$$\begin{aligned} \mathrm{KE}_i \ + \ \mathrm{GPE}_i \quad &= \quad \mathrm{KE}_f \ + \ \mathrm{GPE}_f \\ \frac{1}{2}mv_i^2 \ + \ 0 \quad &= \quad 0 \ + \ mgH \end{aligned}$$

$$\Rightarrow \qquad H \quad = \quad \frac{v_i^2}{2g}$$

$$\begin{aligned} \mathrm{KE}_i \ + \ \mathrm{GPE}_i \quad &= \quad \mathrm{KE}_f \ + \ \mathrm{GPE}_f \\ \frac{1}{2}mv_i^2 \ - \ \frac{GM_Em}{R_i} \quad &= \quad 0 \ - \ \frac{GM_Em}{R_f} \\ \frac{1}{2}mv_i^2 \ - \ \frac{GM_Em}{R_E} \quad &= \quad 0 \ - \ \frac{GM_Em}{\infty} \end{aligned}$$

$$v_i \quad = \quad \sqrt{\frac{2GM_E}{R_E}}$$

$$v_{\mathrm{esc}} \quad = \quad \sqrt{\frac{2GM}{R}}$$

$$R_{BH} \quad = \quad \frac{2GM}{c^2}$$

$$\mathrm{GPE} \quad = \quad - \ \frac{GMm}{R}$$