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Chapter 7

Alpha Decay

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In a series of seminal experiments Ernest Rutherford and his collaborators established the important features of alpha decay. The behavior of the radiations from natural sources of uranium and thorium and their daughters was studied in magnetic and electric fields. The least penetrating particles, labeled "α-rays" because they were the first to be absorbed, were found to be positively charged and quite massive in comparison to the more penetrating negatively charged "β-rays" and the most penetrating neutral "γ-rays." In a subsequent experiment the α-rays from a needle-like source were collected in a very small concentric discharge tube and the emission spectrum of helium was observed in the trapped volume. Thus, alpha rays were proven to be energetic helium nuclei. The α particles are the most ionizing radiation emitted by natural sources (with the extremely rare exception of the spontaneous fission of uranium) and are stopped by as little as a sheet of paper or a few centimeters of air. The particles are quite energetic, (E_α = 4 - 9 MeV), but interact very strongly with electrons as they penetrate into material and stop within 100 μm in most materials.

Understanding these features of α decay allowed early researchers to use the emitted α-particles to probe the structure of nuclei in scattering experiments and later, by reaction with beryllium, to produce neutrons. In an interesting dichotomy, the α-particles from the decay of natural isotopes of uranium, radium and their daughters have sufficient kinetic energies to overcome the Coulomb barriers of light elements and induce nuclear reactions but are not energetic enough to induce reactions in the heaviest elements.

Alpha particles played an important role in nuclear physics before the invention of charged particle accelerators and were extensively used in research. Therefore, the basic features of alpha decay have been known for some time. The process of alpha decay is a nuclear reaction that can be written as:

$$ {}^{A}_{z}(Z)_{N} \rightarrow {}^{A-4}_{z-2}(X)^{2-}_{N-2} + {}^{4}_{2}He^{2+}_{2} + Q_{\alpha} \qquad (7\text{-}1) $$

where we have chosen to write out all of the superscripts and subscripts. Thus the α-decay of ^{238}U can be written

$$ {}^{238}U \rightarrow {}^{234}Th^{2-} + {}^{4}He^{2+} + Q \qquad (7\text{-}2) $$

The Q_{α}-value is positive (exothermic) for spontaneous alpha decay. The helium nucleus emerges with a substantial velocity and is fully ionized, and the atomic electrons on the daughter are disrupted by the sudden change but the whole process conserves charges. We can rewrite the equation in terms of the masses of the neutral atoms:

$$ {}^{A}_{z}(Z)_{N} - {}^{A-4}_{z-2}(X)_{N-2} + {}^{4}_{2}He_{2} + Q_{\alpha} \qquad (7\text{-}3) $$

and then calculate the Q_{α}-value because the *net* change in the atomic binding energies (~65.3 $Z^{7/5}$ - 80 $Z^{2/5}$ eV) is very small compared to the nuclear decay energy.

What causes α-decay? (or, what causes Q_{α} to be positive?) In the language of the semi-empirical mass equation, the emission of an α-particle lowers the Coulomb energy of the nucleus, which increases the stability of heavy nuclei while not affecting the overall binding energy per nucleon because the tightly bound α-particle has approximately the same binding energy/nucleon as the original nucleus.

Two important features of alpha decay are that the energies of the alpha particles are known to generally increase with the atomic number of the parent but yet the kinetic energy of the emitted particle is less than that of the Coulomb barrier in the reverse reaction between the α-particle and the daughter nucleus. In addition, all nuclei with mass numbers greater than A≈150 are thermodynamically unstable against alpha emission (Q_{α} is positive) but alpha emission is the dominant decay process only for the heaviest nuclei, A≥210. The energies of the emitted α-particles can range from 1.8 MeV (^{144}Nd) to 11.6 MeV ($^{212}Po^{m}$) with the half-life of ^{144}Nd being $5x10^{29}$ times

as long as that of $^{212}Po^{m}$. Typical heavy element alpha decay energies are in the range from 4-9 MeV, as noted earlier.

In general, alpha decay leads to the ground state of the daughter nucleus so that the emitted particle carries away as much energy as possible and as little angular momentum as possible. The ground state spins of even-even parents and daughters (including the alpha particle, of course) are zero which makes ℓ=0 alpha particle emission the most likely process for these nuclei. Small branches are seen to higher excited states but such processes are strongly suppressed. Some decays of odd A heavy nuclei populate low-lying excited states that match the spin of the parent so that the orbital angular momentum of the α particle can be zero. For example, the strongest branch (83%) of the alpha decay of ^{249}Cf goes to the 9th excited state of ^{245}Cm because this is the lowest lying state with the same spin and parity as that of the parent. Alpha decay to several different excited states of a daughter nucleus is called *fine structure*, α-decay from an excited state of a parent nucleus to the ground state of the daughter nucleus is said to be *long range alpha emission* because these α-particles are more energetic and thus have longer ranges in matter. The most famous case of long range α-emission is that of $^{212}Po^{m}$ where a 45 s isomeric level at 2.922 MeV decays to the ground state of ^{208}Pb by emitting a 11.65 MeV α-particle.

We will consider the general features of alpha emission and then we will describe them in terms of a simple quantum mechanical model. It turns out that α emission is a beautiful example of the quantum mechanical process of tunneling through a barrier that is forbidden in classical mechanics.

7.1 Energetics of α Decay

As we have seen in the overview of the nuclear mass surface in Chapter II, the alpha particle, or ^{4}He nucleus, is an especially strongly bound particle. This combined with the fact that

the binding energy per nucleon has a maximum value near A≈56 and systematically decreases for heavier nuclei, creates the situation that nuclei with A≥150 have positive Q_α-values for the emission of alpha particles. This behavior can be seen in Figure 7-1. For example, one of the heaviest naturally occurring isotopes, ^{238}U (with a mass defect, Δ of +47.3070 MeV) decays by alpha emission to ^{234}Th (Δ = +40.612 MeV) giving a Q_α-value of:

$$Q_\alpha = 47.3070 - (40.612 + 2.4249) = 4.270 \text{ MeV}$$

Note that the decay energy will be divided between the α particle and the heavy recoiling daughter so that the kinetic energy of the alpha particle will be slightly less. (The kinetic energy of the recoiling ^{234}Th nucleus produced in the decay of ^{238}U is ~0.070 MeV.) Conservation of momentum and energy in this reaction requires that the kinetic energy of the α-particle, T_α, is:

$$T_\alpha = \frac{234}{238} Q_\alpha = 4.198 \text{ MeV}$$

The kinetic energies of the emitted alpha particles can be measured very precisely so we should be careful to distinguish between the Q_α-value and the kinetic energy, T_α. The very small recoil energy of the heavy daughter is very difficult to measure but it is still large compared to chemical bond energies and can lead to interesting chemistry. For example, alpha decay will tend to physically spread radioactive daughter nuclei away from the original source. This can cause serious contamination problems if the daughters are themselves radioactive.

The Q_α-values generally increase with increasing atomic number but the variation in the mass surface due to shell effects can overwhelm the systematic increase (Figure 7-2). The sharp peaks near A=214 are due to the effects of the N=126 shell. When ^{212}Po decays by α-emission, the daughter nucleus is doubly magic-- ^{208}Pb (very stable) with a large energy release. The α decay of ^{211}Pb and ^{213}Po will not lead to such a large Q_α because the products are not doubly magic. Similarly, the presence of the 82 neutron closed shell in the rare earth region causes an increase in

Q_α, allowing observable α-decay half-lives for several of these nuclei (with N=84). Also one has observed short-lived α-emitters near doubly magic ^{100}Sn, including ^{107}Te, ^{108}Te, and ^{111}Xe. For a set of isotopes (nuclei with a constant atomic number) the decay energy generally decreases with increasing mass. These effects can be seen in Figure 7-2. For example, the kinetic energy of alpha particles from the decay of uranium isotopes is typically 4 to 5 MeV, those for californium isotopes are ≈6 MeV, and those for rutherfordium isotopes are ≈8 MeV. However, the kinetic energy from the decay of $^{212}_{84}$Po to the doubly magic $^{208}_{82}$Pb daughter is 8.78 MeV.

The generally smooth variation of Q_α with Z, A of the emitting nucleus can be used to deduce masses of unknown nuclei. One tool in this effort is the concept of closed decay cycles (Figure 7-3). Consider the α- and β-decays connecting $^{237}_{93}$Np, $^{241}_{95}$Am, $^{241}_{94}$Pu, and $^{237}_{92}$U. By conservation of energy, one can state that the sum of the decay energies around the cycle connecting these nuclei must be zero (within experimental uncertainty). In those cases where experimental data or reliable estimates are available for three branches of the cycle, the fourth can be calculated by difference.

Even though the energies released by the decay of a heavy nucleus into an alpha particle and a lighter daughter nucleus are quite substantial, the energies are paradoxically small compared to the energy necessary to bring the alpha particle back into nuclear contact with the daughter. The electrostatic potential energy between the two positively charged nuclei, called the Coulomb potential, can be written as:

$$V_C = \frac{2Z}{R}\,\frac{e^2}{4\pi\varepsilon_0} \qquad (7\text{-}4)$$

where Z is the atomic number of the daughter and R is the separation between the centers of the two nuclei. (As pointed out in Chapter 1, $\frac{e^2}{4\pi\varepsilon_0}$ is 1.440 MeV · fm.) To obtain a rough estimate of the Coulomb energy we can take R to be $1.2(A^{1/3} + 4^{1/3})$ fm, where A is the mass number of the daughter. For the decay of ^{238}U we get:

$$V_C = \frac{(2)(90)(1.440\ \text{MeV fm})}{1.2\ (234^{1/3} + 4^{1/3})\ \text{fm}} \approx \frac{259\ \text{MeV fm}}{9.3\ \text{fm}} = 28\,\text{MeV} \qquad (7\text{-}5)$$

which is 6 to 7 times the decay energy. This factor is typical of the difference between the Coulomb barrier and the Q-value. If we accept for the moment the large difference between the Coulomb barrier and the observed decay energies then we can attribute the two general features of increasing decay energy with increasing atomic number, Z, and decreasing kinetic energy with increasing mass among a set of isotopes to the Coulomb potential. The higher nuclear charge accelerates the products apart and the larger mass allows the daughter and alpha particle to start further apart.

Example of Decay Energies

Calculate the Q_α-value, kinetic energy, T_α, and the Coulomb barrier, V_C, for the primary branch of the alpha decay of ^{212}Po to the ground state of ^{208}Pb. Using tabulated mass defects we have:

$$Q_\alpha = -10.381 - (-21.759 + 2.4249) = 8.953\text{MeV}$$

$$T\alpha = \frac{208}{212} \qquad Q = 8.784\ \text{MeV}$$

and

$$V_C = \frac{(2)(84)(1.440\,\text{MeV fm})}{1.2\ (208^{1/3} + 4^{1/3})\ \text{fm}} \approx 27\ \text{MeV}$$

The ^{212}Po parent also decays with a 1% branch to the first excited state of ^{208}Pb at an excitation energy of 2.6146 MeV. What is the kinetic energy of this alpha particle?

$$Q_\alpha = 8.953 - 2.6146 = 6.339 \text{ MeV}$$

$$T_\alpha = \frac{208}{212}\ 6.1339 = 6.22 \text{ MeV}$$

As discussed previously, many heavy nuclei (A≥150) are unstable with respect to α-decay. Some of them also undergo β^- decay. In Chapter 3, we discussed the natural decay series in which heavy nuclei undergo a sequence of β^- and α-decays until they form one of the stable isotopes of lead or bismuth, ^{206,207,208}Pb or ^{209}Bi. We are now in a position to understand why a particular sequence occurs. Figure 7-4 shows a series of mass parabolas (calculated using the semiempirical mass equation) for some members of the 4n+3 series, beginning with ^{235}U. Each of the mass parabolas can be thought of as a cut through the nuclear mass surface at constant A. ^{235}U decays to ^{231}Th. ^{231}Th then decays to ^{231}Pa by β^- decay. This nucleus, being near the bottom of the mass parabola, cannot undergo further β^- decay, but decays by α-emission to ^{227}Ac. This nucleus decays by β^- emission to ^{227}Th, which must α-decay to ^{223}Ra, etc.

7.2 Theory of α-decay

The allowed emission of alpha particles could not be understood in classical pictures of the nucleus. This fact can be appreciated by considering the schematic potential energy diagram for ^{238}U shown in Figure 7-5. Using simple estimates we have drawn a one dimensional potential energy curve for this system as a function of radius. At the smallest distances, inside the parent nucleus, we have drawn a flat-bottomed potential with a depth of ~-30 MeV (as discussed in Chapter VI). The potential rapidly rises at the nuclear radius and comes to the Coulomb barrier height of $V_C \approx +28$MeV at 9.3 fm. At larger distances the potential falls as $\frac{1}{r}$ according to Coulomb's Law.

Starting from a separated alpha particle and the daughter nucleus, we can determine that the distance of closest approach during the scattering of a 4.2 MeV alpha particle will be ~62 fm. This is the distance at which the alpha particle stops moving towards the daughter and turns around because its kinetic energy has been converted into potential energy of repulsion. Now the paradox should be clear: the alpha particle should not get even remotely near to the nucleus; or from the decay standpoint, the alpha particle should be trapped behind a potential energy barrier that it can not get over.

The solution to this paradox was found in quantum mechanics. A general property of quantum mechanical wavefunctions is that they are only completely confined by potential energy barriers that are infinitely high. Whenever the barrier has a finite size the wavefunction solution will have its main component inside the barrier plus a small but finite part *inside* the barrier (generally exponentially decreasing with distance) and another finite piece outside the barrier. This phenomenon is called tunneling because the classically trapped particle has a component of its wavefunction outside the potential barrier and has some probability to go through the barrier to the outside. The details of these calculations are discussed in Appendix F and in many quantum mechanics textbooks. Some features of tunneling should be obvious: the closer the energy of the particle to the top of the barrier, the more likely that the particle will get out. Also, the more energetic the particle is relative to a given barrier height, the more frequently the particle will "assault" the barrier and the more likely that the particle will escape.

It has been known for some time that probability for α-decay, λ_α, may be related to the square root of the alpha particle energy, T_α, as follows:

$$\log(\lambda) = A + \frac{B}{\sqrt{T_\alpha}} \tag{7-6}$$

where the constants A and B have a Z dependence. This relationship, shown in Figure 7-6, is known

as the Geiger-Nuttall law of α-decay (Geiger and Nuttall, 1911, 1912). This simple relationship describes the data on α-decay, which span over 20 orders of magnitude in decay constant or half-life. Note that a 1 MeV change in α-decay energy results in a change of 10^5 in the half-life. A modern representation of this relationship due to Keller and Munzel has the form

$$\log_{10} t^{\alpha}_{1/2}(s) = a\left(Z/\sqrt{Q_\alpha} - Z^{2/3}\right) + b \tag{7-7}$$

where a and b are fitted to different groups of nuclei. The best-fit values of a and b are: e-e nuclei, $a = 1.61$, $b = -20.261$; e-o nuclei, $a = 1.65$, $b = 20.238$; o-e nuclei, $a = 1.66$, $b = 20.726$; o-o nuclei, $a = 1.77$, $b = 20.657$. This relationship is useful for predicting the expected α-decay half-lives for unknown nuclei.

The theoretical description of alpha emission relies on calculating the rate in terms of two factors. The overall rate of emission consists of the product of the rate at which an alpha particle appears at the inside wall of the nucleus times the (independent) probability that the alpha particle tunnels through the barrier. Thus, the rate of emission, or the partial decay constant λ_α, is written as the product of a frequency factor, f, and a transmission coefficient, T, through the barrier:

$$\lambda_\alpha = fT$$

Some investigators have suggested that this expression should be multiplied by an additional factor to describe the probability of preformation of an alpha particle inside the parent nucleus. Unfortunately, there is no clear way to calculate such a factor but empirical estimates have been made. As we will see below, the theoretical estimates of the emission rates are higher than the observed rates and the preformation factor can be estimated for each measured case. However, there are other uncertainties in the theoretical estimates that contribute to the differences.

The frequency with which an alpha particle reaches the edge of a nucleus can be estimated as the velocity divided by the distance across the nucleus. We can take the distance to be twice the radius (something of a maximum value) but the velocity is a little more subtle to estimate. A lower limit for the velocity could be obtained from the kinetic energy of emitted alpha particle, but the particle is moving inside a potential energy well and its velocity should be larger and correspond to the well depth plus the external energy. Therefore, the frequency can be written:

$$f = \frac{v}{2R} = \frac{\sqrt{2\left(V_0 + Q\right)/\mu}}{2R} \tag{7-8}$$

where we have assumed that the alpha particle is non-relativistic, V_0 is the well depth indicated in Figure 7-5 of approximately 30 MeV, μ is the reduced mass, and R is the radius of the daughter nucleus (because the α-particle needs only reach this distance before it is emitted). We use the reduced mass because the alpha particle is moving inside the nucleus and the total momentum of the nucleus must be zero. The frequency of assaults on the barrier is quite large, usually on the order of 10^{21}/s.

The quantum mechanical transmission coefficient for an α-particle to pass through a barrier is derived in Appendix F. Generalizing the results summarized in equation F-48 to a three dimensional barrier shown in Figure 7-5, we have:

$$T = e^{-2G} \tag{7-9}$$

where the Gamow factor (2G) can be written as:

$$2G = \frac{2}{\hbar}\int_R^b \left[2\mu\left(\frac{Z_\alpha Z_D e^2}{r} - Q_\alpha\right)\right]^{1/2} dr \tag{7-10}$$

where

$$Q_\alpha = \frac{Z_\alpha Z_D e^2}{b} \tag{7-11}$$

and the classical distance of closest approach, b, is given as

$$b = \frac{Z_\alpha Z_D e^2}{Q_\alpha} \tag{7-12}$$

In these equations, e^2 = 1.440 MeV-fm, Q_α is given in MeV, Z_α, Z_D are the atomic numbers of the α-particle and daughter nucleus, respectively.

Rearranging we have

$$2G = \frac{2}{\hbar}\sqrt{2\mu Q_\alpha}\int_R^b \left(\frac{b}{r}\right)^{1/2} dr \tag{7-13}$$

This can be integrated to give

$$2G = \frac{2b}{\hbar}\sqrt{2\mu Q_\alpha}\left(\cos^{-1}\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b}}\sqrt{1-\frac{R}{b}}\right) \tag{7-14}$$

Substituting back for b and simplifying, we have

$$2G = 2\sqrt{\frac{2\mu}{\hbar^2 Q_\alpha}}\left(Z_\alpha Z_D e^2\right)\left(\cos^{-1}\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b}}\sqrt{1-\frac{R}{b}}\right) \tag{7-15}$$

For thick barriers, $\frac{R}{b} << 1 \left(\frac{Q_\alpha}{V_C} << 1\right)$, we can approximate

$$\cos^{-1}\sqrt{\frac{R}{b}} \approx \frac{\pi}{2} - \sqrt{\frac{R}{b}} \tag{7-16}$$

We get

$$2G = 2\sqrt{\frac{2\mu}{\hbar^2 Q_\alpha}}\left(Z_\alpha Z_D e^2\right)\left(\frac{\pi}{2}\right) \tag{7-17}$$

where B is the "effective" Coulomb barrier, i.e.,

$$B = \frac{z_\alpha Z_D e^2}{r_\alpha + R_D} \tag{7-18}$$

Typically, the Gamow factor is large (2G ~60-120) which makes the transmission coefficient T extremely small (~ 10^{-55} - 10^{-27}). Combining the various equations, we have

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{f_T} = \frac{\ln 2}{\frac{\left(2(V_0 + Q_\alpha)/\mu\right)}{2R}e^{-2G}} \tag{7-19}$$

or

$$\log t_{1/2} = a + \frac{b}{\sqrt{Q_\alpha}} \qquad (7\text{-}20)$$

i.e., we get the Geiger-Nuttall law of α decay, where a + b are constants, that depend on Z, etc.

This simple estimate tracks the general behavior of the observed emission rates over the very large range in nature. The calculated emission rate is typically one order of magnitude larger than that observed, meaning that the observed half-lives are shorter than predicted. This has led some researchers to suggest that the probability to find a 'preformed' alpha particle inside a heavy nucleus is on the order of 10^{-1} or less. One estimate of the "preformation factor" is to plot, for even-even nuclei undergoing l=0 decay, the ratio of the calculated half-life to the measured half-life. This is done in Figure 7-7. The average preformation factor is ~ 10^{-2}.

Example of Emission Rate Calculation

Calculate the emission rate and half-life for ^{238}U decay from the simple theory of alpha decay. Compare this to the observed half-life.

$$\lambda = fT$$

where

$$f = \frac{\sqrt{2\ (V_0 + Q)\ /\ \mu}}{2R}$$

$$R = r_0\left(A_1^{1/3} + A_1^{1/3}\right) = 1.2\left(4^{1/3} + 234^{1/3}\right) = 9.3 \text{ fm}$$

Note that since we previously calculated b ≈ 62 fm, $R/b = \frac{8.63}{62} < < 1$

$$\mu = 4 * 234/238 = 3.933 \text{ amu}$$

$$f = \frac{c * \sqrt{2\ (30+4.2)/(3.933*931.5)}}{2*9.3} = (2.26 \times 10^{21})/\text{sec}$$

We know that

$$T = e^{-2G}$$

where

$$2G \approx 2_\left(\frac{2\mu}{\hbar^2 Q_\alpha}\right)^{1/2}(Z_\alpha Z_D e^2)\left(\frac{\pi}{2} - 2\sqrt{\frac{Q}{B}}\right)$$

$$2_\left(\frac{2\mu}{\hbar^2 Q_\alpha}\right) = 2_\left(\frac{2_\ \mu(\text{amu})\ _\ 931.5\left(\frac{\text{MeV}}{\text{amu}}\right)}{(\hbar c)^2\ (\text{MeV-fm})^2\ Q_\alpha(\text{MeV})}\right)^{1/2} = 2\left(\frac{(2)(3.933)(931.5)}{(197.3)^2(4.27)}\right)^{1/2} = 0.420\ (\text{MeV-fm})$$

$$(Z_\alpha Z_D e^2) = (2)(90)(1.440) = 259.2\ \text{MeV-fm}$$

$$\left(\frac{\pi}{2} - 2\sqrt{\frac{Q}{B}}\right) = \frac{\pi}{2} - 2\left(\frac{4.27}{27.9}\right)^{1/2} = 0.788$$

$$T = e^{-85.8} = 5.43 \times 10^{-38}$$

$$\lambda = fT = (2.26\times10^{21})\ (5.43\times10^{-38}) = 1.23\times10^{-16}\ s^{-1}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = 5.65\times10^{15}\text{s} = 1.8\times10^{8}\text{y}$$

The observed half-life of ^{238}U is 4.47 x 10^9 years which is a factor of ~25 times longer than the calculated value. Note the qualitative aspects of this calculation. The α-particle must hit the border of the parent nucleus ~10^{38} times before it can escape. Also note the extreme sensitivity of this calculation to details of the nuclear radius. A 2% change in R changes λ by a factor of 2. In our example, we approximated R as $R_{Th} + R_\alpha$. In reality, the α-particle has not fully separated from the daughter nucleus when they exit the barrier. One can correct for this by approximating $R \approx 1.4A^{1/3}$.

The theory presented above neglects the effects of angular momentum in that it assumes the α-particle carries off no orbital angular momentum ($\ell = 0$). If α decay takes place to or from an excited state, some angular momentum may be carried off by the α-particle with a resulting change in the decay constant. In quantum mechanics, we say that the α-particle has to tunnel through a barrier that is larger by an amount called the centrifugal potential

$$V_\ell = \frac{\ell(\ell+1)\hbar^2}{2\mu R^2} \tag{7-21}$$

where ℓ is the orbital angular momentum of the α-particle, μ is the reduced mass and R is the appropriate radius. This centrifugal potential is added to the potential energy $V_{(r)}$ resulting in a thicker and higher barrier, increasing the half-life (Figure 7-8).

One can evaluate the effect of this centrifugal potential upon α-decay half-lives by simply adding this energy to the Coulomb barrier height. If we define

$$\sigma = \frac{\text{centrifugal barrier height}}{\text{Coulomb barrier height}} \tag{7-22}$$

we can say

$$\sigma = \frac{\ell(\ell+1)\hbar^2}{2\mu R^2} - \frac{R}{Z_\alpha Z_D e^2} = \frac{\ell(\ell+1)\hbar^2}{2\mu R Z_\alpha Z_D e^2} \tag{7-23}$$

Then all we need to do is to replace all occurrences of B by B $(1+\sigma)$. A simple pocket formula that does this is

$$\lambda_{l\neq 0} \approx \lambda_{l=0} \quad \exp\left[-2.027\, l(l+1)\, Z^{-1/2}\, A^{-1/6}\right] \tag{7-24}$$

This centrifugal barrier correction is a very small effect compared to the effect of Q_α or R upon the decay rate.

We should also note that conservation of angular momentum and parity during the α decay process places some constraints on the daughter states that can be populated. Since the α-particle

has no intrinsic spin, the total angular momentum of the α-particle must equal its orbital angular momentum ℓ and the α-particle parity must be $(-1)^{\ell}$. If parity is conserved in α-decay, the final states are restricted. If the parent nucleus has $J^{\pi} = 0^{+}$, then the allowed values of J^{π} of the daughter nucleus are 0^{+} (ℓ=0), 1^{-} (ℓ=1), 2^{+} (ℓ=2). etc. These rules only specify the required spin and parity of the state in the daughter, while the energy of the state is a separate quantity. Recall from Chapter VI that the heaviest elements are strongly deformed and are good rotors. The low lying excited states of even-even nuclei form a low-lying rotational band with spins of 2, 4, 6, etc., while odd angular momenta states tend to lie higher in energy. Because of the decrease in the energy of the emitted α-particle when populating these states, decay to these states will be inhibited. Thus the lower available energy suppresses these decays more strongly than the centrifugal barrier.

Example of Angular Momentum in Alpha Decay

^{241}Am is a long-lived alpha emitter that is used extensively as an ionization source in smoke detectors. The parent state has a spin and parity of $5/2^-$ and cannot decay to the $5/2^+$ ground state of ^{237}Np because that would violate parity conservation. Rather it decays primarily to a $5/2^-$ excited state (85.2%, E*=59.5 keV) and to a $7/2^-$ higher lying excited state (12.8%, E*=102.9 keV). Estimate these branching ratios and compare them to the observed values.

$$Q_\alpha(5/2^-) = 5.578 \text{ MeV}, \; Q_\alpha(7/2^-) = 5.535 \text{ MeV}$$

$$f(5/2^-) = 2.29\text{x}10^{21} \text{ /sec}, \; f(7/2^-) = 2.29\text{x}10^{21} \text{ /sec}$$

$$G(5/2^-) = 33.01, \; G(7/2^-) = 33.84$$

$$\lambda(5/2^-) = 4.89\text{x}10^{-8} \text{ /sec}, \; \lambda(7/2^-) = 9.2\text{x}10^{-9} \text{ /sec}$$

Assuming that the branches to other states are small and do not contribute to the sum of partial half-lives we can write:

$$\text{Branching Ratio}\,(5/2^-) \approx \frac{\lambda(5/2^-)}{\lambda(5/2^-) + \lambda(7/2^-)} = 0.84$$

Note that the observed half-life of 433 yr. is again significantly longer than the predicted half-life of ~3 yr. This difference is attributed to the combined effects of the preformation factor and the hindrance effect of the odd proton in the americium parent (Z=95), see below.

7.3 Hindrance Factors

The one body theory of α-decay applies strictly to e-e alpha emitters only. The odd nucleon α-emitters, especially in ground state transitions, decay at a slower rate than that suggested by the simple one-body formulation as applied to e-e nuclei. Consider the data shown in Figure 7-9

showing the α-decay half-lives of the e-e and odd A uranium isotopes. The odd A nuclei have substantially longer half-lives than their e-e neighbors do.

The decays of the odd A nuclei are referred to as "hindered decays" and a "hindrance factor" may be defined as the ratio of the measured partial half-life for a given α-transition to the half-life that would be calculated from the simple one-body theory applied to e-e nuclides.

In general, these hindrances for odd A nuclei may be divided into five classes:

a. If the hindrance factor is between 1 and 4, the transition is called a "favored" transition. In such decays, the emitted alpha particle is assembled from two low lying pairs of nucleons in the parent nucleus, leaving the odd nucleon in its initial orbital.

To form an α-particle within a nucleus, two protons and two neutrons must come together with their spins coupled to zero and with zero orbital angular momentum relative to the center of mass of the α-particle. These four nucleons are likely to come from the highest occupied levels of the nucleus. In odd A nuclei, because of the odd particle and the difficulty of getting a "partner" for it, one pair of nucleons is drawn from a lower lying level, causing the daughter nucleus to be formed in an excited state.

b. A hindrance factor of 4-10 indicates a mixing or favorable overlap between the initial and final nuclear states involved in the transition.

c. Factors of 10-100 indicate that spin projections of the initial and final states are parallel, but the wave function overlap is not favorable.

d. Factors of 100-1000 indicate transitions with a change in parity but with projections of initial and final states being parallel.

e. Hindrance factors of >1000 indicate that the transition involves a parity change and a spin flip, that is, the spin projections of the initial and final states are antiparallel, which requires substantial reorganization of the nucleon in the parent when the α is emitted.

7.4 Heavy Particle Radioactivity

As an academic exercise one can calculate the Q values for the emission of heavier nuclei than alpha particles and show that it is energetically possible for a large range of heavy nuclei to emit other light nuclei. For example, contours of the Q-values for carbon ion emission by a large range of nuclei are shown in Figure 7-10 calculated with the smooth liquid drop mass equation without shell corrections. Recall that the binding energy steadily decreases with increasing mass (above A~60) and several light nuclei have large binding energies relative to their neighbors similar to the alpha particle. As can be seen in Figure 7-10, there are several nuclei with positive Q values for carbon ion emission. Such emission processes or heavy particle radioactivity have been called "heavy cluster emission."

We should also note that the double shell closures at Z=82 and N=126 lead to especially large positive Q values, as shown in Figure 7-2. Thus, the emission of other heavy nuclei, particularly ^{12}C, has been predicted or at least anticipated for a long time. Notice also that ^{12}C is an even-even nucleus and s-wave emission without a centrifugal barrier is possible. However, the Coulomb barrier will be significantly larger for higher Z nuclei than that for alpha particles.

We can use the simple theory of alpha decay to make an estimate of the relative branching ratios for alpha emission and ^{12}C emission from ^{220}Ra, a very favorable parent that leads to the doubly magic ^{208}Pb daughter. In this case we find Q_α=7.59 MeV and Q_C=32.02 MeV. Using the simple theory and ignoring differences in the preformation factor, the predicted half-life for ^{12}C emission is only longer by a factor of 2!

$$^{220}Ra \Rightarrow {}^{216}Rn + {}^{4}He,\ Q = 7.59,\ \lambda_{calc} = 5.1\text{x}10^{4}\ \text{sec}$$

$$^{220}Ra \Rightarrow {}^{208}Pb + {}^{12}C,\ Q = 32.02,\ \lambda_{calc} = 3.34\text{x}10^{4}\ \text{sec}$$

The encouraging results from simple calculations like this have spurred many searches for this form of radioactivity.

It was only recently that heavy cluster emission was observed at a level enormously lower than these estimates. Even so, an additional twist in the process was discovered when the radiation from a ^{223}Ra source was measured directly in a silicon surface barrier telescope. The emission of ^{14}C was observed at the rate of ~ 10^{-9} times the alpha emission rate and ^{12}C was not observed. Thus, the very large neutron excess of the heavy elements favors the emission of neutron-rich light products. The fact that the emission probability is so much smaller than the simple barrier penetration estimate can be attributed to the very small probability to 'preform' a ^{14}C residue inside the heavy nucleus. This first observation has been confirmed in subsequent measurements with magnetic spectrographs. The more rare emission of other larger neutron-rich light nuclei have been reported in very sensitive studies with nuclear track detectors.

7.5 Proton Radioactivity

For very neutron-deficient (i.e., proton-rich) nuclei, the Q value for proton emission, Q_p, becomes positive. One estimate, based on the semiempirical mass equation, of the line that describes the locus of the nuclei where Q_p becomes positive for ground state decay is shown in Figure 7-11. This line is

known as the *proton-drip line*. Our ability to know the position of this line is a measure of our ability to describe the forces holding nuclei together. Nuclei to the right of the proton dripline in Figure 7-11 can decay by proton emission.

Proton decay should be a simple extension of α-decay with the same ideas of barrier penetration being involved. A simplification with proton decay relative to α-decay is that there should be no preformation factor for the proton. The situation is shown in Figure 7-12 for the case of the known proton emitter ^{151}Lu. One notes certain important features/complications from this case. The proton energies, even for the heavier nuclei, are low (E_p~1 -2 MeV). As a consequence, the barriers to be penetrated are quite thick (R_{out}=80 fm) and one is more sensitive to the proton energy, angular momentum changes, etc.

The measurements of proton decay are challenging due to the low energies and short half-lives involved. Frequently there are interfering α-decays (Figure 7-13). To produce nuclei near the proton dripline from nuclei near the valley of β-stability requires forming nuclei with high excitation energies that emit neutrons relative to protons and α-particles to move toward this proton dripline. This, along with difficulties in studying low energy proton emitters, means that the known proton emitters are mostly in the medium mass –heavy nuclei. A recent review article by Hofmann summarizes the details of proton decay.

References

Textbook discussions of alpha decay that are especially good.

R. Evans, **The Atomic Nucleus** (McGraw-Hill, New York, 1953).

W. Meyerhof, **Elements of Nuclear Physics** (McGraw-Hill, New York, 1967), pp. 135-145.

K. S. Krane, **Introductory Nuclear Physics** (Wiley, New York, 1988), pp. 246-271.

K. Heyde**, Basic Ideas and Concepts in Nuclear Physics** (IOP, Bristol, 1994), pp. 82-103.

S. S. M. Wong, **Introductory Nuclear Physics**, 2nd Edition, (Wiley, New York, 1998).

A more advanced discussion will be found in:

J. O. Rasmussen, "Alpha Decay," in **Alpha-, Beta-, and Gamma-Ray Spectroscopy**, K. Siegbahn, Ed. (North-Holland, Amsterdam, 1965) Chapter XI.

Proton decay is discussed in

S. Hofmann, "Proton Radioactivity", in **Nuclear Decay Modes**, D.N. Poenaru, (IOP, Bristol, 1996)

Problems

1. Using the conservation of momentum and energy, derive a relationship between Q_α and T_α.
2. All nuclei with A>210 are α-emitters yet very few emit protons spontaneously. Yet both decays lower the Coulomb energy of the nucleus. Why isn't proton decay more common?
3. Use the Geiger-Nuttall rule to estimate the expected α-decay half-lives of the following nuclei: ^{148}Gd, ^{226}Ra, ^{238}U, ^{252}Cf, and ^{262}Sg.
4. Use the one-body theory of α-decay to estimate the half-life of ^{224}Ra for decay by emission of a ^{14}C ion or a ^{4}He ion. The measured half-life for the ^{14}C decay mode is 10^{-9} relative to the ^{4}He decay mode. Estimate the relative preformation factors for the α-particle and ^{14}C nucleus in the parent nuclide.
5. ^{212}Po^{m} and ^{269}110 both decay by the emission of high energy α-particles (E_α = 11.6 and 11.1 MeV, respectively). Calculate the expected lifetime of these nuclei using the one-body theory of α-decay. The observed half-lives are 45.1 s and 170μs, respectively. Comment on any difference between the observed and calculated half-lives.

6. Consider the decay of $^{278}112$ to $^{274}110$. The ground state Q_α value is 11.65 MeV. Calculate the expected ratio of emission to the 2^+, 4^+, 6^+ states of 110.

7. What is the wavelength of an α-particle confined to a ^{238}U nucleus?

8. ^{8}Be decays into two α-particles with $Q_\alpha = 0.094$ MeV. Calculate the expected half-life of ^{8}Be using one body theory and compare this estimate to the measured half-life of 2.6×10^{-7} s.

9. Calculate the kinetic energy and velocity of the recoiling daughter atom in the α-decay of ^{252}Cf.

10. Calculate the hindrance factor for the α-decay of ^{243}Bk to the ground state of ^{239}Am. The half-life of ^{245}Bk is 4.35 hours, the decay is 99.994% EC and 0.006% α-decay. 0.0231% of the α-decays lead to the ground state of ^{241}Am. Q_α for the ground state decay is 6.874 MeV.

11. Calculate Q_α for gold. Why don't we see α-decay in gold?

12. The natural decay series starting with ^{232}Th has the sequence αββα. Show why this is the case by plotting the mass parabolas (or portions thereof for A=232,228 and 224.

13. Using the semiempirical mass equation, verify that Q_α becomes positive for $A \geq 150$.

14. Calculate the heights of the centrifugal barrier for the emission of α-particles carrying away two units of angular momentum in the decay of ^{244}Cm. Assume $R_0 = 1.x10^{-13}$ cm. What fraction of the Coulomb barrier height does this represent?

15. Use one-body theory to calculate the expected half-life for the proton decay of ^{185}Bi.

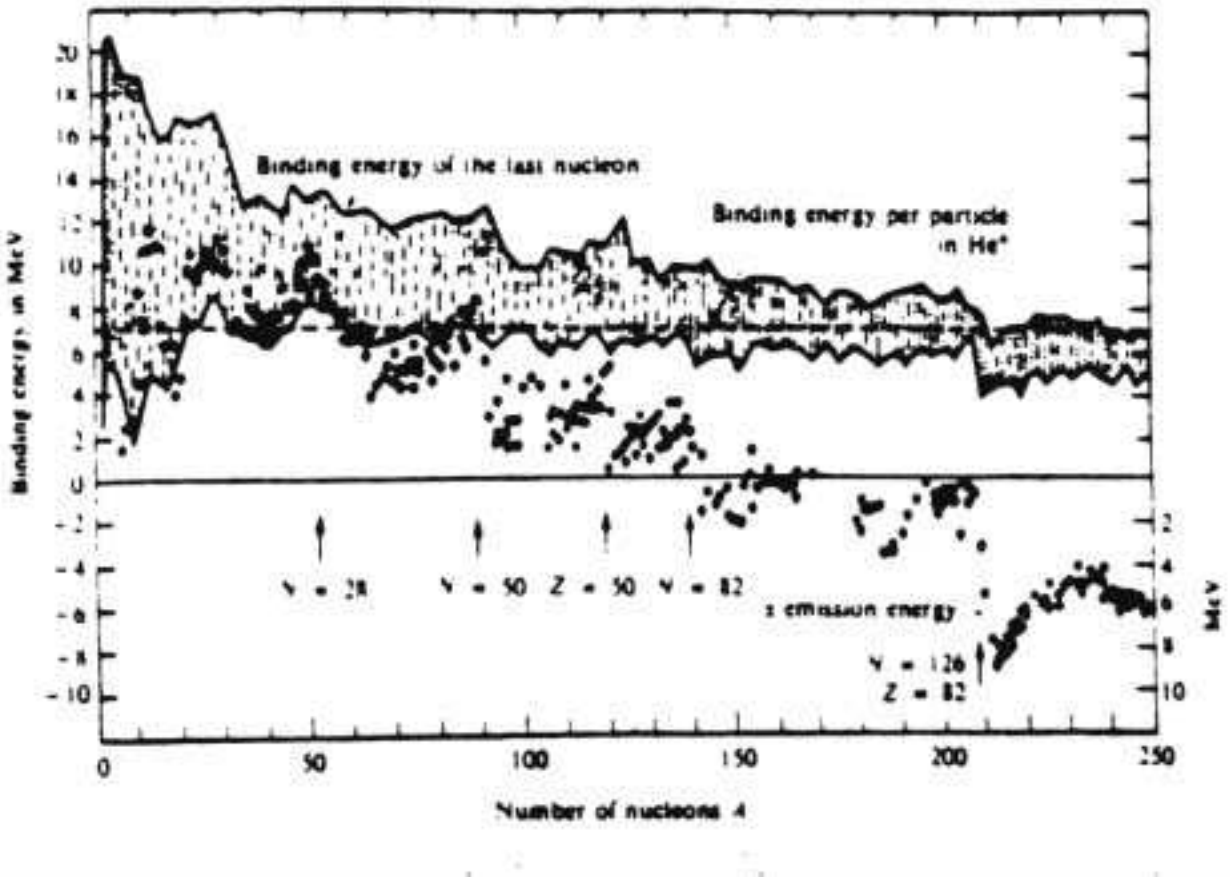


Figure 7-1. The variation of the alpha particle separation energy as a function of mass number.

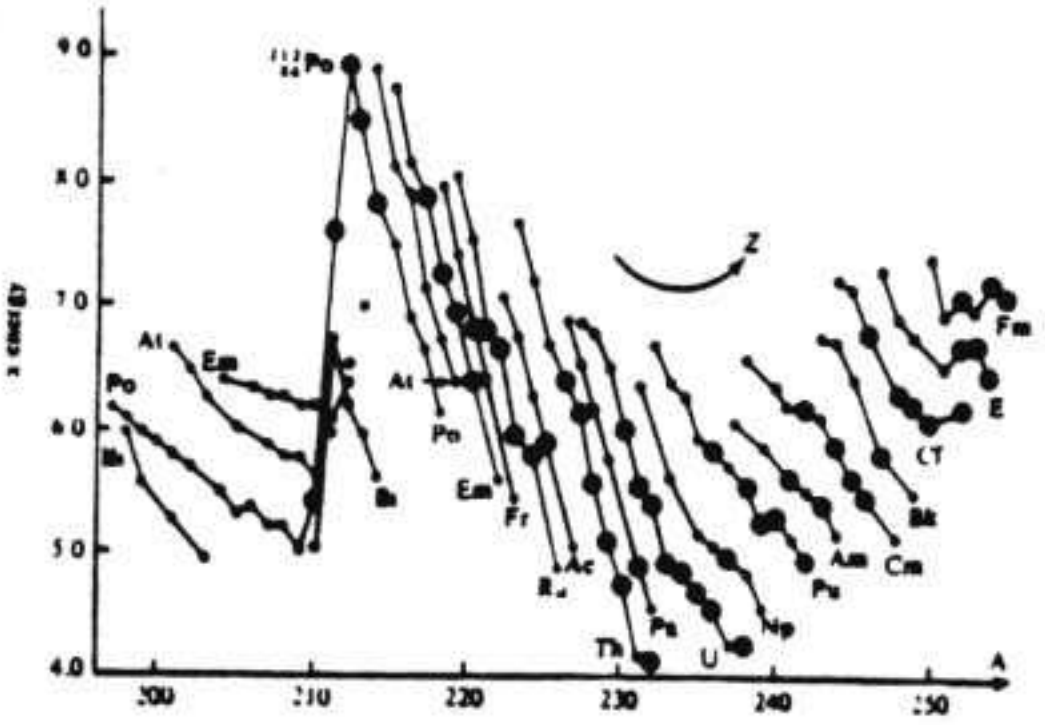


Figure 7-2. The variation of alpha decay energies indicating the effect of the N=126 and Z=82 shell closures.

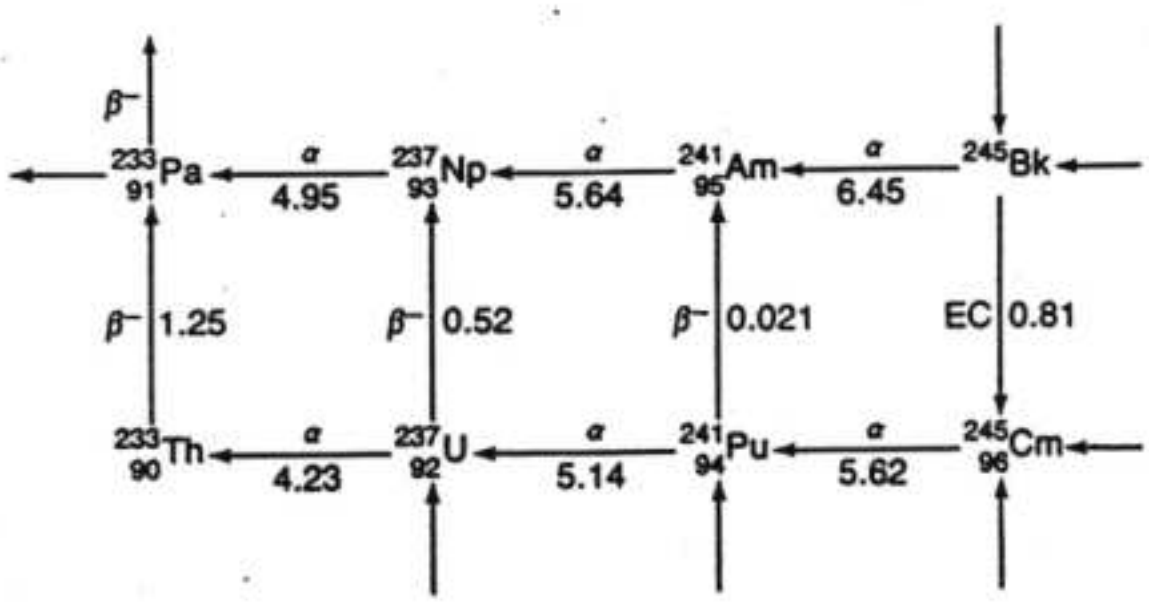


Figure 7-3. Decay cycles for part of the 4n+1 family. Modes of decay are indicated over the arrows; the numbers indicate total decay energies in MeV.

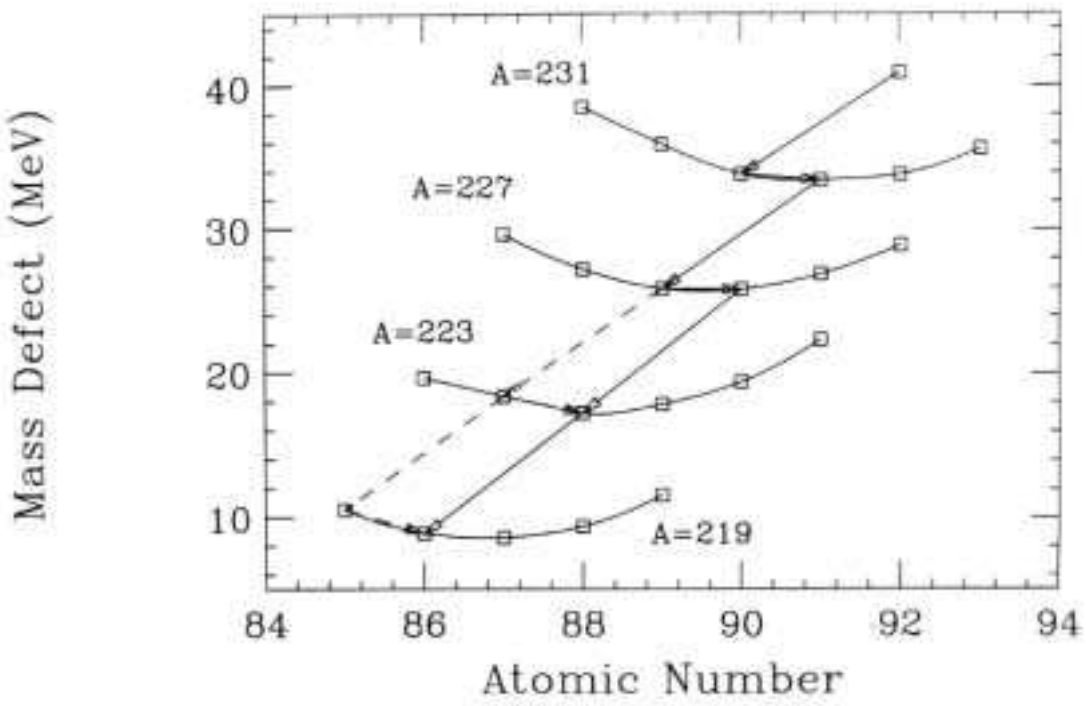


Figure 7-4. Mass parabolas for some members of the 4n+3 natural decay series. The main decay path is shown by a solid line while a weak branch is indicated by a dashed line.

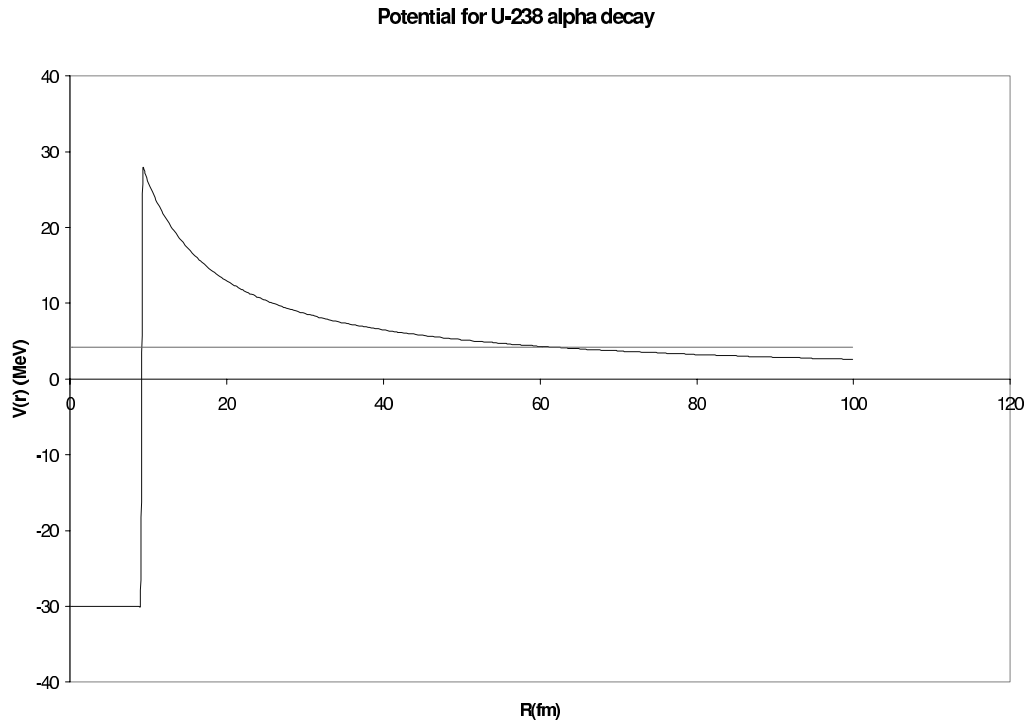


Figure 7-5. A (reasonably accurate) one dimensional potential energy diagram for ^{238}U indicating the energy and calculated distances for alpha decay into ^{234}Th. Fermi energy ≈30 MeV, Coulomb barrier ≈28 MeV at 9.3 fm, Q_α 4.2 MeV, distance of closest approach 62 fm.

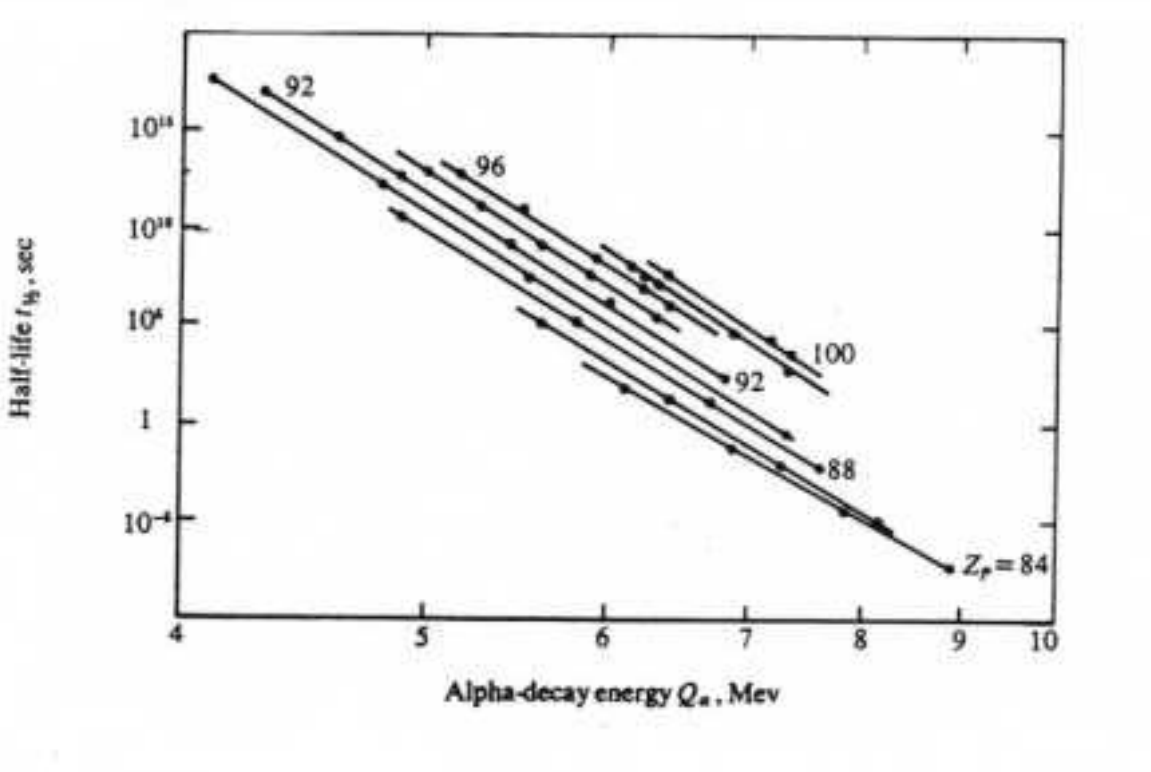


Figure 7-6. A Geiger-Nuttall plot.

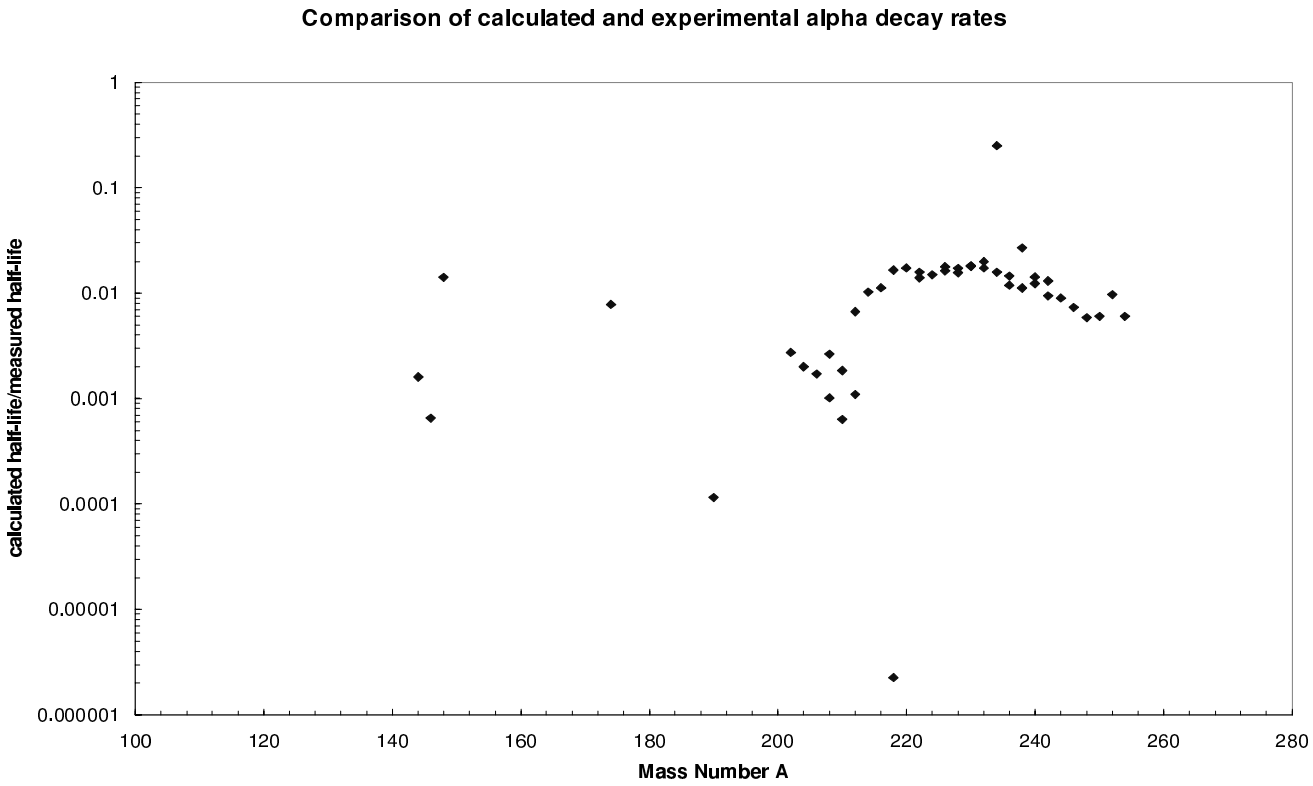
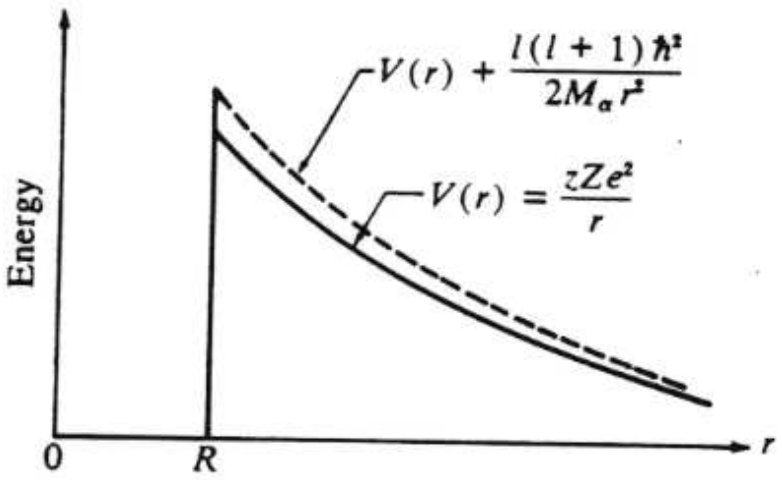


Figure 7-7. Plot of the ratio of the calculated partial α-decay half-life for ground state l=0 transitions of even-even nuclei to the measured half-lives. The calculations were made using the simple theory of α-decay.



(b)

Figure 7-8. Modification of the potential energy in α-decay due to the centrifugal potential.

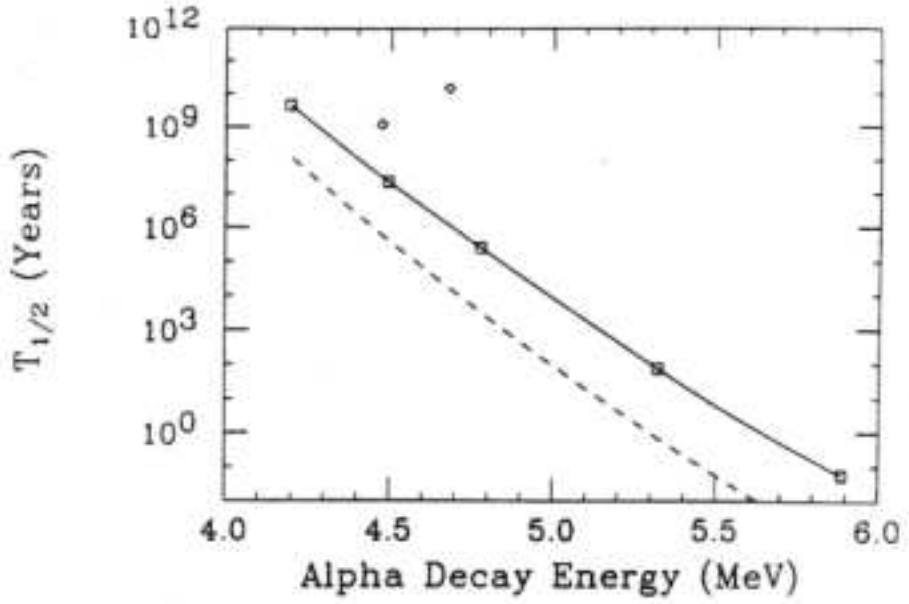


Figure 7-9. The α-decay half-lives of the e-e (squares) and odd A (diamonds) isotopes of uranium. The measured values are connected by the solid line; the medians are shown by the dashed line.

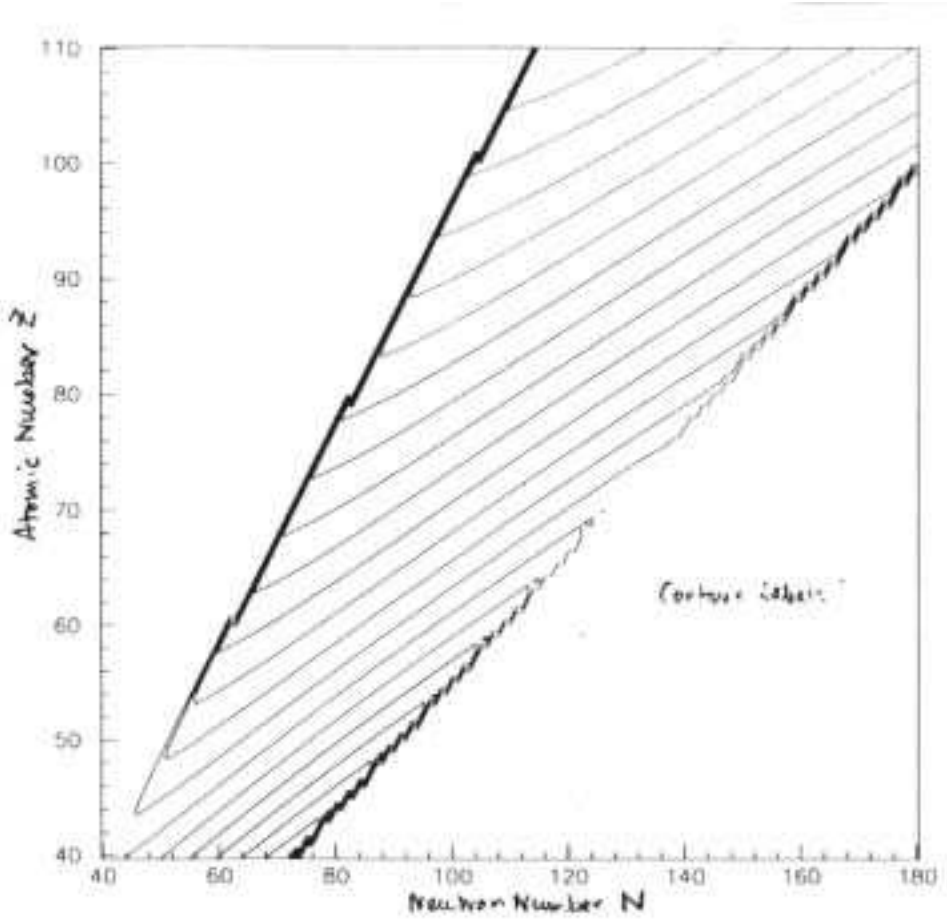


Figure 7-10. Contours of Q value for carbon ion emission by nuclei as calculated by the liquid drop model. Contour levels are 1 MeV apart.

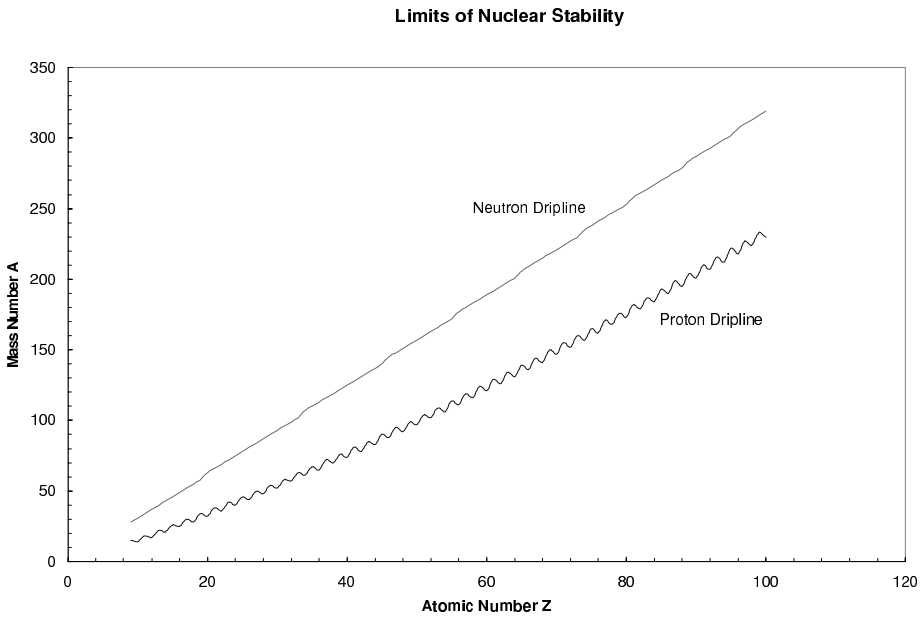


Figure 7-11 Locus of neutron and proton driplines as predicted by the liquid drop model.

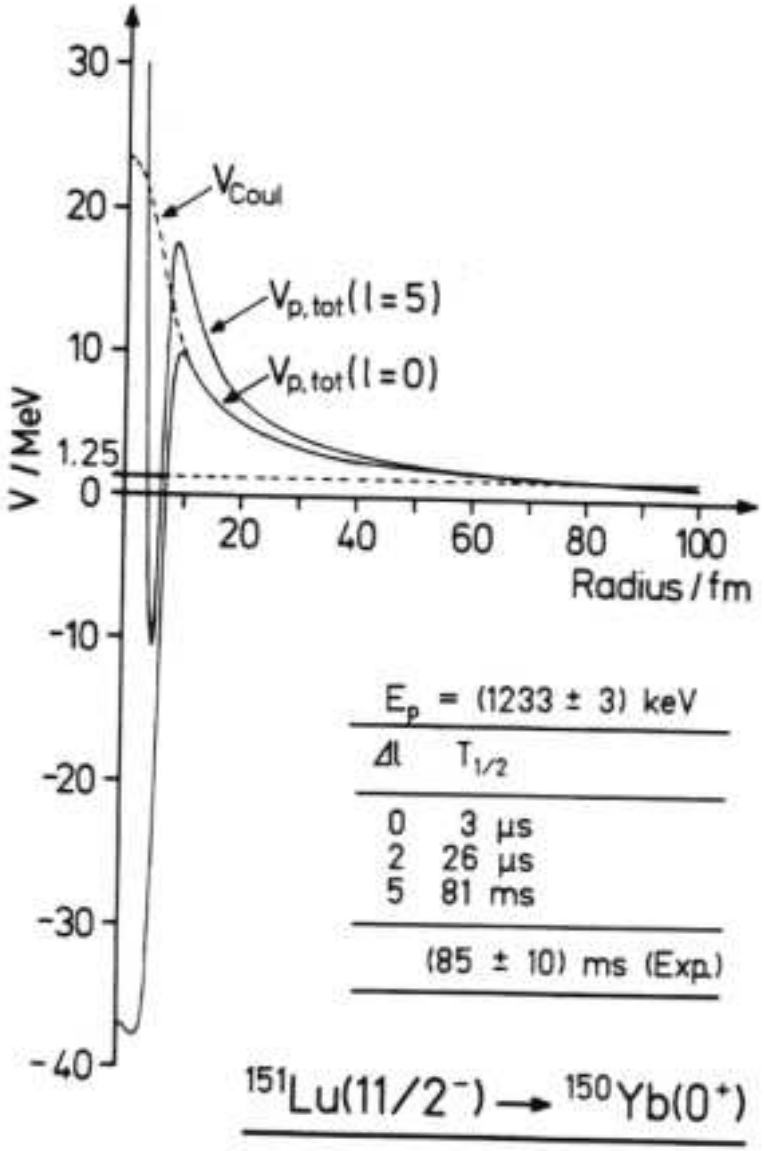


Figure 7-12 Proton-nucleus potential for the semi classical calculation of the ^{151}Lu partial proton half-life.

From Hofmann

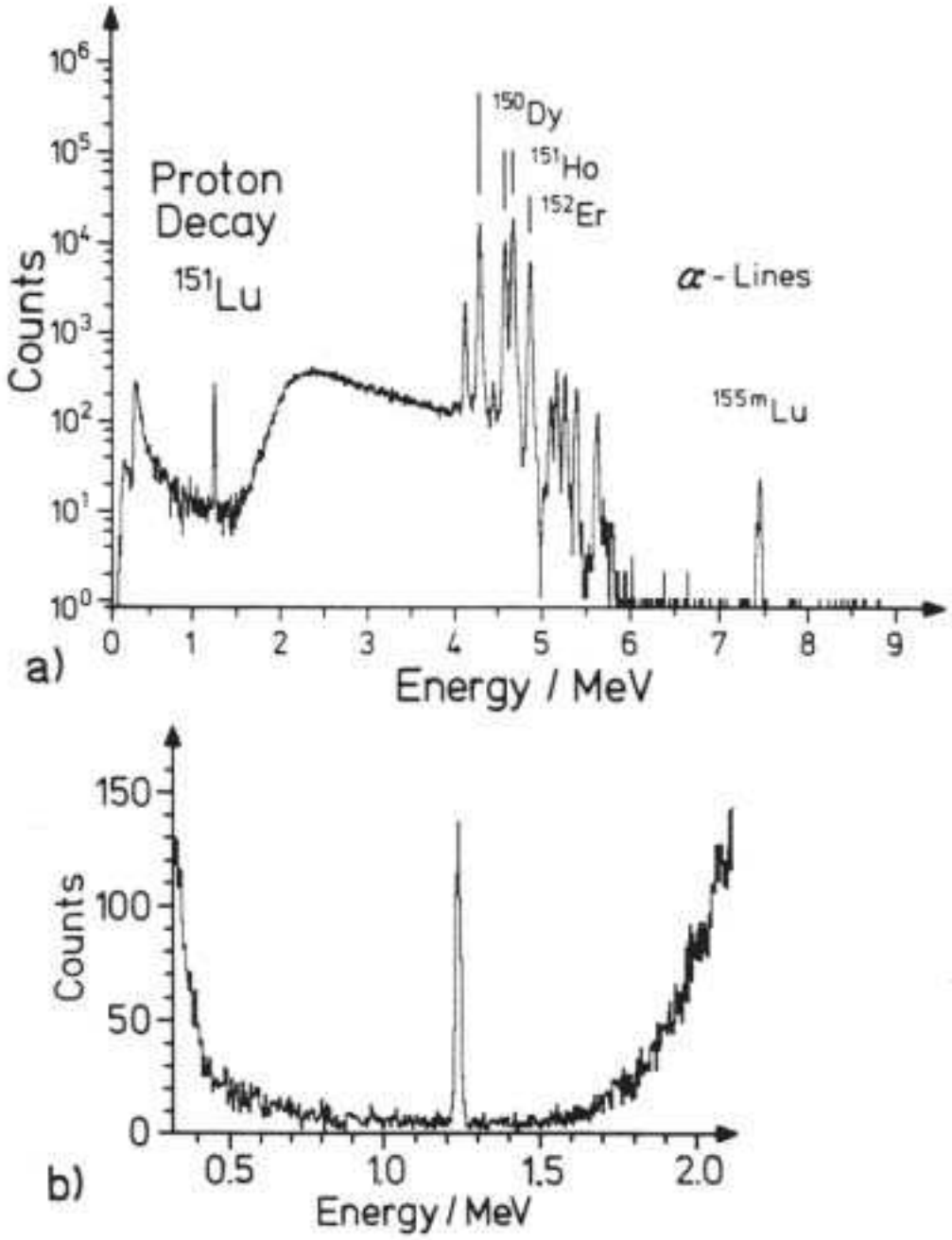

Proton
Decay
151Lu
150Dy
151Ho
152Er
α - Lines
155mLu
Counts
Energy / MeV
a)
b)

Figure 7-13 (a) Energy spectrum obtained during the irradiation of a ^{96}Ru target with 261 MeV ^{58}Ni projectiles. (b) Expanded part of the spectrum showing the proton line from ^{151}Lu decay. From Hofmann.