HW 1, #1

a) A star emits as a perfect blackbody at $T = 8500$ Kelvin. How much energy does it emit per sq. meter per second?

$$L = \sigma T^4 = \left(5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}\right)(8500\,K)^4$$

$$\boxed{L = 2.96 \times 10^8 \frac{W}{m^2}}$$

b) The peak wavelength is

$$\lambda = \frac{0.0029\ m \cdot K}{8500\ K} = 3.41 \times 10^{-7}\ m$$
$$= 341\ nm$$
$$= 3410\ \text{Å}$$

c) How much of the energy emitted per sq. meter lies within the range 5070 - 5960 Å?

In order to solve this problem, one must perform numerical integration of the blackbody function $B_\lambda(T)$ over the given range. One finds

$$\boxed{L_V = 3.18 \times 10^7 \frac{W}{m^2}} = \approx 11\% \text{ of total}$$

d) The star has radius $R = 2.0 \times 10^9\,m$. Its total luminosity in V-band is

$$L_V = 4\pi (2.0 \times 10^9\,m)^2 \left(3.18 \times 10^7 \frac{W}{m^2}\right)$$

$$= \boxed{1.60 \times 10^{27}\ W}$$

e) The star is a distance $d = 10$ pc from Earth. What is the flux in the V-band above Earth's atmosphere?

$$\text{flux}_V = \frac{L_V}{4\pi d^2}$$

$$= \frac{1.60 \times 10^{27}\ \text{W}}{4\pi \left(10\ \text{pc} \cdot \frac{3.09 \times 10^{16}\ \text{m}}{\text{pc}}\right)^2}$$

$$= 1.33 \times 10^{-9}\ \frac{\text{W}}{\text{m}^2} = 1.33 \times 10^{-6}\ \frac{\text{erg}}{\text{cm}^2 \cdot \text{s}}$$

f) Joe's telescope is 50 cm in diameter. How many photons per second will his telescope collect?

cheap method: assume all photons have $\lambda = 5515$ Å, the middle of the V band. Then each photon has energy

$$E_{ph} = \frac{hc}{\lambda} = 3.60 \times 10^{-19}\ \text{J}$$

The energy collected is

$$E_{col} = 1.33 \times 10^{-9}\ \frac{\text{W}}{\text{m}^2} \cdot \pi (0.25\ \text{m})^2$$

$$= 2.61 \times 10^{-10}\ \text{W}$$

So # photons is

$$N = \frac{E_{col}}{E_{ph}} = 7.25 \times 10^8\ \frac{\text{photon}}{\text{s}}$$

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f) We can also answer this question "the right way" by modifying the numerical integration to compute the number of photons in the passband, rather than the energy. We find

$$\frac{\#\text{photons}}{m^2 \cdot s} = 8.80 \times 10^{25}$$ from each m^2 of the star

then $$\frac{\#\text{photons}}{s} = 4\pi R^2 \cdot 8.80 \times 10^{25}$$

$$= 4.42 \times 10^{45} \ \frac{\text{photons}}{s}$$ in V-band

The number striking Joe's telescope is

$$4.42 \times 10^{45} \ \frac{\text{photon}}{s} \cdot \frac{1}{4\pi (10\,pc)^2} \cdot \pi (0.25\,m)^2$$

$$= \boxed{7.24 \times 10^{8} \ \frac{\text{photons}}{s}}$$

which is within 1% of the "cheap" method.

BONUS: The star's flux in V-band was $1.33 \times 10^{-6} \ \frac{erg}{cm^2 \cdot s}$; the flux of Vega, a bright star with $V=0$, is about $3.6 \times 10^{-9} \ erg/cm^2 \cdot s \cdot Å$. So across the V-band, of width ≈ 900 Å, Vega produces about $3.2 \times 10^{-6} \ erg/cm^2 \cdot s$. In other words, this star is about 1/3 as bright as Vega, or mag $V \approx 1$.

HW 1, #2

a) Hydrogen atoms have energy levels

$n=1$	$-13.6\text{ eV}\left(\frac{1}{1}\right)^2$	$= -13.6\text{ eV}$
$n=2$	$\left(\frac{1}{2}\right)^2$	$= -3.4\text{ eV}$
$n=3$	$\left(\frac{1}{3}\right)^2$	$= -1.51\text{ eV}$
$n=4$	$\left(\frac{1}{4}\right)^2$	$= -0.85\text{ eV}$
$n=5$	$\left(\frac{1}{5}\right)^2$	$= -0.544\text{ eV}$
$n=6$	$\left(\frac{1}{6}\right)^2$	$= -0.378\text{ eV}$

Transitions down to $n=2$, the Balmer lines, lie in the optical range. We may expect to see

$H\alpha$	$n=3 \rightarrow n=2$	$\Delta E = 1.89\text{ eV}$	$\lambda = 6563\text{ Å}$
$H\beta$	$n=4 \rightarrow n=2$	2.55 eV	4863 Å
$H\gamma$	$n=5 \rightarrow n=2$	2.856 eV	4342 Å
$H\delta$	$n=6 \rightarrow n=2$	3.022 eV	4103 Å

The spectrum clearly shows $H\alpha, H\beta, H\gamma, H\delta$.
It may show $H\epsilon$ as well.

b) Using a graphics editor to measure positions of lines on the given spectrum, I find a measured wavelength

$$\lambda_{obs} = 4864\text{ Å} \quad \text{for } H\beta$$

The rest wavelength of $H\beta$ is

$$\lambda_{rest} = 4861\text{ Å}$$

$\rightarrow$

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b) continued

So we can use the Doppler formula to compute at least an upper limit on the speed of the nebula relative to Earth:

$$\frac{v}{c} <= \frac{4864 - 4861}{4861} = 0.00062$$

$$\Rightarrow v \leq 185 \ \frac{km}{s}$$ away from Earth (positive shift)

This is a very rough estimate, as the given spectrum has a resolution too poor to measure the wavelength of the lines precisely.

Hw 1, #3

a) The given spectrum has a peak at a wavelength of

$$\lambda_{peak} \approx 650 \text{ nm} = 6500 \text{ Å}$$

if one smooths the curve to find the large-scale shape. Therefore,

$$T = \frac{0.0029 \text{ m} \cdot \text{K}}{6500 \times 10^{-10} \text{ m}} \cong \underline{\underline{4460 \text{ K}}}$$

b) If sodium atoms have energy levels

n =	energy (eV)
1	−5.14
2	−3.04
3	−1.96
4	−1.52

then some transitions yielding absorption lines are

	ΔE	λ	
$n=4 \Rightarrow n=1$	3.62 eV	3425 Å	
3 1	3.18	3899	← maybe this one
2 1	2.10	5905	← this one for sure
$n=4 \Rightarrow n=2$	1.52	8158 Å	
4 3	0.44	28180	
$n=3 \Rightarrow n=2$	1.08	11481 Å	