$$P_{\rm rad} \quad = \quad \frac{4}{3c}\,\sigma T^4 \quad = \quad \frac{1}{3}\,aT^4$$

$$\frac{dP_{\rm rad}}{dr} \quad = \quad \frac{1}{3c}\,\sigma T^3\,\frac{dT}{dr} \quad = \quad \frac{4}{3}\,aT^3\,\frac{dT}{dr}$$

$$\frac{dP_{\rm rad}}{dr} \quad = \quad -\frac{\bar\kappa\rho}{c}\,F_{\rm rad}$$

$$-\frac{\bar\kappa\rho}{c}\,F_{\rm rad} \quad = \quad \frac{4}{3}\,aT^3\,\frac{dT}{dr}$$

$$F_{\rm rad} \quad = \quad \frac{L_r}{4\pi r^2}$$

$$-\frac{\bar\kappa\rho}{c}\,\frac{L_r}{4\pi r^2} \quad = \quad \frac{4}{3}\,aT^3\,\frac{dT}{dr}$$

$$\frac{dT}{dr} \quad = \quad -\frac{3\bar\kappa\rho L_r}{16\pi a\,c\,r^2\,T^3}$$

$$PV \quad = \quad NkT \qquad \Rightarrow \qquad P \quad = \quad nkT \quad = \quad \frac{\rho}{\bar m}\,kT$$

$$P \quad = \quad \frac{k}{\bar m}\,\rho T \quad , \quad T \quad = \quad \frac{\bar m}{k}\,\frac{P}{\rho} \quad , \quad \rho \quad = \quad \frac{\bar m}{k}\,\frac{P}{T}$$

$$\frac{dP}{dr} \quad = \quad \frac{k}{\bar m}\,\frac{d\rho}{dr}\,T \;+\; \frac{k}{\bar m}\,\rho\,\frac{dT}{dr}$$

$$\frac{dP}{dr} \quad = \quad \frac{P}{\rho}\,\frac{d\rho}{dr} \;+\; \frac{P}{T}\,\frac{dT}{dr}$$

$$P \quad = \quad C\,\rho^\gamma$$

$$\frac{dP}{dr} \quad = \quad C\,\gamma\rho^{\gamma-1}\,\frac{d\rho}{dr} \quad = \quad \gamma C\,\frac{\rho^\gamma}{\rho}\,\frac{d\rho}{dr} \quad = \quad \gamma\,\frac{P}{\rho}\,\frac{d\rho}{dr}$$

$$\frac{P}{\rho}\,\frac{d\rho}{dr} \;+\; \frac{P}{T}\,\frac{dT}{dr} \quad = \quad \gamma\,\frac{P}{\rho}\,\frac{d\rho}{dr}$$

$$\frac{dT}{dr} \quad = \quad \frac{T}{P}\,\left[(\gamma-1)\,\frac{P}{\rho}\,\frac{d\rho}{dr}\right]$$

$$\frac{dT}{dr} \quad = \quad \frac{T}{P}\,\left[\gamma\,\frac{P}{\rho}\,\frac{d\rho}{dr} \;-\; \frac{P}{\rho}\,\frac{d\rho}{dr}\right]$$

$$\frac{dT}{dr} = \frac{T}{P}\left[\frac{dP}{dr} - \frac{1}{\gamma}\frac{dP}{dr}\right]$$

$$\frac{dT}{dr} = \frac{T}{P}\left(1 - \frac{1}{\gamma}\right)\frac{dP}{dr}$$

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

$$\frac{dT}{dr} = -\left(\frac{\bar{m}}{k}\right)\left(1 - \frac{1}{\gamma}\right)\frac{GM_r}{r^2}$$

$$\left|\frac{dT}{dr}\right|_{\mathrm{actual}} >= \left|\frac{dT}{dr}\right|_{\mathrm{adiabat}} = \left(\frac{\bar{m}}{k}\right)\left(1 - \frac{1}{\gamma}\right)\frac{GM_r}{r^2}$$

$$P \propto T^{\alpha} \quad \Rightarrow \quad \ln P = \alpha \ln T$$

$$\frac{d\ln P}{d\ln T} = \alpha$$

$$\frac{d\ln P}{d\ln T} = \frac{\gamma}{\gamma - 1}$$

$$\frac{d\ln P}{d\ln T} < \frac{\gamma}{\gamma - 1}$$