Hw 3, #1

a) $1 \leq (B-R) \leq 2$ = "yellow" 6602 stars
$3 \leq (B-R) \leq 5$ = "red" 2930 stars

b) see graph "dr2_hr_15_shell.png"

c) Both sets of stars increase with distance as one might expect for uniformly distributed objects; the red stars are slightly more numerous. Rey's claim is supported more than Bob's.

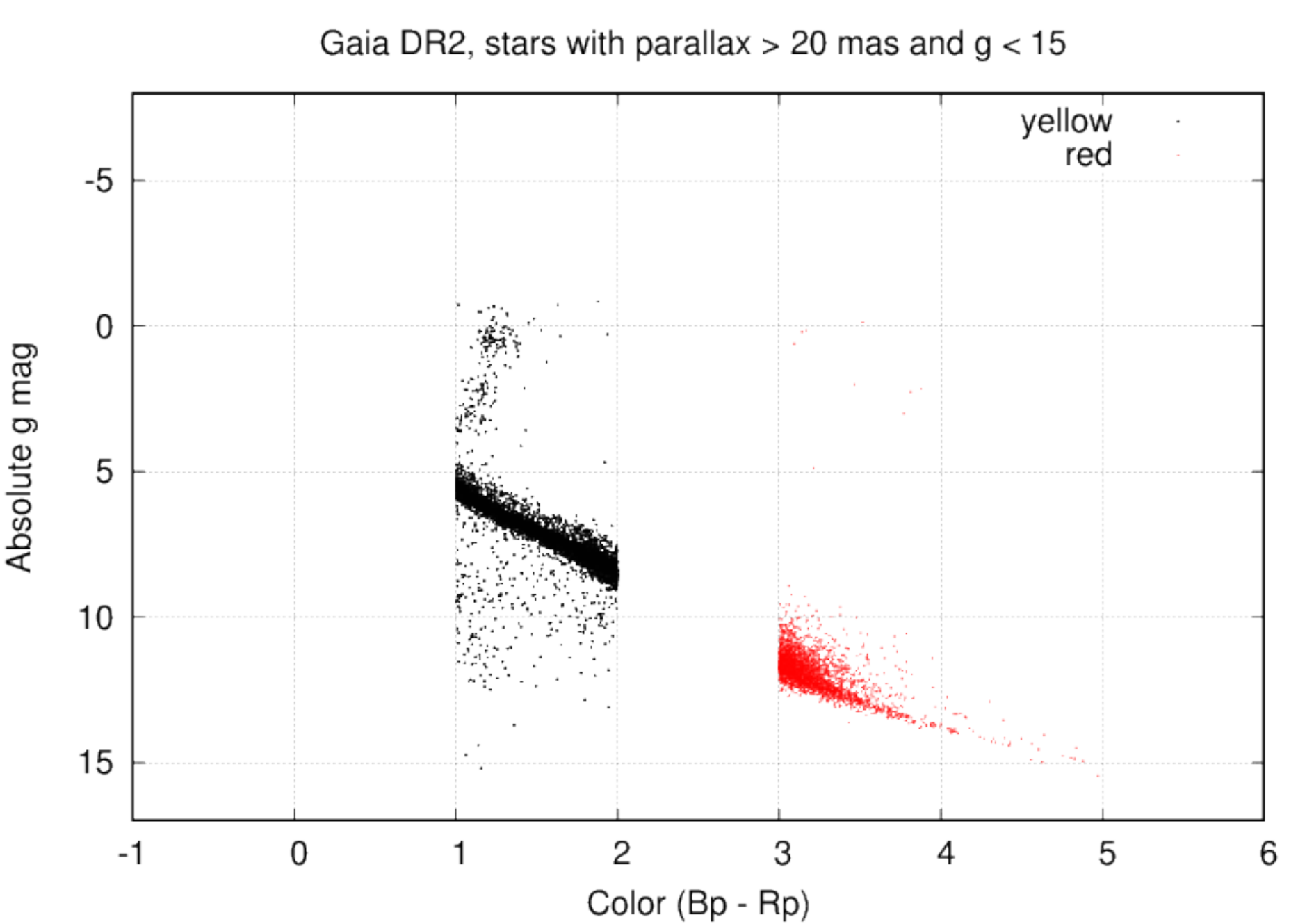
d) see graph "dr2_hr_50_shell.png". Here, the yellow stars continue to increase w/ distance, but the red stars decrease in number after 25 pc or so. It is likely that the red stars may still be present at $d > 25$ pc, but they are too faint to be detected.

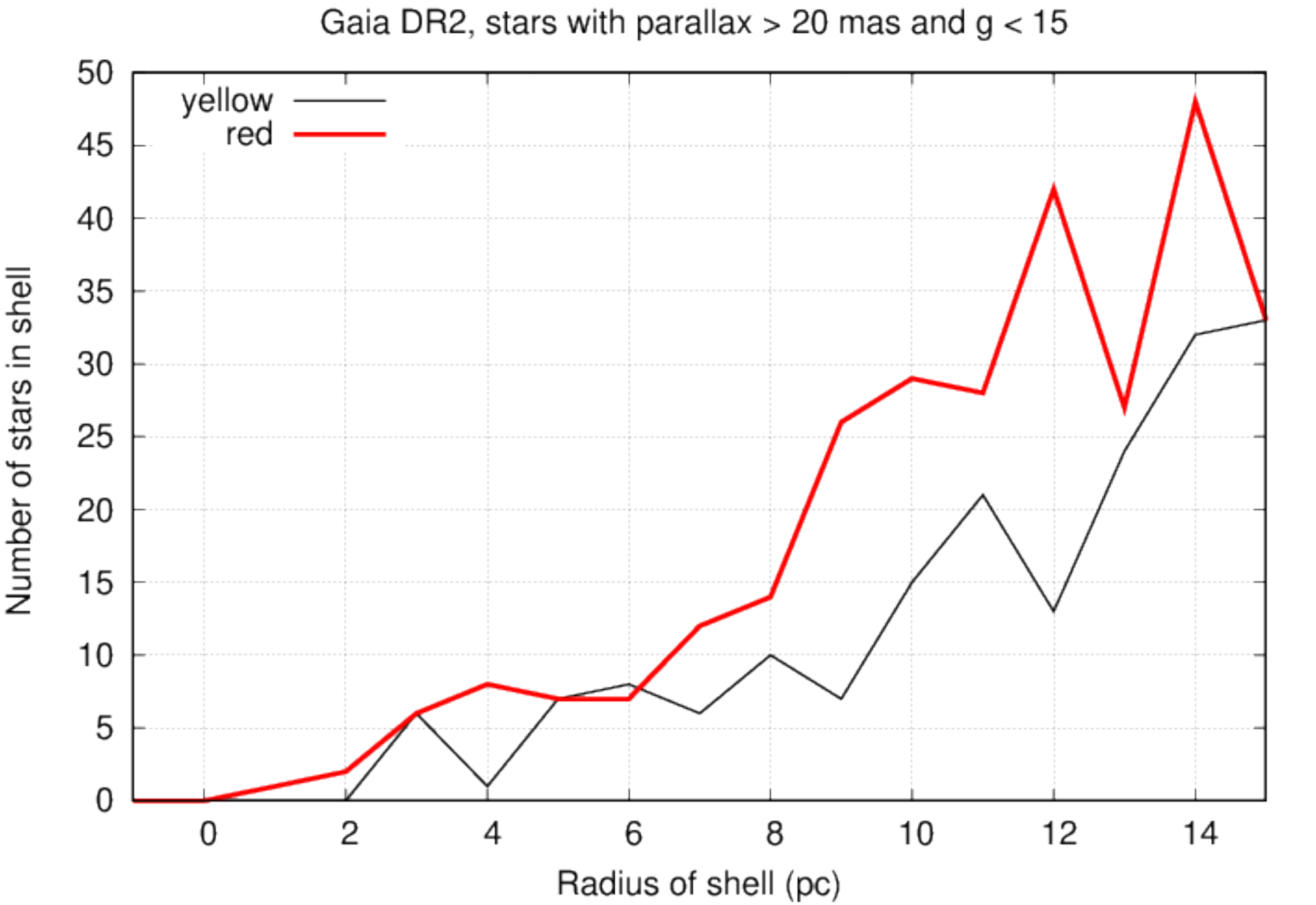
e) Consider red stars in the middle of the red range, at $(B-R) = 4$. They have absolute mag $M \approx 14$; so, at a distance of $d = 10$ pc, their apparent mag is also 14. But at a distance of 20 pc,

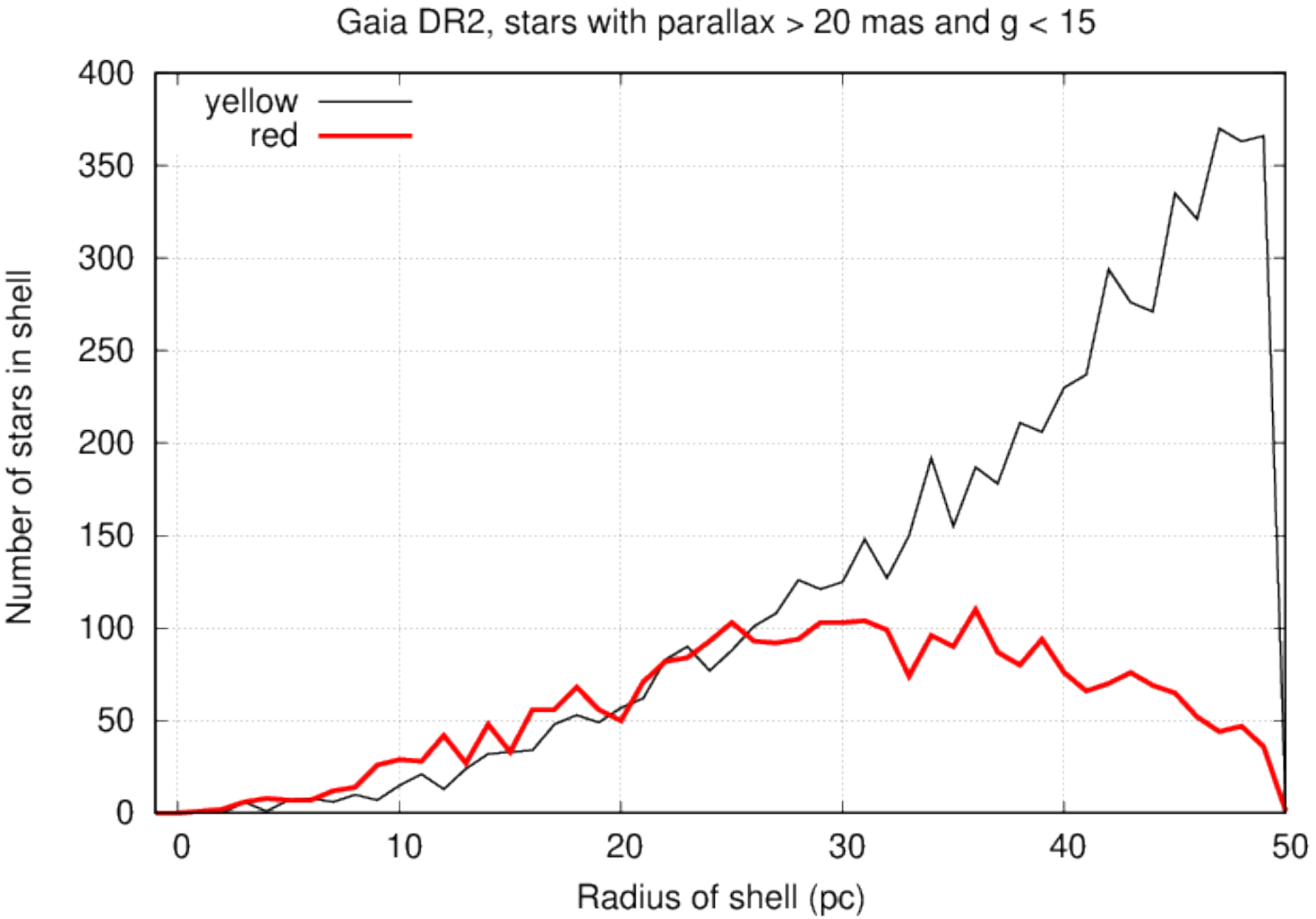
$$m - M = 5 \log 20 - 5 = 1.5$$
$$\Rightarrow m = M + 1.5 = 15.5$$

which moves them out of the sample. So, the red sample grows invisible at distances beyond 20–30 pc.

f) Bob's claim was false. The reason the sample has more yellow stars is because the yellow stars are more luminous and can be seen at greater distances. Red stars are more common in a volume-limited sample.


Gaia DR2, stars with parallax > 20 mas and g < 15
yellow
red
Absolute g mag
-5
0
5
10
15
-1
0
1
2
3
4
5
6
Color (Bp - Rp)


Gaia DR2, stars with parallax > 20 mas and g < 15
yellow
red
Number of stars in shell
Radius of shell (pc)
0
5
10
15
20
25
30
35
40
45
50
0
2
4
6
8
10
12
14


Gaia DR2, stars with parallax > 20 mas and g < 15
yellow
red
Number of stars in shell
Radius of shell (pc)
0
50
100
150
200
250
300
350
400
0
10
20
30
40
50

HW 3, # 2.

Given the solar core has

opacity $\kappa = 20\ m^2/kg$

density $\rho = 2 \times 10^4\ kg/m^3$

a) mean free path is

$$s = \frac{1}{\rho\kappa} = \frac{1}{20\ \frac{m^2}{kg} \cdot 2\times10^4\ \frac{kg}{m^3}}$$

$$= 2.5\times10^{-6}\ m$$

b) If this mfp were the same everywhere in the Sun, then in order to travel radius R from core to surface, the number of steps would be

$$N = \left(\frac{R}{s}\right)^2 = \left(\frac{6.95\times10^8\ m}{2.5\times10^{-6}\ m}\right)^2$$

$$= 7.7\times10^{28}\ \text{steps}$$

c) The journey would require a total time $T$

$$T = 7.7\times10^{28}\ \text{steps} \times \frac{2.5\times10^{-6}\ m}{3\times10^8\ \frac{m}{s}}$$

$$= 6.4\times10^{14}\ s \approx 2\times10^7\ yr$$

$$= 20 \text{ million years}$$

d) In a vacuum, the trip would take

$$T' = \frac{6.95\times10^8\ m}{3\times10^8\ \frac{m}{s}} = 2.3\ \text{sec.}$$

Bonus! Are there any natural materials on the surface of the Earth with densities greater than the Sun's core?

solar core $\rho = 2 \times 10^4 \frac{kg}{m^3} = 20 \frac{g}{cm^3}$

$= 20 \times$ water's density

The densest pure elements are the platinum-group elements:

| | |
|---|---|
| platinum | 21.45 $g/cm^3$ |
| iridium | 22.4 |
| osmium | 22.6 |

Rhenium is also above 20 $g/cm^3$, as is neptunium - but Np does not occur in macroscopic quantities naturally.

Since platinum does occur naturally (if rarely) as solid nuggets, it should qualify as "naturally-found material denser than Solar core's material."

HW 3, #3

a) Soot particles are spheres of diameter $d = 1 \times 10^{-6}$ m and density $\rho = 100\ kg/m^3$. Cross section area $\sigma$ is

$$\sigma = \pi \left(\frac{d}{2}\right)^2 = 7.85 \times 10^{-13}\ m^2$$

b) The particles fill the air with a density $n = 10^{10}\ \frac{particles}{m^3}$.

The mean free path of a photon is

$$mfp = \frac{1}{n\sigma} = \frac{1}{10^{10}\ \frac{part}{m^3} \cdot 7.85 \times 10^{-13}\ m^2/part}$$

$$= 127\ m$$

c) Ranger Rick's flashlight emits 2% of its 10 watts in optical photons. Assume all such photons are $\lambda = 550$ nm. The threshold for detection of the human eye is a flux $f = 10^9\ \frac{photon}{m^2 \cdot s}$. How far from Rick could a person see his flashlight if the air were perfectly clear?

In that case, only inverse square law is needed.

$$f = \frac{L}{4\pi d^2}$$

The luminosity of flashlight, in photons/sec, is

$$L = \frac{0.02 \cdot 10\ J/s}{hc/\lambda} = \frac{0.2\ J/s}{3.61 \times 10^{-19}\ J/photon}$$

$$= 5.54 \times 10^{17}\ \frac{photon}{s}$$

Hw 3, #3 continued

c) The critical distance for human detection occurs when

$$f = 10^9 \frac{\text{photon}}{m^2 \cdot s} = \frac{5.54 \times 10^{17} \frac{\text{photon}}{s}}{4\pi d^2}$$

Solve for d.

$$d = \sqrt{\frac{5.54 \times 10^{17} \frac{\text{photon}}{s}}{4\pi \cdot 10^9 \frac{\text{photon}}{m^2 \cdot s}}}$$

$$\boxed{d \cong 6,600 \text{ m} = 6.6 \text{ km}}$$

d) When air is full of soot, extinction occurs in addition to the inverse square law. After traveling a distance d, extinction by itself reduces intensity by

$$\frac{I}{I_0} = e^{-\tau} = e^{-d/mfp}$$

So effective flux becomes

$$f = 10^9 \frac{\text{photon}}{s \cdot m^2} = \frac{5.54 \times 10^{17} \frac{\text{photon}}{s}}{4\pi d^2} \cdot e^{-d/127 m}$$

One can solve for d in several ways. Simple graphing or trial-and-error yield

$$\boxed{d = 607 \text{ m}}$$

Considerably less than the 6600 m without soot.

HW 3, #4

Analyze a slice across the surface of the Sun, using an image from the Big Bear Solar Observatory.

a-b. Done

c. See graph "big_bear_slice_a.ps". I have normalized the graph, using a peak central intensity of

$$I_0 = 245 \text{ counts}$$

d. See graph "big_bear_slice_b.ps".

e. A fit to the data with

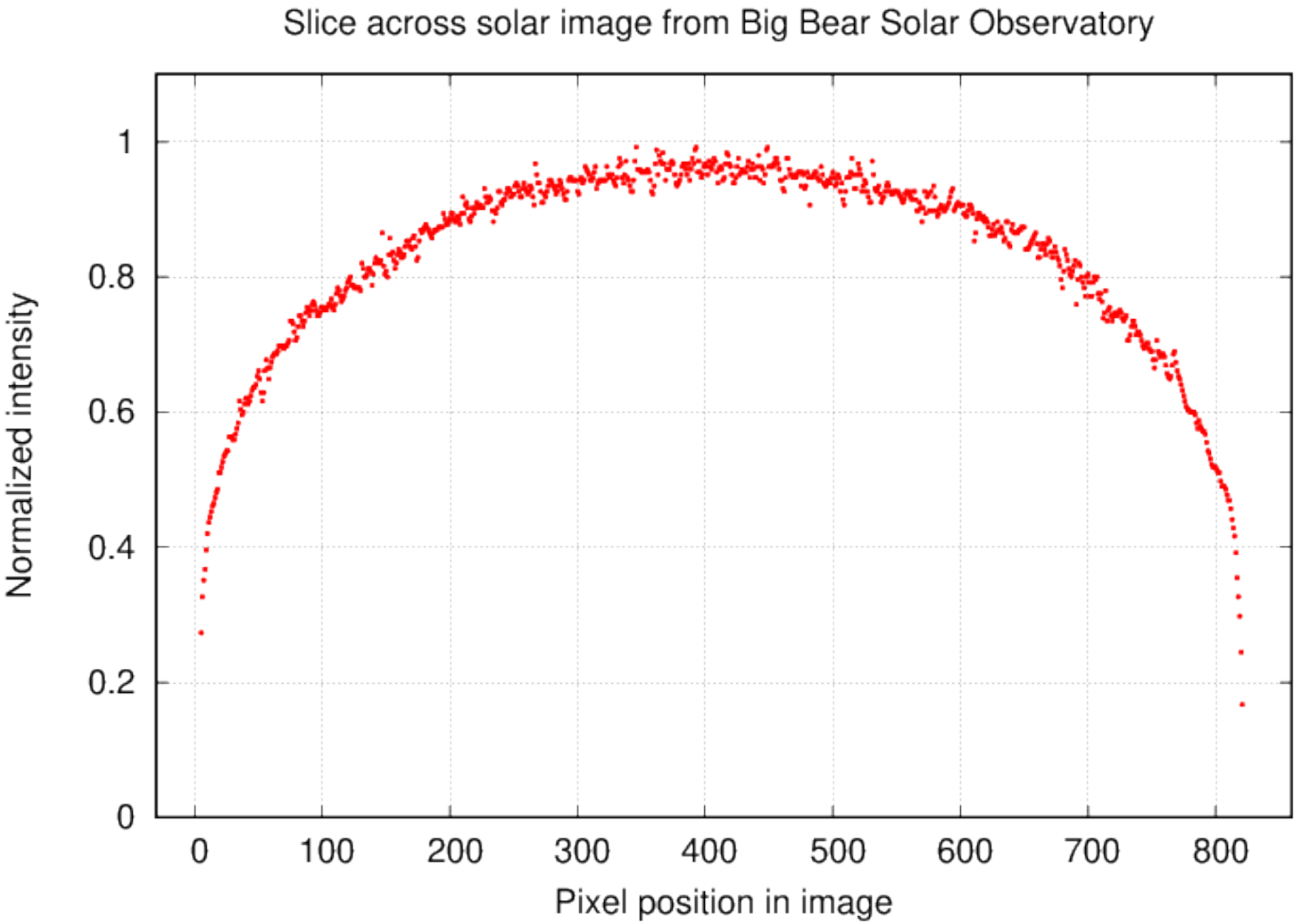
$$|\theta| < 60^\circ \Rightarrow \cos(\theta) > 0.5$$

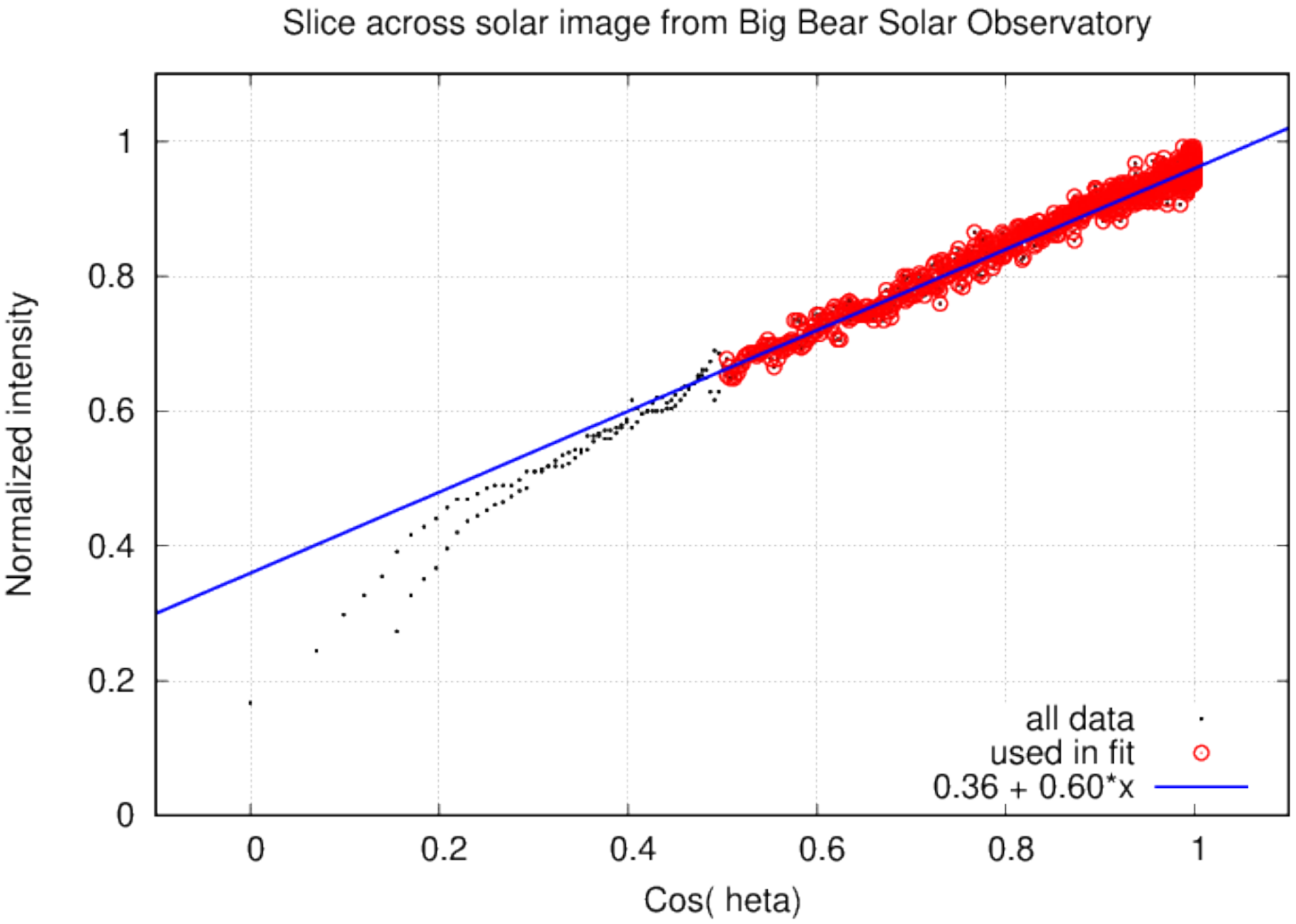
yields

$$\frac{I}{I_0} = 0.36 + 0.60 * \cos(\theta)$$

which is very close to the prediction of

$$\frac{I}{I_0} = 0.40 + 0.60 * \cos(\theta)$$


Slice across solar image from Big Bear Solar Observatory
Normalized intensity
Pixel position in image
0
0.2
0.4
0.6
0.8
1
0
100
200
300
400
500
600
700
800


Slice across solar image from Big Bear Solar Observatory
Normalized intensity
Cos( heta)
0
0.2
0.4
0.6
0.8
1
all data
used in fit
0.36 + 0.60*x