$$\text{mean free path } L = \frac{1}{\pi\,(2r)^2\,n}$$

$$\text{before approach } E = KE_A + KE_B + GPE = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 - \frac{Gmm}{\infty}$$

$$E_{\text{initial}} = mv^2$$

$$\text{during approach } E = KE_A + KE_B + GPE = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 - \frac{Gmm}{R_{\min}}$$

$$\frac{Gm^2}{R_{\min}} = \frac{1}{2}\,E_{\text{initial}}$$

$$R_{\min} = \frac{2Gm}{v^2}$$

$$\text{duration of force } \Delta t = \frac{b}{v}$$

$$\text{size of force } F = \frac{Gm_1 m_2}{b^2}$$

$$\text{impulse} \quad m_1\,\Delta v_y = F\,\Delta t = \frac{Gm_1 m_2}{b^2}\left(\frac{b}{v}\right)$$

$$\Delta v_y = \frac{Gm_2}{b\,v}$$

$$\text{distance travelled } \Delta y = \text{(velocity) (time)} = \left(\frac{Gm_2}{b\,v}\right)\left(\frac{b}{v}\right)$$

$$\Delta y = \frac{Gm_2}{v^2}$$

$$2\,\Delta y \ll b \quad \Rightarrow \quad \frac{2\,Gm_2}{v^2} \ll b$$

$$\text{volume } V = \text{(length) (area of ring)} = (v\,t)\,(2\pi b\,db)$$

$$N = V\,n = (2\pi\,vt\,b\,db)\,(n)$$

$$\text{contribution at impact parameter } b \quad = \quad (\text{number of encounters }) \; \left(\Delta {v_y}^2\right)$$

$$= \quad (2\pi \, vt \, n \, b \, db) \; \left(\frac{Gm}{vb}\right)^2$$

$$\text{total } |(\Delta v_y)^2| \quad = \quad \int_{b_{\min}}^{b_{\max}} (2\pi \, vt \, n \, b \, db) \; \left(\frac{Gm}{vb}\right)^2$$

$$\text{total } |(\Delta v_y)^2| \quad = \quad \left(\frac{2\pi G^2 m^2 n}{v}\right) \; t \; \int_{b_{\min}}^{b_{\max}} \frac{db}{b}$$

$$\text{total } |(\Delta v_y)^2| \quad = \quad \left(\frac{2\pi G^2 m^2 n}{v}\right) \; t \; \left(\ln \frac{b_{\max}}{b_{\min}}\right)$$

$$\Lambda \quad \equiv \quad \left(\frac{b_{\max}}{b_{\min}}\right)$$

$$\ln \Lambda \quad \equiv \quad \ln \left(\frac{b_{\max}}{b_{\min}}\right)$$

$$|(\Delta v_y)^2| \quad = \quad |v^2|$$

$$v^2 \quad = \quad \left(\frac{2\pi G^2 m^2 n}{v}\right) \; t \; \ln \Lambda$$

$$t_{\text{relax}} \quad = \quad \frac{v^3}{2\pi \, G^2 m^2 \, n \, \ln \Lambda}$$