HW 6 #4

Earth has mass $M_E = 5.98 \times 10^{24}$ kg and radius $R = 6.37 \times 10^{6}$ m. Assume it has uniform density. What is its gravitational binding energy?

$$GBE = -\frac{3}{5}\frac{GM_E^2}{R_E} = -2.25 \times 10^{32}\ \text{J}$$

Asteroid moves toward Earth from very far away, with $V_{far} = 30{,}200$ m/s.

BEFORE

V_{far}

$$r \approx \infty, \Rightarrow GPE \approx 0$$

E

$$E_{tot} = \frac{1}{2} m V_{far}^2 + \cancel{\frac{GM_E m}{\infty}} - \frac{3}{5}\frac{GM_E^2}{R_E}$$

In order for the collision to break up the Earth and scatter the pieces to infinity,

$$E_{tot} \geq 0 \Rightarrow \frac{1}{2} m V_{far}^2 \geq \frac{3}{5}\frac{GM_E^2}{R_E}$$

We can compute the mass of asteroid, m, required to cause total energy to be positive.

$$m \geq \left(\frac{6}{5}\frac{GM_E^2}{R_E}\right)\left(\cdot\frac{1}{V_{far}^2}\right)$$

$$\geq \left(4.5 \times 10^{32}\ \text{J}\right)\left(\frac{1}{30{,}200\ \text{m/s}}\right)^2$$

$$\geq 4.9 \times 10^{23}\ \text{kg} \approx 0.08\, M_E$$

#4 continued

Suppose the density of the asteroid is

$$\rho = 2010 \frac{kg}{m^3}$$

Then the minimum size of an Earth-destroying asteroid is

$$\frac{4}{3}\pi R_{ast}^3 \cdot \rho = 4.9 \times 10^{23} \, kg$$

$$\longrightarrow \quad R_{ast} = \left[\frac{4.9 \times 10^{23} \, kg}{\frac{4}{3}\pi \left(2010 \, kg/m^3\right)} \right]^{1/3}$$

$$= 3.9 \times 10^6 \, m$$

There are no asteroids this large, nor this massive:

most massive

Eris	1.7×10^{22} kg	Kuiper Belt
Pluto	1.3×10^{22} kg	
Ceres	9.4×10^{20} kg	main belt

largest size

Pluto	2.39×10^6 m	Kuiper Belt
Eris	2.33×10^6 m	
Ceres	9.39×10^5	main belt

Nothing in inner solar system is even close to big enough.