MODERN PHYSICS II HOMEWORK 1 *March 2006*

You may assume that the mass of the neutron and the proton are the same for the purposes of this homework.

1. Use Bohr's model of the Hydrogen atom to show that when an atom makes a transition from the state *n* to the state *n-1*, the frequency of the emitted light is given by

$$f = \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{(n-1)^2 n^2}\right]$$

   a) Show that as $n \to \infty$ the preceding expression varies as $1/n^3$.

   b) Suppose that all the excited states of hydrogen had lifetimes of 10-8 seconds. As we go to larger and larger n values the energy levels get closer and closer together until they are so close in energy that the uncertainty in energy of each state becomes as large as the energy spacing between the states and they then cannot be resolved. Use the expression you derived above for the energy difference for adjacent states large n find the value of n for which this occurs

   c) What is the radius of such and atom?

   d) Compare the classical frequency of revolution of an electron with the frequency of the photons emitted in transitions from *n* to the state *n-1*, for

      (a) n= 10
      (b) n = 100
      (c) n = 1000
      (d) n = 10000

[Hint; To calculate the frequency remember that the frequency of revolution is $v/2\pi r$). [This problem is about the correspondence principal]

2. The negative pion π^- is a subatomic particle with the same charge as the electron but with a mass $m_\pi = 273 m_e$. A π^- can be captured into a Bohr orbit around an atom

   a) What is the orbital radius for a π^- captured into the n = 1 orbit of a carbon nucleus

   b) Given that the carbon nucleus has a radius $R \cong 3\times 10^{-15}$ m can this orbit be formed.

c) Repeat parts a) and b) for a lead nucleus ($R \cong 7\times 10^{-15}$ m)

3. (a) A sample of Hydrogen atoms are all in the n = 5 state. If all the atoms return to the ground state how many different photon energies will be emitted assuming <u>all possible</u> transitions occur?

   (b) If there are 500 atoms in the sample and assuming that from any state <u>all possible</u> downward transitions are equally probable, what is the total number of photons that will be emitted when all the atoms have returned to the ground state?

4. A spectral line Radiated by a Helium ion He^{+} is found to be nearly in wavelength to the $H\alpha$ line (the first line of the Balmer series)

   a) Between what two states (values of n) does the transition in the Helium ion occur [Hint: Don't worry about the difference between R_H and R_{He} at this stage]?

   b) Is the wavelength greater or smaller than that of the $H\,\alpha$ line [Hint: Now you do need to consider the difference between R_H and R_{He}]?

   c) Calculate the difference between the two wavelengths [Hint: consider $\Delta\lambda/\lambda_H$].

5. The general quantization of motion in circular orbits is obtained by combining the equation of motion $mv^2/r = \left|dU(r)/dr\right|$ with the angular – momentum quantization condition $mvr = n\hbar$. Here $U(r)$ is the potential energy. Use this procedure to calculate the spectrum for circular motion in the potential $U = u_0 r$

This Question is to do with Sommerfields modification to the Bohr model

6. (a) Find the energy difference between the circular n=2 orbit (k=1) and the elliptical n=2 orbit (k=2) of the hydrogen atom. The energies are given by the formula

   $$E = -\frac{1}{2}\alpha^2 mc^2\left[1+\frac{\alpha^2}{n}\left(\frac{1}{k}-\frac{3}{4n}\right)\right]\frac{1}{n^2}$$

   where α is the fine structure constant

   (b) If we neglect the splitting of the n=3 states we would expect the n=3→n=2 emission line to have two components. Assuming the energy difference is small find the wavelength difference between the two components [Hint: think $\lambda = \frac{hc}{E}$]

7. Mosley showed that the general formula for the K series of x-ray wavelengths is

   $$\frac{1}{\lambda_k} = R_H(z-1)^2\left[1-\frac{1}{n^2}\right]$$

An unknown element is used as a target in an x-ray tube. Measurements show that the characteristic spectral lines with the longest wavelengths are 0.155nm and 0.131 nm. What is the element?

8. (a) A particle of mass *m* is confined to an infinitely deep potential well of width L. An external force is used to push together the walls of the well. Estimate the minimum amount of work that would have to be done in order to reduce the width if the well to $\frac{1}{2}$L.

   (b) Estimate the amount of work required if the particles are rubidium atoms with atomic mass 85.5 (look up the SI value of an amu in the book) if the well has a width of 20mm, and if there are 2000 particles in the well. Give your answer in electron volts.

9. The wavelengths of the lines of the K series of ^{74}W are for K_α ,λ = 0.210Å for K_β,, λ = 0.184Å for K_γ, ,λ = 0.0.179Å. The wavelength corresponding to the K absorption edge is ,λ = 0.178Å. Use this information to construct an x-ray energy level diagram for ^{74}W.

10. A particle is confined to a 3-D box infinite square well that has sides of length L_1, L_2= $2L_1$ and L_3=$4L_1$.
    a) Give the sets of quantum numbers n_1, n_2 and n_3 that correspond to the lowest 10 energy levels of this box
    b) What set of quantum numbers correspond to degenerate energy levels?.
    c) What are the wavefunctions of these 10 quantized energy states?