute magnitudes calculated from the mean parallaxes and magnitudes of the groups of stars investigated by Kapteyn, Campbell, and Boss, concerning which data were given in Table III. The larger circles represent Boss's results, the smaller circles Kapteyn's, and the large crosses Campbell's.

It is evident that the conclusions previously drawn from Fig. 1 are completely corroborated by these new and independent data. Most of the members of these clusters are dwarf stars, and it deserves particular notice that the stars of different clusters, which are presumably of different origin, are similar in absolute magnitude. But there are also a few giant stars, epecially of Class K (among which are the well-known bright stars of this type in the Hyades); and most remarkable of all is Antares, which, though of Class M, shares the proper motion and radial velocity of the adjacent stars of Class B, and is the brightest star in the group, giving out about two thousand times the light of the sun.

16 *A. N.*, 4422, 1910.

17 *Zeitschrift für Wissenschaftliche Photographie*, vol. iii., p. 442, 1905.

It is also clear that the naked-eye stars, studied by Boss, Campbell, and Kapteyn, are, for the most part, giants. With this in mind, we are now in a position to explain more fully the differences between the results of these investigators.

All the stars of Class B are giants, and, so far as we may judge from the Scorpius cluster, they do not differ from one another very greatly in absolute brightness. It is therefore natural that the results of all three investigators are in this case fairly similar, though Campbell, in employing stars that averaged brighter to the eye than did the others, has evidently been working with stars that are really brighter. In Class A the giants and dwarfs differ so little, and are so thoroughly intermingled, that the situation is about the same. In Class M, even the nearest and brightest of the dwarf stars are invisible to the naked eye: hence the stars of this class studied by the three investigators are all giants, and once more their results agree.

A number of the dwarf stars of Class K are visible to the naked eye; but these all lie very near us, and have such large proper motions that they are excluded as "abnormal" by both Campbell and Boss. The results of the two agree in indicating that the stars studied by them are typical giants. The few dwarfs, however, have such large parallaxes and proper-motions that their inclusion more than doubles the mean proper-motion, and presumably, also, the mean parallax of the whole, as shown by Kapteyn's figures in Table III. For Class G, the dwarf stars average much brighter, and a much greater number of them is visible to the naked eye. These have large parallaxes and proper-motions, and raise the average for all the stars of this class to greater values than for any other. But Boss's rigorous limitation to small proper-motions weeds them practically all out, leaving giant stars once more. Campbell's less drastic procedure omits only the nearer of the dwarfs (to be precise, those with the larger proper-motions), and his result lies about half-way between the others. In the case of Class F, the dwarf stars are still brighter—intermingling, in fact, with the giants. We can therefore see them farther off, and we get more of them in our catalogues, in proportion to the giants, than in any other class. Their mean parallax is, however, smaller than for the dwarfs of Classes G and K, and hence the mean proper-motion and parallax of all the stars of this class is less than for Class G. Campbell's criterion here excludes very few stars, and even Boss's admits a good many of the remoter and slower moving dwarfs, causing his mean parallax and proper-motion to be considerably greater for this class than for any other.

It should finally be added that Kapteyn's discussion

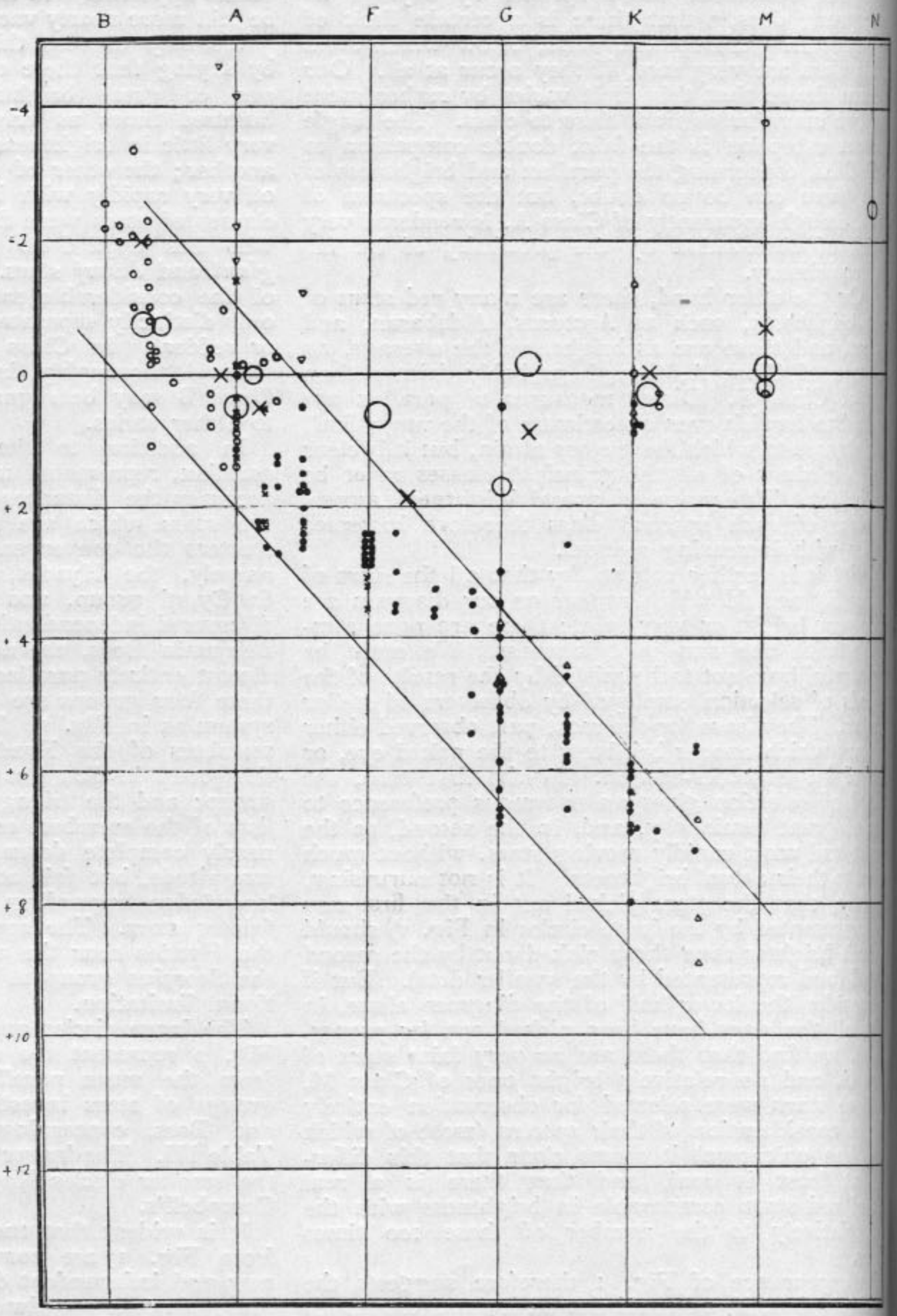


FIG. 2.

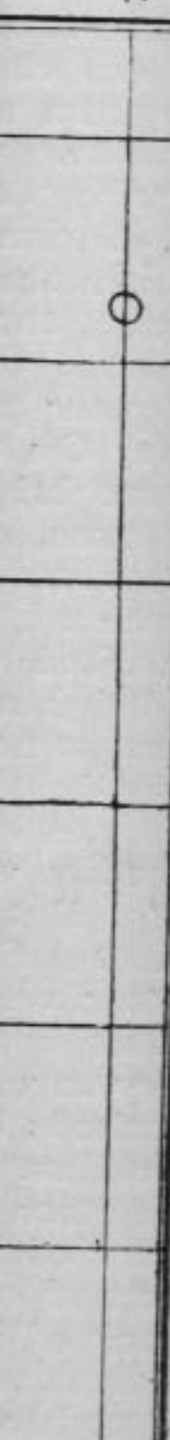
shows that the stars of Class N are exceedingly bright, possibly surpassing any of the other giant stars.

We are now in a position to define more precisely the brightness of a typical giant or dwarf star of a given class of spectrum, and also to obtain a measure of the degree of divergence of the individual stars from this typical brightness. Taking first the stars of Class B and the dwarf stars of the other classes,

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we find, for the mean absolute magnitudes of all the stars of each class, the following values:—

TABLE V.
Mean Absolute Magnitudes.

Spectrum	Stars of measured parallax No.	Abs. mag.	Formula	O–C m.	Stars in clusters No.	Abs. mag.	Formula	O–C m.
B2	—	—	—	—	21	−1·2	−1·1	−0·1
B8	—	—	—	—	8	+0·3	+0·2	+0·1
A0	6	+1·4	+1.4	0·0	13	0·5	0 6	−0·1
A4	7	2·5	2·3	+0·2	26	1·7	1·5	+0·2
F0	—	—	—	—	15	2·4	2·7	−0·3
F1	5	4·2	3·7	+0·5	—	—	—	—
F3	—	—	—	—	7	3·3	3·3	0·0
F5	9	4·3	4·5	−0·2	—	—	—	—
F8	8	5·1	5·2	−0·1	5	4·2	4·4	−0·2
G0	29	5·7	5·6	+0·1	18	5·0	4 8	+0·2
G5	19	5·7	6·6	−0·9	9	5·1	5·8	−0 7
K0	28	7·1	7·7	−0·6	9	6·4	6·9	−0·5
K4	19	9·2	8·6	+0·6	7	+7·0	+7·7	(−0·7)
Ma	10	+9·9	+9 8	+0·1	—	—	—	—

The rate of decrease of brightness with increasing redness is very nearly the same for the stars with directly measured parallaxes and the stars in clusters, but the latter appear, with remarkable consistency, to be about 0·8m. brighter than the former. This seems at first sight very puzzling, but it is undoubtedly due to the way in which the stars observed for parallax were selected. Most observers, in preparing their working lists, have included mainly those stars which were brighter than a given magnitude and had proper-motions exceeding some definite limit. Of the stars above this limiting magnitude, those of greater actual luminosity will be, on the average, farther away, and have smaller proper-motions, than those of small luminosity, and selection by proper-motion favours the latter. The limitation of our present lists to stars the parallaxes of which have been determined with a probable error not exceeding 42 per cent. of their own amounts, though necessary to diminish the effects of casual errors of observation, works in the same direction, for, among the stars of any given visual magnitude, those of greatest luminosity have the smallest parallaxes, and are least likely to pass the test. The difference shown in our table need not therefore alarm us, but it is clear that the stars in clusters, rather than the others, should be taken as typical of the dwarf stars as a whole. For both sets of stars the absolute magnitude appears to be very nearly a linear function of the spectral class (if B is regarded as 1, A as 2, etc.) The columns headed "formula" in Table V. give the values calculated from the expressions M = 1·4m. + 2·1m. (Sp.−A) for the stars of directly measured parallax, and M = 0·6m. + 2·1m. (Sp.−A) for the stars in clusters. The residuals from these empirical formulæ, for the mean absolute magnitudes of the observed stars of different classes, average ±0·33m. in the first case and ±0·29m. in the second. They appear to be accidental in character, though in some cases (notably in Class G5) the residuals for the stars of the two sets are similar in sign and magnitude. The large negative residuals for Classes K and K5 in the clusters arise from the fact that in the Hyades, which contribute most of these stars, only the brighter ones have had their proper-motions determined, and get into our lists, as is clear from examination of Fig. 2.

Among the dwarf stars, therefore, a typical star of any spectral class is about seven times fainter than one of the preceding class, and seven times brighter than one of the following class.

The giant stars of all the spectral classes appear to be of about the same mean brightness, averaging a little above absolute magnitude zero, that is, about a hundred times as bright as the sun. Since the stars of this series which appear in Fig. 2 have been selected by apparent brightness, which gives a strong preference to those of the greatest luminosity, the average brightness of all the giant stars in a given region of space must be less than this, perhaps considerably so.

By tabulating the residual differences between the absolute magnitudes of the individual dwarf stars and the values given by the formulæ just described, we find that the average difference, regardless of sign, for the stars of measured parallax is ±0·88m. for spectra A to F8, ±1·02m. for spectra G and G5, and ±1·15m. for K and M. For the stars in clusters, the average differences are ±0·70m. for spectra BO to B9, ±0·66m. for A and A5, ±0·56m. for spectra F to F8, and ±0·80m. for G and G5.

These differences are larger for the stars of measured parallax than for the others (probably on account of the greater average uncertainty of the individual parallaxes and spectra in this case), but show no marked systematic variation with the class of spectrum. Their distribution follows very approximately the law of accidental errors, as is shown by Table VI., in which the observed numbers lying between certain limits are compared with those given by this law

TABLE VI.
Distribution of Differences from the Typical Absolute Magnitudes.

Stars with measured parallax Limits	Observed	Theory	Stars in clusters Limits	Observed	Theory
m. m.			m. m.		
±0·0 to ±0·8	65	61	±0·0 to ±0·5	59	58
±0·8 to ±1·6	41	44	±0·5 to ±1·0	42	42
±1·6 to ±2·4	21	23	±1·0 to ±1·5	21	24
±2·4 to ±3·2	10	9	±1·5 to ±2·0	10	8
±3·2 to ±4·0	3	3	±2·0 to ±2·5	4	4

The theoretical distribution for the stars in clusters corresponds to a probable error of ±0·61m., and that for the others to one of ±0·94m. Correction for the known influence of uncertainties of the parallaxes and spectra would reduce the latter to about ±0·75m. It appears, therefore, that the absolute magnitude of a dwarf star can be predicted with surprising accuracy from a mere knowledge of its spectrum. Half of all the dwarf stars are not more than twice as bright or as faint as the typical stars of their spectral classes. The corresponding uncertainty in the estimated parallax would be about one-third of its amount.

The parallaxes of the giant stars are so small, in comparison with the errors of even the best present methods of observation, that direct observations are not well adapted to determine to what degree they differ in brightness among themselves. An indirect method of determining this is, however, practicable, among those classes in which all the naked-eye stars are giants, by comparing the parallactic motions of those stars the proper-motions of which at right angles to the direction of the parallactic drift are large and small. A discussion by this method of the typical case of Class M (the details of which will be given elsewhere) shows that, if the distribution of the absolute magnitudes of these stars also follows the "law of errors," the probable error corresponding to it is approximately ±0·6m.—almost exactly the same as has already been found for the dwarf stars. The mean absolute magnitude of all the stars of this class which are visible to the naked eye is −0·5, and that of all the stars in a given region of space is +0·6. This method can scarcely be applied to the naked-eye stars of the other spectral classes (unless some way can be devised for weeding out the dwarf stars from among the giants); but it seems probable

that they do not differ greatly from the stars of Classes B and M as regards the degree of their similarity to one another in brightness With such a probable error of distribution of the absolute magnitudes as has here been derived, the giant and dwarf stars would overlap perceptibly in Class G, be just separated in Class K, and widely so in Class M, as the observational data indicate.

The questions now arise: What differences in their nature or constitution give rise to the differences in brightness between the giant and dwarf stars? and Why should these differences show such a systematic increase with increasing redness or "advancing" spectral type?

We must evidently attack the first of these questions before the second. The absolute magnitude (or the actual luminosity) of a star may be expressed as a function of three physically independent quantities —its mass, its density, and its surface-brightness. Great mass, small density, and high surface-brightness make for high luminosity, and the giant stars must possess at least one of these characteristics in a marked degree, while the dwarf stars must show one or more of the opposite attributes.

A good deal of information is available concerning all these characteristics of the stars. The masses of a considerable number of visual and spectroscopic binaries are known with tolerable accuracy, the densities of a larger number of eclipsing variable stars have recently been worked out, and the recent investigations on stellar temperatures lead directly to estimates of the relative surface brightness of the different spectral classes (subject, of course, to the uncertainty whether the stars really radiate like black bodies, as they are assumed to do). We will take these matters up in order.

First, as regards the masses of the stars, we are confined to the study of binary systems, which may or may not be similar in mass to the other stars. There appears, however, to be no present evidence at all that they are different from the other stars, and in what follows we will assume them to be typical of the stars as a whole.

The most conspicuous thing about those stellar masses which have been determined with any approach to accuracy is their remarkable similarity. While the range in the known luminosities of the stars exceeds a millionfold, and that in the well-determined densities is nearly as great, the range in the masses so far investigated is only about fiftyfold. The greatest known masses are those of the components of the spectroscopic binary and eclipsing variable V Puppis, which equal nineteen times that of the sun; the smallest masses concerning which we have any trustworthy knowledge belong to the faint components of ζ Herculis and Procyon, and are from one-third to one-fourth of the sun's mass. These are exceptional values, and the components of most binary systems are more nearly similar to the sun in mass.

There appears, from the rather scanty evidence at present available, to be some correlation between mass and luminosity. Those stars which are known to be of small mass (say, less than half the sun's) are all considerably fainter than the sun. On the other hand, Ludendorff[18] has shown conclusively that the average mass of the spectroscopic binaries of spectrum B (which are all of very great luminosity) is three times as great as that of the spectroscopic binaries of other spectral types, and may exceed ten times that of the sun. Further evidence in favour of this view is found in the fact that the components of a binary, when equal in brightness, are nearly equal in mass, while in unequal pairs the brighter star is almost (if not quite) always the more massive, but the ratio of the masses very rarely exceeds 3:1, even when one component is hundreds of times as bright as the other. Very large masses (such as one hundred times the sun's mass) do not appear, though they would certainly be detected among the spectroscopic binaries if they existed. It is equally remarkable that there is no trustworthy evidence that any visible star has a mass as small as one-tenth that of the sun. The apparent exceptions which may be found in the literature of the subject may be shown to arise from faulty determinations of parallax, arbitrary estimates of quantities unobtainable by observation (such as the ratio of the densities of the two components of Algol), and even numerical mistakes.

It follows from this similarity of mass that we can obtain a very fair estimate of the parallax of any visual binary (called by Doberck the hypothetical parallax) by guessing at its mass, and reversing the familiar relation between mass and parallax. If we assume that the mass of the system is twice that of the sun (about the average value), our hypothetical parallaxes, as the existing evidence shows, will usually be well within 40 per cent. of the truth, and the deduced absolute magnitudes of the components will rarely be more than one magnitude in error. We may thus extend our study of the relation between absolute magnitude and spectrum to all the visual binaries for which orbits have been computed. The hypothetical absolute magnitudes which we will obtain for them will indeed be somewhat in error, owing to the differences in their masses; but, for our present purpose, *the hypothetical values are actually more useful than the true values would be*. This sounds remarkable; but it is easy to show that, if we assume that the brighter components of the systems have all the same mass (say that of the sun), the resulting hypothetical absolute magnitudes will be the actual absolute magnitudes of stars identical in density and surface-brightness with the real stars, but all of the assumed mass. In other words, *the effects of differences of mass among the stars are eliminated from these hypothetical absolute magnitudes*, leaving only those of differences in density and surface-brightness. (This is simply a statement in different form of a theorem which has been known for many years.) It is therefore desirable to extend our study to as many binary stars as possible. The number for which binary orbits have been computed is relatively small, but by a simple statistical process we may include all those pairs which are known to be connected really physically, however slow their relative motion may be.[19]

Consider any pair of stars, of combined mass m times that of the sun, at a distance of r astronomical units, and with a relative velocity of v astronomical units per annum. By gravitational theory, we have

$$v^2 r = (2\pi)^2 m(2 - r/a) = 39{\cdot}7m(2 - r/a),$$

where a is the semi-major axis of the orbit. Now let π be the parallax of the system, s the observed distance in seconds of arc, w the observed relative motion in seconds of arc per annum, and i_1 and i_2 the angles which r and v make with the line of sight. Then $s = r\pi \sin i_1$, $w = v\pi \sin i_2$, and our equation becomes

$$sw^2 = 39{\cdot}7\pi^3 m \sin i_1 \sin^2 i_2 (2 - r/a).$$

In the individual case, the last three factors of the second member are unknown, and we are no wiser

[18] *A. N.*, 4520, 1911.

[19] An outline of this method was given by the speaker at the meeting of the Astronomical and Astrophysical Society of America at Ottawa, August 25, 1911, and published in *Science*, N.S., vol. xxxiv., pp. 523–25, October 20, 1911. A similar method was worked out quite independently and almost simultaneously by Hertzsprung, and published in *A. N.*, December 19, 1911 (the date of writing being October 11, 1911).

than at the start; but the average value which their product should have, in a large number of cases, and the percentage of these cases in which it should lie within any given limits, may be computed on the principles of geometrical probability. It is thus found that the formula $\pi^3 = sw^2/14{\cdot}6m$ gives values for the hypothetical parallax the average for a large number of cases of which will be correct, and that, while in individual cases these values will be too large or too small, half of them will be within 19 per cent. of the true values, and the numbers of larger errors will fall off in very nearly the manner corresponding to this probable error. If we compute absolute magnitudes from these parallaxes, their average for all the stars will be a little too bright (since the cases in which the computed parallax comes out too small have more influence than those in which it is too large). This may be allowed for by adding 0·15m. to all the hypothetical magnitudes so computed—an amount almost negligibly small for our present purpose.

We thus obtain a series of hypothetical absolute magnitudes the average for a large number of cases of which will be correct. In 59 per cent. of the individual cases the error arising from the statistical process—that is, from the substitution of a mean value of

$$\sin i_1 \sin^2 i_2 (2 - r/a)$$

for the true value—will affect the deduced magnitude by less than ±0·5m., and in 89 per cent. of all cases the error will not exceed ±1·0m. The approximation is therefore quite sufficient for our purpose. It should, however, be noted that, while the error of the statistical process can never make the computed absolute magnitude of any star too faint by more than 1·5m., it may in rare cases make it too bright by any amount whatever—more than 2·0m. in one case in sixty, more than 3·0m. once in 250 cases, and so on.

We may now proceed to compute hypothetical absolute magnitudes for all the physical pairs which show even a trace of relative motion—including many which are ordinarily described as "fixed," but, on careful study of the observations, show very slow relative change. With the aid of the splendid collection of observational data contained in Burnham's great catalogue and other recent works on double stars, and of many observations of spectra made at Harvard in generous response to requests for information, it has been possible to derive results for more than 550 stars. Assuming that the brighter component of each of these (which is usually the only one of which the spectrum is known) is equal in mass to the sun, estimating that of the fainter component on the basis of the difference of brightness (with the data for the systems in which the mass-ratio is known as a sufficient guide), and proceeding as indicated above, we obtain the data plotted in Fig. 3. The co-ordinates have here the same meaning as in the previous diagrams, and the figure shows at a glance the relations which would exist between the absolute magnitudes and spectra of these 550 stars if all differences of mass were eliminated, leaving only those of density and surface-brightness operative. Binaries for which orbits have been computed are shown by solid dots, and physical pairs, to which the statistical process has been applied, by open circles.

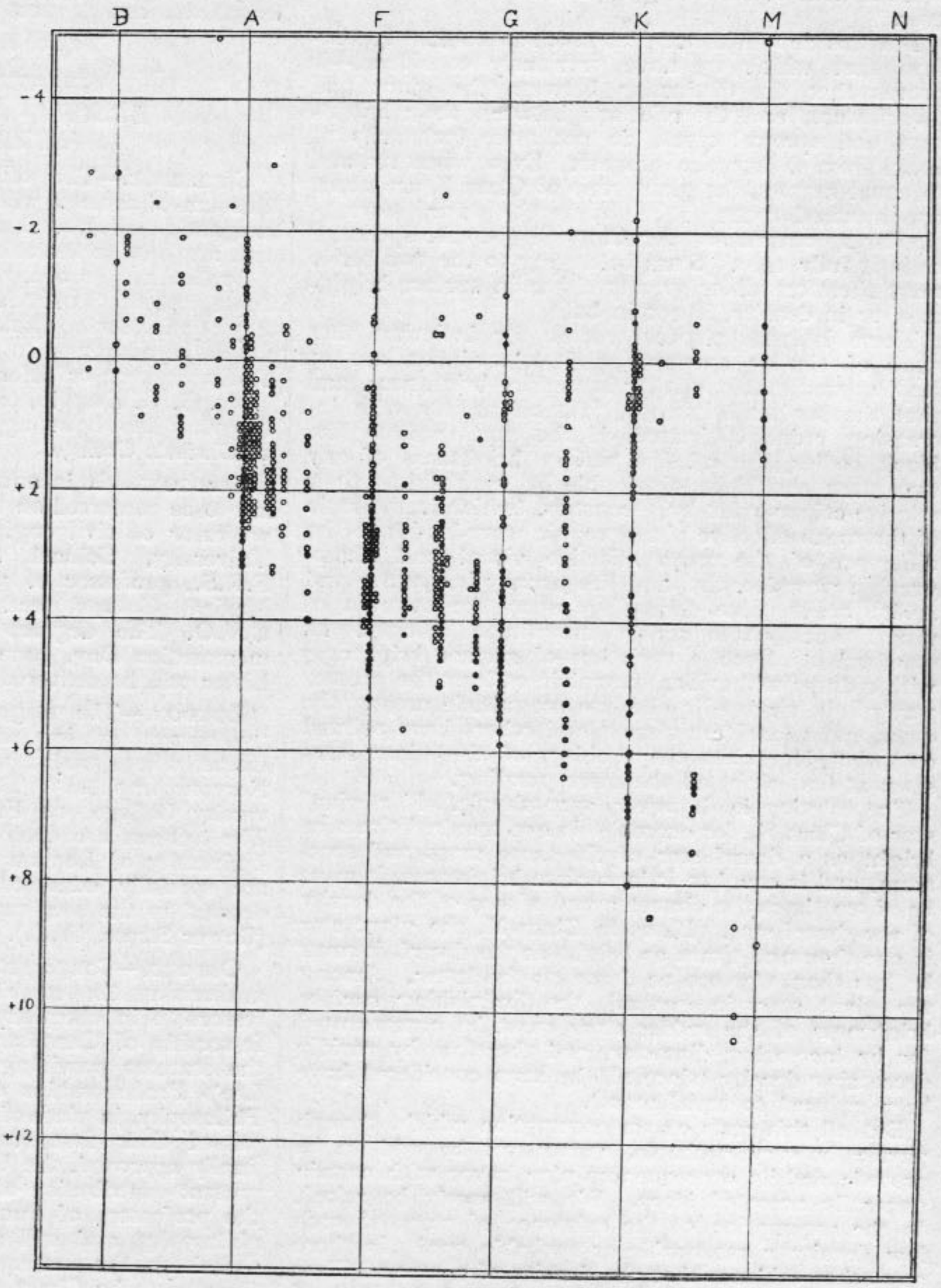


FIG. 3

Our new diagram is strikingly similar in appearance to the previous ones, even in its minor details.

The two series of giant and dwarf stars appear once more; the giants are all of about the same brightness, except that those of Class B are brighter than the rest; the dwarf stars diminish in brightness by about two magnitudes for each spectral class; the two series overlap up to Class G and separate at Class K, and so on. We have clearly come, for the third time, and again from independent data, upon the same phenomena as before; and, with the more extensive observational material, some of the characteristics and relations of the two groups are shown better than ever.

But this new evidence does much more than to confirm that which we have previously considered—it proves that the distinction between the giant and dwarf stars, and the relations between their brightness and spectral types, do not arise (primarily at least) from differences in mass. Even when reduced to equal masses, the giant stars of Class K are about one hundred times as bright as the dwarf stars of similar spectrum, and for Class M the corresponding ratio is fully 1000. Stars belonging to the two series must therefore differ greatly either in surface brightness or in density, if not in both.

There is good physical reason for believing that stars of similar spectrum and colour-index are at least approximately similar in surface brightness, and that the surface brightness falls off rapidly with increasing redness. Indeed, if the stars radiate like black bodies, the relative surface brightness of any two stars should be obtainable by multiplying their relative colour-index by a constant (which is the ratio of the mean effective photographic wave-length to the difference of the mean effective visual and photographic wave-lengths, and lies usually between 3 and 4, its exact value depending upon the systems of visual and photographic magnitude adopted as standards). Such a variation of surface brightness with redness will evidently explain at least the greater part of the change in absolute magnitude among the dwarf stars (as Hertzsprung and others have pointed out), but it makes the problem of the giant stars seem at first sight all the more puzzling.

The solution is, however, very simple. If a giant star of Class K, for example, is one hundred times as bright as a dwarf star of the same mass and spectrum, and is equal to it in surface brightness, it must be of ten times the diameter and 1/1000 of the density of the dwarf star. If, as in Class M, the giant star is one thousand times as bright as the dwarf, it must be less than 1/30,000 as dense as the latter. Among the giant stars in general, the diminishing surface brightness of the redder stars must be compensated for by increasing diameter, and therefore by rapidly decreasing density (since all the stars considered have been reduced to equal mass).

But all this rests on an assumption which, though physically very probable, cannot yet be said to be proved; and its consequences play havoc with certain generally accepted ideas. We will surely be asked, Is the assumption of the existence of stars of such low density a reasonable or probable one? Is there any other evidence that the density of a star of Class G or K may be much less than that of the stars of Classes B and A? Can any other evidence than that derived from the laws of radiation be produced in favour of the rapid decrease of surface brightness with increasing redness?

We can give at once one piece of evidence bearing on the last question. The twelve dwarf stars of Classes K2 to M, shown in Fig. 3, have, when reduced to the sun's mass, a mean absolute magnitude of 7·8—three magnitudes fainter than the sun. If of the sun's surface brightness, they would have to be, on the average, of one-fourth its radius, and their mean density would be sixty-four times that of the sun, or ninety times that of water—which is altogether incredible. A body of the sun's mass and surface brightness, even if as dense as platinum, would only be two magnitudes fainter than the sun, and the excess of faintness of these stars beyond this limit can only be reasonably ascribed to deficiency in surface brightness. For the four stars of spectra K8 and M, the mean absolute magnitude of which, reduced to the sun's mass, is 9·5, the mean surface brightness can at most be one-tenth that of the sun.

(To be continued.)

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

CAMBRIDGE.—The office of superintendent of the museum of zoology will shortly become vacant by the resignation of Dr. Doncaster. The stipend at present attached to the office is 200*l*. per annum.

Applications to occupy the University's table in the Zoological Station at Naples, and that in the laboratory of the Marine Biological Association at Plymouth, should be addressed to Prof. Langley, The Museum, Cambridge, on or before June 4.

Mr. C. G. Darwin, eldest son of the late Sir George Darwin, has been appointed mathematical lecturer at Christ's College.

GLASGOW.—It is announced that honorary degrees are to be conferred on Dr. Archibald Barr, late regius professor of civil engineering and mechanics in the University, Colonel Sir William B. Leishman, F.R.S., professor of pathology in the Royal Army Medical College, and Sir Ernest H. Shackleton, C.V.O. The degrees will be conferred on Commemoration Day, June 23, when an oration on Lord Lister will be delivered by Sir Hector C. Cameron.

LONDON.—The Page-May Memorial Lectures for the current session will be delivered by Dr. Keith Lucas, whose subject will be "The Conduction of the Nervous Impulse." The course will be held at University College, on Fridays, beginning on May 15. The lectures are open to all internal students of the University of London and to such other persons as are specially admitted. Applications should be addressed to the secretary, University College, London (Gower Street, W.C.).

OXFORD.—Congregation on May 5 passed a statute authorising the establishment of an additional professorship in chemistry, to be called Dr. Lee's Professorship of Chemistry. In the same Congregation the statutes providing for the establishment of Dr. Lee's Professorships of Anatomy and Experimental Philosophy, in place of the existing Lee's Readerships, passed their first stage. Should these statutes be finally approved, the University will be relieved of its present contribution of 1470*l*. towards the stipends of the professors of human anatomy and experimental philosophy, and will gain an additional professor of chemistry, the consequent charges being borne in all these cases by Christ Church.

The Halley Lecture for 1914 will be delivered by Colonel C. F. Close, director of the Ordnance Survey, at the Examination Schools at 8.30 p.m. on May 21. Subject, "The Geodesy of the United Kingdom."

The celebration of the seven hundredth anniversary of the birth of Roger Bacon will be held on Wednesday, June 10.

BROWN UNIVERSITY, Rhode Island, is to receive a visit in November next from Prof. W. H. Bragg.

RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS.*

III.

TO proceed farther we must have recourse to the study of eclipsing variable stars. Methods for computing their orbits have been developed at Princeton during the last few years,[20] the main motive for the investigation being the astrophysical importance of the results. Dr. Shapley, using the methods devised by the speaker, has obtained elements for eighty-seven such systems,[21] for each one of which the density of the components may be calculated. The values here employed are those which result from the assumption that the stars present discs darkened toward the edge, like the sun, but to a still higher degree, and the principal uncertainty of the results (which in any case cannot be very serious) arises from our present ignorance of the actual degree of this darkening. For our present purpose, they may be best utilised by computing the absolute magnitudes which the brighter component of each system would have if its mass and surface brightness were equal to those of the sun, leaving outstanding the differences due to density alone.

The results for the eighty eclipsing variable the elements and spectra of which are known are plotted in Fig. 4, on the same system as in the preceding figures. The black dots represent those stars for which the photometric data are most trustworthy, the open circles those of less precision. We are once more greatly indebted to Prof. Pickering and Miss Cannon for information regarding the spectra of these stars. To the absolute magnitude 4·0 on this diagram corresponds a density 1/3 that of the sun; to 3·0, 1/11; to 2·0, 1/45; to 1·0, 1/180 of the sun's density, and so on.

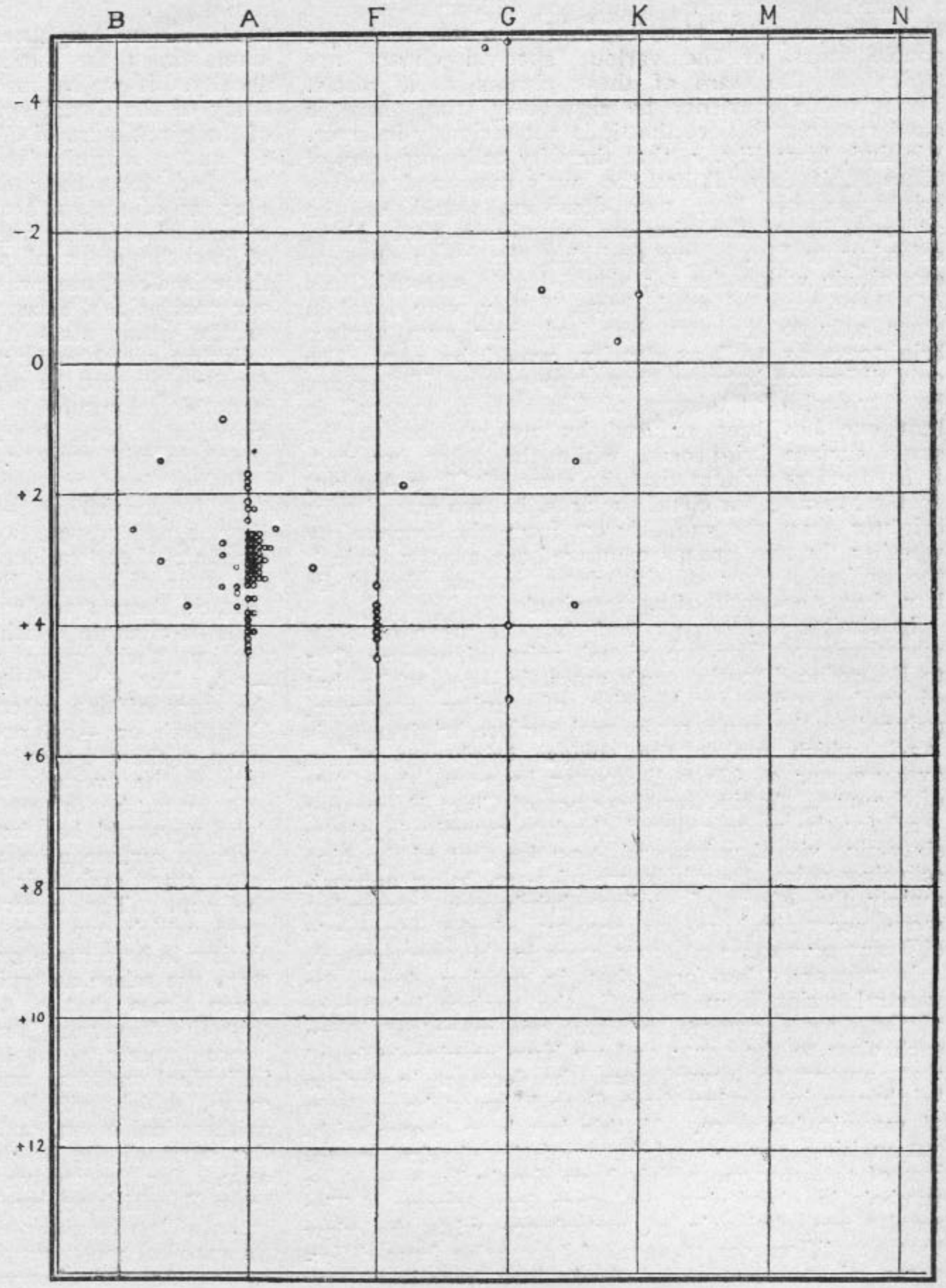


FIG. 4.

This diagram bears at first sight but small resemblance to the previous ones, but a little study brings out several important things. First, though the majority of these eclipsing variables are of Class A, every class from B to K is represented, and there are eight stars of Class G or redder. Secondly, all but one of these eighty stars are less dense than the sun, though but few of them are of less than 1/100 the sun's density. Thirdly, the stars of Classes A and B are fairly similar to one another in density, the great majority having densities between 1/3 and 1/45 of the sun's; those of Class F show a compact group of high density, and an isolated star of low density; but in Classes G and K the range of density is enormously great—from nearly twice that of the sun (W Ursæ Majoris) to one-millionth of the sun's density (W Crucis, at the top of the diagram). Fourthly, among the stars of density less than 1/200 that of the sun (corresponding to about +1·0m. on the diagram), only one of the seventy-one stars of Classes B to F appears, while four of the eight stars

* An address delivered before a joint meeting of the Astronomical and Astrophysical Society of America and Section A of the American Association for the Advancement of Science, at Atlanta, Georgia, December 30, 1913, with a few additions, by Prof. H. N. Russell. Continued from p. 258.

[20] *Astrophys. Jour.*, vol. xxxv., p. 315, and vol. xxxvi., pp. 54, 239, 385, 1912.

[21] *Astrophys. Jour.*, vol. xxxviii., pp. 159–73, 1913.

of Classes G and K are included. We may now answer decisively, and in the affirmative, the first two questions which were put a few moments ago. Some stars actually have densities quite as low as any that might be required to explain the great brightness of the reddest giant stars; and these stars of low density show a very marked preference for the "later" spectral classes, while practically all the stars of "earlier" type are far denser.

We can answer the third question as well, and in a quantitative fashion, if we are willing to assume that the eclipsing binaries, and also the telescopic double stars, of the various spectral classes are typical of the stars of these classes as a whole. Though this may not be rigorously true, there is good reason to believe that it is not seriously in error. We find, from Fig. 4, that the fifty eclipsing stars of Class A, if they all had the sun's mass and surface brightness, but their own densities, would, on the average, be of the absolute magnitude 3·06. Now, referring to Fig. 3, we find that the mean absolute magnitude which the 115 visual double stars of Class AO there recorded would have, if they were equal in mass only to the sun, but had their own surface brightness as well as density, would be 1·07. The only difference between these two groups (if they are both typical of the stars of Class A in general) is that one has been reduced by computation to the sun's surface brightness, while the other has not. It is therefore evident that the stars of Class A must, on the average, for equal surfaces, be two magnitudes brighter than the sun. Apart from the uncertainty whether the two groups compared are exactly typical, the probable error of this determination should be less than one-tenth of a magnitude.

In similar fashion, we find that the mean absolute magnitude of fifty-two visual pairs of spectra Oe5 to B9, reduced to the sun's mass, is −0·4, while that of twelve eclipsing binaries of similar spectrum, reduced to the sun's mass, and surface brightness, is +2·8, which makes the surface brightness of an average star of Class B greater by 3·2m. than that of the sun. Again, for the stars of Class F, we get +2·6 for the mean reduced absolute magnitude of the sixty-nine visual pairs, and +3·7 for that of the nine eclipsing pairs, the difference of 1·1m. being approximately the effect of surface brightness (somewhat more uncertain here, on account of the apparently different proportion of giant stars in the two groups).

It appears, therefore, that in passing down the spectral series from B to G, the surface brightness of the stars decreases by about one magnitude from each class to the next; and we have previously found that, among the dwarf stars, the decrease in surface brightness in passing from G to M must be at least 2½ magnitudes more. All this has been shown without making any use whatever of the physical meaning of the spectra, which have simply been used as symbols in classifying the stars into groups. The results are obviously in accordance with the view that the differences of spectral type arise from differences of temperature. Indeed, they constitute new and important evidence in its favour. How well they agree with other independent lines of evidence is shown by comparing the relative surface brightness just computed with the colour-index for the various classes. Taking A as a standard, we have:—

Spectrum	B	A	F	G	K	M
Surface brightness	−1·2	0·0	+0·9	+2·0	—	+4·5 (at least)
Colour-index ...	−0·3	0·0	+0·3	+0·7	+1·2	+1·6

The computed differences in surface brightness are in all cases about three times the colour-indices, in good agreement with the theoretical ratio.

We may now estimate the density of the redder giant stars. It appears from Fig. 3 that the mean absolute magnitude of the giant stars, if reduced to the sun's mass, is +0·6 for Class G, +0·5 for Class K, and 0·0 for Classes K5 and M. The differences between these values are small, and we may well take the general mean, +0·44, as typical of the whole. This corresponds to about fifty times the sun's luminosity. Such a giant star of Class G, if of the sun's surface brightness, would have to be of about seven times the sun's radius, and of 1/350 of its density. If we assume, on the basis of the foregoing study of the dwarf stars, that the surface intensities of the giant stars of Classes K and M are respectively 1·5 and 3 magnitudes fainter than that of the sun, we find that their densities must be 1/2800 and 1/25,000 of the sun's density. The observed densities of several eclipsing variables of Classes G and K are of just the order of magnitude here found, so that there is direct observational evidence in favour of all our conclusions, except the very low density assigned to the giant stars of Class M (among which no eclipsing variables have yet been found, so that their densities cannot be directly determined). But there is nothing improbable about so low a density, for we know of at least one star—W Crucis—the density of which is still smaller.

Before leaving these diagrams we should notice that, by comparing the data of Fig. 3 with those of Figs. 1 and 2, we may obtain the average masses of the stars of the various types. Consider, for example, the stars of spectra B to B5. From Fig. 3 we see that, if these stars were reduced to the sun's mass without changing either their surface brightness or density, their mean absolute magnitude would be −0·6. But the actual mean absolute magnitude of the stars of this spectral class is −2·0 according to Campbell, or −0·8 according to Boss. Taking the mean of these determinations, we find that these stars are, on the average, 2·1 times as bright as stars of unit mass, but of the same surface brightness and density, would be, from which it follows that their average surface area must be 2·1 times that of the latter stars, and their average mass 3·0 times that of the sun. The uncertainty whether the groups of stars which we are comparing are really exactly similar is here more serious than usual; if Campbell's stars are taken as typical, the mean mass comes out seven times that of the sun. It should be noticed that the "average" mass here obtained corresponds approximately to the average of the logarithms of the individual masses, and hence to their geometrical mean, which will be somewhat smaller than their arithmetical mean, and that we are here dealing with the mass of the brighter component of each system only. For the twelve spectroscopic binaries of spectrum B, which are available for comparison, the mean mass of the brighter components is about 9, and the geometrical mean probably about 7·5, times the sun's mass. As the observational selection in this case undoubtedly favours the larger masses, there is no serious discrepancy between the two results.

Proceeding similarly for the stars of the other spectral classes, we obtain the results collected in Table VII. The observed absolute magnitudes of the stars in clusters have been taken in preference to those of the stars of directly measured parallax, for the reasons already stated, and for the giant stars the mean of the results of Boss and Campbell has been used (except for Class G, for which Boss's value alone really represents them).

TABLE VII.

Mean Masses of Bright Components of Double Stars.

Spectrum	Observed absolute mag.	Abs. mag. reduced to sun's mass	Resulting average mass
B2	−1·4	−0·6	3·0
Ao	+0·5	+1·1	2·3
A5, dwarf	+1·5	+1·6	1·2
Fo ,, ...	+2·4	+2·8	1·7
F3 ,, ...	+3·3	+3·1	0·8
F8 and Go ...	+4·8	+4·0	0·5
G5, dwarf	+5·1	+4·2	0·3
Ko ,, ...	+6·4	+5·5	0·3
K5 and M, dwarf	+8·9	+7·7	0·2
G and G5, giant	−0·2	+0·6	3·0
Ko, giant ...	+0·2	+0·5	1·5
K5 and M, giant	−0·3	0·0	1·5

The general similarity in mass among the stars of such widely different characteristics is very striking. In view of the small numbers of stars in some of the groups, the differences between the individual values should not be greatly stressed, but the gradual decrease of average mass among the dwarf stars is in accordance with the results of direct measurement. The geometrical mean of the computed masses of the bright components of the eight visual binaries of spectra A to F5, the parallaxes of which have been determined with tolerable accuracy, is 1·8 times the sun's mass; for the ten similar stars of spectra F8 to K it is 0·8. The greater mass of the stars of Class B is scarcely shown by these figures, but on this matter the testimony of the spectroscopic binaries deserves much the greater weight. The important conclusion which may be drawn from Table VII. is that, although the range in mean luminosity among the various groups of stars exceeds ten-thousandfold, the range in the mean masses probably does not exceed twenty-, or at most thirty-fold.

We may now summarise the facts which have been brought to light, as follows:—

(1) The differences in brightness between the stars of different spectral classes, and between the giant and dwarf stars of the same class, do not arise (directly at least) from differences in mass. Indeed, the mean masses of the various groups of stars are extraordinarily similar.

(2) The surface brightness of the stars diminishes rapidly with increasing redness, changing by about three times the difference in colour-index, or rather more than one magnitude, from each class to the next.

(3) The mean density of the stars of Classes B and A is a little more than one-tenth that of the sun. The densities of the dwarf stars increase with increasing redness from this value through that of the sun to a limit which cannot at present be exactly defined. This increase in density, together with the diminution in surface brightness, accounts for the rapid fall in luminosity with increasing redness among these stars.

(4) The mean densities of the giant stars diminish rapidly with increasing redness, from one-tenth that of the sun for Class A to less than one twenty-thousandth that of the sun for Class M. This counteracts the change in surface brightness, and explains the approximate equality in luminosity of all these stars.

(5) The actual existence of stars of spectra G and K, the densities of which are of the order here derived, is proved by several examples among the eclipsing variables, all of which are far less dense than any one of the more numerous eclipsing stars of "early" spectral type, with the sole exception of β Lyræ.

Evolution.

These facts have evidently a decided bearing on the problem of stellar evolution, and I will ask your indulgence during the few minutes which remain for an outline of the theory of development to which it appears to me that they must inevitably lead.

Of all the propositions, more or less debatable, which may be made regarding stellar evolution, there is probably none that would command more general acceptance than this—that as a star grows older it contracts. Indeed, since contraction converts potential energy of gravitation into heat, which is transferred by radiation to cooler bodies, it appears from thermodynamic principles that the general trend of change must, in the long run, be in this direction. It is conceivable that at some particular epoch in a star's history there might be so rapid an evolution of energy—for example, of a radio-active nature—that it temporarily surpassed the loss by radiation and led to an expansion against gravity; but this would be, at most, a passing stage in its career, and it would still be true in the long run that the order of increasing density is the order of advancing evolution.

If, now, we arrange the stars which we have been studying in such an order, we must begin with the giant stars of Class M, follow the series of giant stars, in the reverse order from that in which the spectra are usually placed, up to A and B, and then, still with increasing density, though at a slower rate, proceed down the series of dwarf stars, in the usual order of the spectral classes, past the sun, to those red stars (again of Class M), which are the faintest at present known. There can be no doubt at all that this is the order of increasing density; if it is also the order of advancing age, we are led at once back to Lockyer's hypothesis that a star is hottest near the middle of its history, and that the redder stars fall into two groups, one of rising and the other of falling temperature.[22] The giant stars then represent successive stages in the heating up of a body, and must be more primitive the redder they are; the dwarf stars represent successive stages in its later cooling, and the reddest of these are the farthest advanced. We have no longer two separate series to deal with, but a single one, beginning and ending with Class M, and with Class B in the middle, all the intervening classes being represented, in inverse order, in each half of the sequence.

The great majority of the stars visible to the naked eye, except perhaps in Class F, are giants; hence for most of these stars the order of evolution is the reverse of that now generally assumed, and the terms "early" and "late" applied to the corresponding spectral types are actually misleading.

This is a revolutionary conclusion; but, so far as I can see, we are simply shut up to it, with no reasonable escape. If stars of the type of Capella, γ Andromedæ, and Antares represent later stages of development of bodies such as δ Orionis, α Virginis and Algol, we must admit that, as they grew older and lost energy, they have expanded, in the teeth of gravitation, to many times their original diameters, and have diminished many hundred-, or even thousand-fold in density. For the same reason we cannot regard the giant stars of Class K as later stages of those of Class G, or those of Class M as later stages of either of the others, unless we are ready to admit that they have expanded against gravity in a similar fashion. We may, of course, take refuge in the belief that the giant stars of the

[22] Phil. Trans., vol. clxxxiv., p. 688, 1902; Proc. Roy. Soc., vol. lxv., p. 186, 1899.

various spectral classes have no genetic relations with one another—that no one class among them represents any stage in the evolution of stars like any of the others—but this is to deny the possibility of forming any general scheme of evolution at all.

We might be driven to some such counsel of despair if the scheme suggested by the observed facts should prove physically impossible; but, as a matter of fact, it is in conspicuous agreement with the conclusions which may be reached directly from elementary and very probable physical considerations.

There can be very little doubt that the stars, in general, are masses of gas, and that the great majority of them, at least, are at any given moment very approximately in stable internal equilibrium under the influence of their own gravitation, and very nearly in a steady state as regards the production and radiation of heat, but are slowly contracting on account of their loss of energy. Much has been written upon the behaviour of such a mass of gas by Lane, Ritter, and several later investigators,[23] and many of their conclusions are well established and well known. So long as the density of the gaseous mass remains so low that the ordinary "gas laws" represent its behaviour with tolerable accuracy, and so long as it remains built upon the same model (*i.e.*, so long as the density and temperature at geometrically homologous points vary proportionally to the central density or temperature), the central temperature (and hence that at any series of homologous points) will vary inversely as the radius. This is often called Lane's law. If, after the contraction, the star is built only approximately on the same model as before, this law will be approximately, but not exactly, true.

The temperature of the layers from which the bulk of the emitted radiation comes will also rise as the star contracts, but more slowly, since the increase in density will make the gas effectively opaque in a layer the thickness of which is an ever-decreasing fraction of the radius. The temperature of the outer, nearly transparent gases, in which the line absorption takes place, will be determined almost entirely by the energy density of the flux of radiation through them from the layers below—that is, by the "black-body" temperature corresponding to this radiation as observed at a distance.

As the gaseous mass slowly loses energy and contracts, its effective temperature will rise, its light will grow whiter, and its surface brightness increase, while corresponding modifications will occur in the line absorption in its spectrum. Meanwhile, its diameter and surface will diminish, and this will at least partially counteract the influence of the increased surface brightness, and may even overbalance it. It cannot therefore be stated, without further knowledge, in which direction the whole amount of light emitted by the body will change.

This process will go on until the gas reaches such a density that the departures of its behaviour from the simple laws which hold true for a perfect gas become important. Such a density will be first reached at the centre of the mass. At the high temperatures with which we are dealing, the principal departure from the simple gas laws will be that the gas becomes more difficultly compressible, so that a smaller rise in temperature than that demanded by the elementary theory will suffice to preserve equilibrium after further contraction. The rise in temperature will therefore slacken, and finally cease, first at the centre, and later in the outer layers. Further contraction will only be possible if accompanied by a fall of temperature, and the heat expended in warming the mass during the earlier stages will now be gradually transmitted to the surface and liberated by radiation, along with that generated by the contraction. During this stage, the behaviour of the mass will resemble, roughly, that of a cooling solid body, though the rate of decrease of temperature will be far slower. The diameter and surface brightness will now both diminish, and the luminosity of the mass will fall off very rapidly as its light grows redder. It will always be much less than the luminosity of the body when it reached the same temperature while growing hotter, on account of the contraction which has taken place in the interval, and this difference of luminosity will be greater the lower the temperature selected for the comparison. Sooner or later, the mass must liquefy, and then solidify (if of composition similar to the stellar atmospheres), and at the end it will be cold and dark; but these changes will not begin, except perhaps for a few minor constituents of very high boiling point, until the surface temperature has fallen far below that of the stars of Class M (about 3000° C.).

The "critical density" at which the rise of temperature will cease can only be roughly estimated. It must certainly be much greater than that of ordinary air, and (at least for substances of moderate molecular weight), considerably less than that of water. Lord Kelvin,[24] a few years ago, expressed his agreement with a statement of Prof. Perry's that "speculation on this basis of perfectly gaseous stuff ought to cease when the density of the gas at the centre of the star approaches one-tenth of the density of ordinary water in the laboratory."

It is clear from the context that this refers rather to the beginning of sensible departures from Lane's law than to the actual attainment of the maximum temperature, which would come later; and it seems probable, from the considerations already mentioned, that the maximum temperature of the surface would be attained at a somewhat higher density than the maximum central temperature.

The resemblance between the characteristics that might thus be theoretically anticipated in a mass of gas of stellar dimensions, during the course of its contraction, and the actual characteristics of the series of giant and dwarf stars of the various spectral classes is so close that it might fairly be described as identical. The compensating influences of variations in density and surface brightness, which keep all the giant stars nearly equal in luminosity, the rapid fall of brightness among the dwarf stars, and the ever-increasing difference between the two classes, with increasing redness, are all just what might be expected. More striking still is the entire agreement between the actual densities of the stars of the various sorts and those estimated for bodies in the different stages of development, on the basis of the general properties of gaseous matter. The densities found observationally for the giant stars of Classes G to M are such that Lane's law must apply to them, and they must grow hotter if they contract; that of the sun (a typical dwarf star) is so high that the reverse must almost certainly be true; and the mean density of the stars of Classes B and A (about one-ninth that of the sun, or one-sixth that of water) is just of the order of magnitude at which a contracting mass of gas might be expected to reach its highest surface temperature.

We may carry our reasoning farther. Another deduction from the elementary theory (as easily proved as Lane's law, but less generally known) is that, in two masses of perfect gas, similarly constituted and of equal radius, the temperatures at homologous points are directly proportional to their masses. As in the previous case, the effective surface temperature of the

[23] An excellent summary may be found in Emden's *Gaskugeln*.

[24] NATURE, vol. lxxv., p. 368, 1907.

more massive body will be the greater, though to a less degree than the central temperature. A large mass of gas will therefore arrive at a higher maximum temperature, upon reaching its critical density, than a small one. The highest temperatures will be attained only by the most massive bodies, and all through their career these will reach any given temperature at a lower density, on the ascent, and return to it at a higher density, on the descending scale, than a less massive body. They will therefore be of much greater luminosity, for the same temperature, than bodies of small mass if both are rising toward their maximum temperatures. On the descending side the difference will be less conspicuous. Bodies of very small mass will reach only a low temperature at maximum, which may not be sufficient to enable them to shine at all.

All this, again, is in excellent agreement with the observed facts. The hottest stars—those of Class B—are, on the average, decidedly more massive than those of any other spectral type. On the present theory, this is no mere chance, but the large masses are the necessary condition—one might almost say the cause—of the attainment of unusually high temperature. Only these stars would pass through the whole series of the spectral classes, from M to B and back again, in the course of their evolution. Less massive bodies would not reach a higher temperature than that corresponding to a spectrum of Class A; those still less massive would not get above Class F, and so on. This steady addition of stars of smaller and smaller mass, as we proceed down the spectral series, would lower the average mass of all the stars of a given spectral class with "advancing" type, in the case of the giants as well as that of the dwarfs. This change is conspicuously shown among the dwarf stars in Table VII., and faintly indicated among the giant stars. The average masses of the giant and dwarf stars appear, however, to be conspicuously different, which at first sight seems inconsistent with the theory that they represent different stages in the evolution of the same masses. But the giant stars which appear in these lists have been picked out in a way that greatly favours those of high luminosity, and hence, as we have seen, those of large mass, while this is not the case among the dwarf stars. The observed differences between them are therefore in agreement with our theory, and form an additional confirmation of it.

It is now easy, too, to understand why there is no evidence of the existence of luminous stars of mass less than one-tenth that of the sun. Smaller bodies presumably do not rise, even at maximum, to a temperature high enough to enable them to shine perceptibly (from the stellar point of view), and hence we do not see them. The fact that Jupiter and Saturn are dark, though of a density comparable with that of many of the dwarf stars, confirms this view.[25]

We may once more follow the lead of our hypothesis into a region which, so far as I know, has been previously practically untrodden by theory. It is well known that the great majority of the stars in any given region of space are fainter than the sun, and that there is a steady and rapid decrease in the number of stars per unit volume, with increasing luminosity. The dwarf stars, especially the fainter and redder ones, really greatly outnumber the giants, the preponderance of which in our catalogues arises entirely from the egregious preference given them by the inevitable method of selection by apparent brightness.

What should we expect to find theoretically? To get an answer, we must make one reasonable assumption, namely, that the number of stars, in any sufficiently large region of space, which is, at the present time, in any given stage of evolution, will be (roughly at least) proportional to the lengths of time which it takes a star to pass through the respective stages.[26] While a star is growing hotter it is large and bright, is radiating energy rapidly, and is also storing up heat in its interior; while, on account of its low density, contraction by a given percentage of its radius liberates a relatively small amount of gravitational energy. It will therefore pass through these stages with relative rapidity. Its passage through its maximum temperature will obviously be somewhat slower. During the cooling stages its surface is relatively small and its rate of radiation slow; it is dense, and a given percentage of contraction liberates a large amount of energy, and the great store of heat earlier accumulated in its interior is coming out again. It must therefore remain in these stages for very much longer intervals of time, especially in the later ones, when the rate of radiation is very small.

This reproduces, in its general outlines, just what is observed—the relative rarity of giant stars, the somewhat greater abundance of those of Class A near the maximum of temperature, and the rapidly increasing numbers of dwarf stars of smaller and smaller brightness. The well-known scarcity of stars of Class B, per unit of volume, is further accounted for if we believe, as has been already explained, that only the most massive stars reach this stage.

In this connection we will very probably be asked, What precedes or follows Class M in the proposed evolutionary series, and why do we not see stars in still earlier or later stages? With regard to the latter, it is obvious that dwarf stars still fainter than the faintest so far observed (which are of Class M) would, even if among our very nearest neighbours, be apparently fainter than the tenth magnitude. We cannot hope to find such stars until a systematic search has been made for very large proper-motions among very faint stars. The extreme redness of such stars would, unfortunately, render such a search by photographic methods less profitable than in most cases.

But a giant star of Class M, a hundred times as bright as the sun, certainly cannot spring into existence out of darkness. In its earlier stages it must have radiated a large amount of energy, though perhaps less than at present. But as the temperature of a radiating body falls below 3000° C., the energy-maximum in its spectrum moves far into the infra-red, leaving but a beggarly fraction of the whole radiation in the visible region. Stars in such stages

[25] In the foregoing presentation of the theory, to avoid interference with the progress of the main argument, no mention has been made of certain considerations which should be discussed here.

(1) It is probable that at stellar temperatures the gaseous matter is very considerably dissociated and ionised. But this will not affect its gaseous nature. For our present purpose it amounts to little more than a diminution of the mean molecular weight. This will lower the temperature corresponding to a given density and pressure, and so tend to lower the maximum attainable temperature; but as the degree of dissociation is likely to vary gradually with the temperature, it should not affect the orderly sequence of changes which form the basis of the previous arguments.

(2) It is also probable that the available potential energy of a star is not entirely gravitational, but partly, if not mainly, of radio-active or similar atomic origin. If, as in the relatively very small range accessible to experimental investigation, the rate of liberation of this energy is independent of the temperature and pressure, it would simply supply a constant annual addition to the energy derived from gravitational contraction, and the only difference in the course of events would be that a star, on cooling, would approach, not complete extinction, but a steady state, of very long duration, in which as much energy was annually radiated away as was supplied by atomic disintegration. If the rate of disintegration is increased under the extremely violent molecular collisions which must occur in the interior of a star, a great liberation of energy may occur when the interior has got hot enough, thus increasing the maxima temperature and prolonging its duration. But, even on this hypothesis, the number of the violent collisions which liberate the atomic energy would increase gradually as the temperature of the interior rose, and the general character of the evolutionary changes, including the relation of the mass and density of the body to the time of their occurrence, would not be radically altered.

It seems, therefore, probable that the previous reasoning would require no essential modification on account of either of these factors in the problem.

[26] Hertzsprung, *Zeitschrift für Wissenschaftliche Photographie*, vol. iii, p. 442, 1905.

would therefore emit much less light than they would do later, and stand a poor chance of being seen.[27] We know, as yet, very little about the colour-index and temperature of stars of those varieties of Class M (Mb and Mc) which are evidently furthest along in the spectral series, and it may well be that a star usually reaches the temperature corresponding to these stages by the time that it begins to shine at all brightly. In any case, stars in these very early stages should be of small or moderate luminosity, and rare per unit of volume, and hence very few of them would be included in our catalogues.

The great luminosity and extreme redness of the stars of Class N suggest that they belong at the beginning of the series of giant stars; but the relations of this very distinct spectral type to the others are not yet quite clear, and it would be premature to give it a definitive place in the sequence. It seems clear, however, that these stars must be in a very primitive condition, rather than in a very late one, as believed by Lockyer. The stars of Class O (Wolf-Rayet stars) are of very great average luminosity, and probably lie beyond those of Class B at the apex of the temperature scale, as Lockyer supposes. But in the absence of data concerning their masses, densities, and the like, we cannot place them definitively, except that Oe5 and Oe come almost certainly just above Bo.

One further application of the theory may be very briefly mentioned. If we have a large number of contracting masses of gas endowed with various moments of momentum, more and more of them will split up into pairs as they grow denser, and the pairs latest formed will have the shortest periods. A large percentage of spectroscopic binaries, especially of short period, is therefore direct evidence of a fairly advanced state of evolution, and the occurrence of this condition among the stars of Classes B and A supports—indeed, almost by itself compels—the view that they are far removed from a primitive condition. Most of the stars which have been investigated for radial velocity are giants, and the absence of spectroscopic binaries of short period among the redder stars is in agreement with the view that they are in earlier stages of evolution.

The distribution of the visual binaries and physical pairs among the various spectral classes depends mainly upon a quite different factor, namely, the resolving power of our telescopes, which allows us to separate the closer pairs of short period only among the nearer stars, so that the systems for which orbits have been determined are nearly all dwarf stars.

I have endeavoured in the past hour to set before you the present state of knowledge concerning the real brightness, masses, densities, temperatures, and surface brightness of the stars, and to sketch the theory of stellar evolution to which the study of these things has led me. This theory is inconsistent with the generally accepted view. Its fundamental principle is identical with that of Lockyer's classification, but it differs radically from the latter in the principles according to which it assigns individual stars, and even whole classes of stars, to the series of ascending or descending temperature. (For example, Lockyer puts such conspicuously giant stars as Canopus, Capella, Arcturus, and β Cygni, and all the stars of Class N, into the descending series, and places β Hydri and δ Pavonis (which are clearly dwarf stars) in the ascending series.)

Two things have gone farthest to convince me that it may be a good approximation to the truth—the way in which it explains and coordinates characteristics of the different spectral types which previously appeared to be without connection or reason, and the way in which a number of apparent exceptions to its indications have disappeared, one by one, as more accurate information concerning spectra, orbits of double stars, and the like, became available, until only one doubtful case remains.

I have purposely made no attempt at this time to touch upon certain other interesting matters, such as the difference of the mean peculiar velocities of stars of the various spectral classes, although, with the aid of simple and very reasonable assumptions, they may be added to the list of things explainable by the new theory. My reason for this has been less for lack of time than because there is at present no definite reason, assignable from general considerations in advance, why we should expect an old star to be moving faster or slower than a younger one, while there is such a reason why we should suppose that a dense star is in a later stage of evolution than one of low density. It seems better to find out what we can about the order of evolution from data of the latter kind, and then apply our results to the study of problems of the former sort, than to attack them without such aid or by means of unproved assumptions. The assumptions that are necessary on the new theory are simple and probable enough, but they do not form an integral part of the theory, and cannot be established directly from general considerations, and so I will not discuss them now.

The new theory will not explain everything about the stars—I should be rather afraid of it if it did; for example, it leaves the phenomena of preferential motion, or "star-streaming," as puzzling as ever. I have only tried to interpret some of the facts most obviously capable of evolutionary explanation, on the fundamental assumption that the properties of matter, and the forces in operation, among the stars, are the same as those with which we are familiar in the laboratory. He would be a bold man indeed who would assert that this assumption is entirely true, but it seems clear that it should be thoroughly tried out before the existence of new forces can legitimately be postulated.

If the ideas to which you have so kindly listened to-day shall prove of any help toward removing the need for belief in unknown forces, and extending the domain of those already known, my labour will be far more than repaid; but it should not be forgotten that the real labourers have been those who, through long and weary nights, accumulated bit by bit, and, through monotonous days, prepared for the use of others the treasures of observational knowledge with which it has been my pleasurable lot to play in the comfort of my study.

I need scarcely add that, if what I have said proves of interest to any of you, your frank and unsparing criticism will be the greatest service which you can render me.

[27] Russell, *Science*, N.S. Vol. xxxvii, p. 646, 1913.

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

BIRMINGHAM.—At its last meeting the University council passed the following resolution:—"That the council desires to record its deep sorrow at the death of Prof. Poynting, who so faithfully served the Mason College and the University for thirty-four years. During his distinguished career as professor of physics he was not only an inspiring teacher and investigator, but bore a considerable part in the development of the college and of the University. His keen interest in all that concerned the University, its staff, and its students, his genial and attractive personality, will be long and affectionately remembered; his death leaves a gap which it will be most difficult to fill."