HW 4, #1

A cloud of pure H_2 has

$$n = 1750 \text{ mol}/\text{cm}^3 = 1.75 \times 10^9 \text{ mol}/\text{m}^3$$
$$T = 20 \text{ K}$$

Approximate a hydrogen molecule as a sphere of diameter

$$d = 1.6 \times 10^{-10} \text{ m}$$

The mean free path is

$$mfp = \frac{1}{\pi d^2 n}$$

$$= 7.1 \times 10^9 \text{ m}$$

In AU

$$mfp = \frac{7.1 \times 10^9 \text{ m}}{1.496 \times 10^{11} \text{ m/AU}} = 0.0475 \text{ AU}$$

The mass of one H_2 molecule is $2 m_H = 2 \cdot 1.67 \times 10^{-27}$ kg

So

$$V = \sqrt{\frac{3kT}{(2m_H)}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(20 \text{ K}}{2 * 1.67 \times 10^{-27} \text{ kg}}}$$

$$= 498 \text{ m/s}$$

Time between collisions is

$$t = \frac{mfp}{V} = \frac{7.1 \times 10^9 \text{ m}}{498 \text{ m/s}} = 1.42 \times 10^7 \text{ s}$$

HW 4, #2

From the Figure in lecture showing surface density of molecular gas as a function of galactocentric radius, let's compute the total mass of H_2 in the inner Milky Way.

inner radius = 2 kpc = 2000 pc
outer radius = 13 kpc = 13,000 pc

The average surface density of H_2 is about

$$\sigma_{avg} \approx 2.5\ M_\odot \text{ per sq. parsec}$$

So the total mass of H_2 must be

$$M = \pi \left(R_{outer}^2 - R_{inner}^2 \right) \sigma_{avg}$$

$$= \pi \left(13{,}000^2\ pc - 2000^2\ pc \right) 2.5 \frac{M_\odot}{pc^2}$$

$$= 1.3 \times 10^9\ M_\odot$$

HW 4, #3

Read values of chemical abundances from a graph shown in class.

element	reading on graph	abundance rel to H *
sodium = Na	≈ 6.2	1.6×10^{-6}
iron = Fe	≈ 7.5	3.2×10^{-5}
copper = Cu	≈ 4.3	2.0×10^{-8}
uranium = U	≈ -0.5	3.2×10^{-13}

* note the method to compute abundance:

$$\text{abundance rel to H} = 10^{[\text{element's reading} - \text{H reading}]}$$

$$= 10^{[\text{element's reading} - 12.0]}$$

Hw 4, #4

Compute the properties of some rotational transitions of the CO molecule. The energy of the rotational state "J" is

$$E_J = B_o \left(J[J+1] \right) \qquad J = 0, 1, 2, \ldots$$

where

$$B_o = 0.00023825 \text{ eV}$$

$J = 1 \to 0$ $\quad \Delta E = 4.765 \times 10^{-4}$ eV

$$\lambda = 1240 \text{ eV.nm} / \Delta E = 2.60 \times 10^{6} \text{ nm}$$

$$= 2.60 \text{ mm}$$

$$\nu = \frac{c}{\lambda} = 1.15 \times 10^{11} \text{ Hz} = 115 \text{ GHz}$$

$J = 2 \to 1$ $\quad \Delta E = B_o \{ 6 - 2 \} = 9.53 \times 10^{-4}$ eV

$$\lambda = 1.30 \times 10^{+6} \text{ nm} = 1.30 \text{ mm}$$

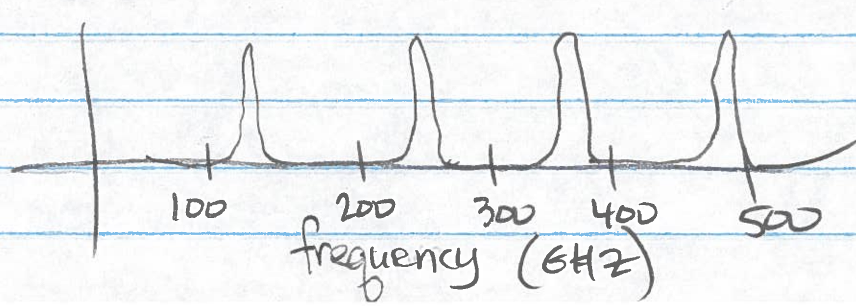
$$\nu = 2.30 \times 10^{11} \text{ Hz} = 230 \text{ GHz}$$

$J = 3 \to 2$ $\quad \Delta E = B_o \{ 12 - 6 \} = 1.43 \times 10^{-3}$ eV

$$\lambda = 8.67 \times 10^{5} \text{ nm} = 0.867 \text{ mm}$$

$$\nu = 3.46 \times 10^{11} \text{ Hz} = 346 \text{ GHz}$$

Note that emission lines appear at regular intervals of frequency: hence the name "bands".



HW 4, #5

CO is a useful tracer of <u>cold</u> H_2 clouds because very little energy is needed to excite CO into a state from which it emits photons.

Consider rotational $J=1 \rightarrow J=0$ transition.

a) how much energy to excite into $J=1$?

$$E = 0.00023825 \text{ eV} \{(1)(2) - (0)(1)\}$$
$$= 4.765 \times 10^{-4} \text{ eV}$$
$$= 7.67 \times 10^{-23} \text{ J}$$

Suppose a CO molecule sits in a cloud at $T = 11$ K. The typical kinetic energy of a CO molecule is

b) $$KE = \frac{3}{2} kT = 2.277 \times 10^{-22} \text{ J}$$
$$= 1.42 \times 10^{-3} \text{ eV}$$

Yes enough energy to excite to $J=1$

c) <u>Yes</u>, a typical CO-CO collision can excite a molecule into the $J=1$ state.

Could photons from the CMB excite a CO molecule? CMB has a blackbody spectrum.

d) temp of CMB $T = 2.73$ K

e) at spectrum's peak, $\lambda = \frac{0.00290 \text{ m}\cdot\text{K}}{2.73 \text{ K}}$
$$= 1.06 \text{ mm}$$

$$\Rightarrow E = \frac{hc}{\lambda} = 1.17 \times 10^{-3} \text{ eV}$$