

HW 6

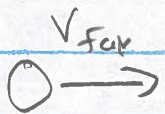
#4

Earth has mass $M_E = 5.98 \times 10^{24}$ kg and radius $R = 6.37 \times 10^6$ m. Assume it has uniform density. What is its gravitational binding energy?

$$GBE = -\frac{3}{5} \frac{GM_E^2}{R_E} = -2.25 \times 10^{32} \text{ J}$$

Asteroid moves toward Earth from very far away, with $V_{\text{far}} = 30,200$ m/s.

BEFORE



$$r \approx \infty, \Rightarrow GPE \approx 0$$

$$E_{\text{tot}} = \frac{1}{2} m V_{\text{far}}^2 + \frac{GM_E m}{\infty} - \frac{3}{5} \frac{GM_E^2}{R_E}$$

(E)

In order for the collision to break up the Earth and scatter the pieces to infinity,

$$E_{\text{tot}} \geq 0 \Rightarrow \frac{1}{2} m V_{\text{far}}^2 \geq \frac{3}{5} \frac{GM_E^2}{R_E}$$

We can compute the mass of asteroid, m , required to cause total energy to be positive.

$$m \geq \left(\frac{6}{5} \frac{GM_E^2}{R_E} \right) \left(\frac{1}{V_{\text{far}}^2} \right)$$

$$\geq (4.5 \times 10^{32} \text{ J}) \left(\frac{1}{(30,200 \text{ m/s})^2} \right)$$

$$\geq 4.9 \times 10^{23} \text{ kg} \approx 0.08 M_E$$

#4 continued

Suppose the density of the asteroid is

$$\rho = 2010 \frac{\text{kg}}{\text{m}^3}$$

Then the minimum size of an Earth-destroying asteroid is

$$\frac{4}{3} \pi R_{\text{ast}}^3 \cdot \rho = 4.9 \times 10^{23} \text{ kg}$$

$$\rightarrow R_{\text{ast}} = \left[\frac{4.9 \times 10^{23} \text{ kg}}{\frac{4}{3} \pi (2010 \text{ kg/m}^3)} \right]^{\frac{1}{3}}$$

$$= 3.9 \times 10^6 \text{ m}$$

There are no asteroids this large, nor this massive:

most massive

Eris	$1.7 \times 10^{22} \text{ kg}$	) Kuiper Belt
Pluto	$1.3 \times 10^{22} \text{ kg}$	
----- Ceres	$9.4 \times 10^{20} \text{ kg}$	) main belt

largest size

Pluto	$2.39 \times 10^6 \text{ m}$	) Kuiper Belt
Eris	$2.33 \times 10^6 \text{ m}$	
----- Ceres	9.39×10^5	) main belt

Nothing in inner solar system is even close to big enough.