$$N_p = \left(\frac{4\pi}{3}\,R^3\right)\; n_p$$

$$N_{\rm recomb} = \frac{N_p}{t_r} = \frac{\left(\frac{4\pi}{3}\,R^3\right)\; n_p}{t_r}$$

$$N_{\rm UV} = N_{\rm recomb} = \frac{\left(\frac{4\pi}{3}\,R^3\right)\; n_p}{t_r}$$

$$R_{eq} = \left(\frac{3\,N_{\rm UV}}{4\pi n_p}\right)^{1/3}\; (t_r)^{1/3}$$

$$\text{volume swept} = \sigma v_e t_r$$

$$\text{volume per electron} = \frac{1}{n_e}$$

$$\sigma v_e t_r = \frac{1}{n_e}$$

$$t_r = \frac{1}{\sigma v_e n_e}$$

$$v_e = \sqrt{\frac{3kT}{m_e}}$$

$$\sigma \simeq 5.9\times 10^{-25}\ \mathrm{m}^2$$

$$\sigma_{\rm ionize} \simeq 6\times 10^{-22}\ \mathrm{m}^2$$

$$\text{mfp of ionizing photon} = \frac{1}{\sigma_{\rm ionize}\; n_H}$$

$$R_{\rm Stromgren} = \left(\frac{3\,N_{\rm UV}}{4\pi}\right)^{1/3}\left(\frac{1}{\sigma v_e}\right)^{1/3}\; n_p^{-2/3}$$