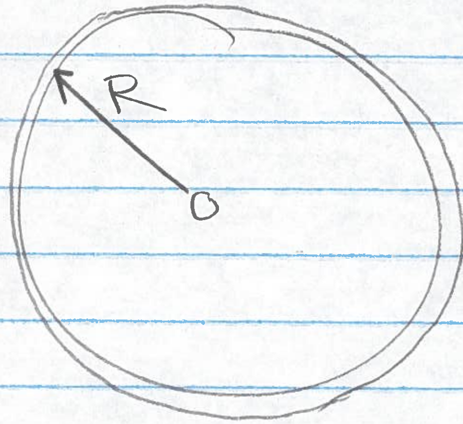
HW 6 #1

A star emits luminosity $L = 11.2 L_\odot$. A distance $R = 90$ AU a ring of dust grains orbits.



Each grain is spherical w/ radius $a = 2.4$ mm

The material has density $\rho = 1325$ kg/m³.

What is equilibrum temperature of dust?

$$T = \left[\frac{L}{16\pi R^2 \sigma}\right]^{1/4}$$

$$= \left[\frac{11.2 * 3.83\times10^{26}\ \mathrm{J/s}}{16\pi\left[90 * 1.5\times10^{11}\,\mathrm{m}\right]^2\left(5.67\times10^{-8}\,\frac{\mathrm{J}}{\mathrm{s\cdot m^2\cdot K^4}}\right)}\right]$$

$$= 54\ \mathrm{K}$$

The peak wavelength of blackbody emission is

$$\lambda_{peak} = \frac{0.0029\ \mathrm{m\cdot K}}{T} = 5.4\times10^{-5}\ \mathrm{m}$$

$$= 5.4\ \mu\mathrm{m}$$

The total luminosity of the dust is

$$L_d = 1.593\times10^{24}\ \frac{\mathrm{J}}{\mathrm{s}}$$

What is the mass of all the dust grains?

#1, continued

The total luminosity of the grains must be

$$L_d = (\text{area of all grains}) \cdot \sigma T^4$$

$$= (4\pi a^2 \cdot N) \cdot \sigma T^4$$

where N = number of grains. We can solve for N:

$$N = \frac{L_d}{4\pi a^2 \sigma T^4}$$

$$= \frac{1.593 \times 10^{24} \ \frac{J}{s}}{4\pi \left(2.4 \times 10^{-3} \, m\right)^2 \left(5.67 \times 10^{-8} \ \frac{J}{s \cdot m^2 \cdot K^4}\right) (54 \, K)^4}$$

$$= 4.7 \times 10^{31} \text{ grains}$$

So the mass of dust is

$$M = \left(\frac{4}{3}\pi a^3 \cdot \rho\right) \cdot N$$

$$= \frac{4}{3}\pi \left(2.4 \times 10^{-3} \, m\right)^3 \left(1325 \ \frac{kg}{m^3}\right) \left(4.7 \times 10^{31} \text{ grains}\right)$$

$$= 3.6 \times 10^{27} \, kg \cdot \frac{1 \, M_E}{5.98 \times 10^{24} \, kg}$$

$$= \sim 600 \, M_E$$