HW #5, Prob 1

How thick is the layer of neutral H outside an HII region? To a rough approximation, it should be ≈ 1 mean free path of a photon.

Given

$$\text{density } n = 9020 \text{ atom/cm}^3$$
$$= 9020 \times 10^6 \text{ atom/m}^3$$

The cross section for ionizing photons in ISM is shown on a figure:

$$\sigma = 6 \times 10^{-18} \text{ cm}^2$$
$$= 6 \times 10^{-22} \text{ m}^2$$

So

$$\text{mean free path } L = \frac{1}{n \cdot \sigma}$$
$$= \frac{1}{(9020 \times 10^6 \frac{1}{m^3})(6 \times 10^{-22} \text{ m}^2)}$$

$$\boxed{L \approx 1.85 \times 10^{11} \text{ m}}$$

If the HII region has a radius $R = 13.0$ pc, how thick is the layer of HI compared to it?

$$L = 1.85 \times 10^{11} \text{ m}$$
$$R = 13.0 \text{ pc} * \frac{3.08 \times 10^{16} \text{ m}}{\text{pc}} = 4.0 \times 10^{17} \text{ m}$$

$$\rightarrow \frac{L}{R} \sim 5 \times 10^{-7} \quad \text{very thin!}$$

HW #5, Prob 2

How large is HII region around the Sun? We adopt

$$N_{uv} = 2.9 \times 10^{35} \ \frac{\text{ionizing photon}}{\text{sec}}$$

density of gas $\quad n = 91 \ \frac{\text{atom}}{\text{cm}^3} = 91 \times 10^6 \ \frac{\text{atoms}}{\text{m}^3}$

temp of gas $\quad T = 10^4$ K

In these circumstances,

speed of electrons $\quad V_e = 6.7 \times 10^5$ m/s

cross section recomb $\quad \sigma = 5.9 \times 10^{-25}$ m^2

Stromgren radius is

$$R_s = \left[\frac{3 N_{uv}}{4\pi}\right]^{1/3} \left(\frac{1}{\sigma V_e}\right)^{1/3} n^{-2/3}$$

$$= 2.77 \times 10^{12} \text{ m} \cong 18 \text{ AU}$$

HW #5, Prob 3

The cross-section for electron-proton recombination is

$$\sigma = 5.9 \times 10^{-25} \ m^2$$

If protons and electrons were hard little spheres, the cross section of a proton would be πr^2 — where "r" is a bogus radius. What would it be?

$$\text{"bogus"} \quad r = \sqrt{\frac{\sigma}{\pi}} = 4.3 \times 10^{-13} \ m$$
$$= 430 \ fm$$

But experiments with particle accelerators show that the actual radius of a proton is approximately

$$\text{actual} \quad r \approx 0.8 \ fm$$

HW #5, Problem 4

Based on measurements of H-alpha emission from galaxy UGC 8460, we can compute a number of its properties.

a) rest wavelength $H\alpha = 656.3$ nm

b) the max shift in wavelength - from graph - is

$$\Delta\lambda_{max} = 0.3 \text{ nm}$$

$$\Rightarrow \frac{v}{c} = \frac{\Delta\lambda_{max}}{\lambda} = \frac{0.3 \text{ nm}}{656.3 \text{ nm}} = 4.57 \times 10^{-4}$$

$$\Rightarrow V_{max} = 4.57 \times 10^{-4}\, c = 1.37 \times 10^{5} \text{ m/s}$$

c) max angular distance of gas

$$\theta_{max} \simeq 40'' = 0.011 \text{ degrees}$$

d) the galaxy is at a distance of $D = 90.1$ Mpc. The linear separation corresponding to θ_{max} is

$$L = D \cdot \tan\theta_{max} = 1.92 \times 10^{-4}\,(90.1 \text{ Mpc})$$

$$= 17.3 \text{ kpc}$$

$$= 5.33 \times 10^{20} \text{ m}$$

HW #5, Problem 4 continued

Now we can compute the mass of the galaxy – at least, out to the radius at which we can see the gas.

$$\frac{V^2}{R} = \frac{GM}{R} \Rightarrow M = \frac{V^2 R}{G}$$

$$M = \frac{\left(1.37 \times 10^5 \ m/s\right)^2 \left(5.33 \times 10^{20} \ m\right)}{6.67 \times 10^{-11} \ N \cdot m^2/kg^2}$$

$$= 1.5 \times 10^{41} \ kg = 7.5 \times 10^{10} \ M_\odot$$