$$\mathrm{Prob}(x_0 < x < x_0 + dx) = |\Psi(x_0)|^2 dx$$

$$\mathrm{Prob}(r_0 < r < r_0 + dr) = |\Psi(r_0)|^2 dV$$

$$= |\Psi(r_0)|^2 r^2 dr$$

$$\mathrm{Prob}(R_1 < r < R_2) = \int_{R_1}^{R_2} |\Psi(r)|^2 r^2 dr$$

$$E = -\frac{\hbar^2}{2ma^2} = -\frac{{m_e}^2 e^4}{32\pi^2 {\epsilon_0}^2 \hbar^2}$$

$$\Psi(r) \;=\; Q e^{-r/a}$$

$$\mathrm{Prob}\;(r_1 < r < r_2) \;=\; \int_{r_1}^{r_2} \Psi(r)^2 \, dr = \int_{r_1}^{r_2} Q^2 e^{-2r/a} \, dr$$

$$dV \;=\; dx$$

$$dV \;=\; dx\, dy$$

$$dV \;=\; dr\, r d\theta$$

$$dV \;=\; dx\, dy\, dz$$

$$
\begin{aligned}
dV &= (dr)\,(r d\theta)\,(r \sin\theta\, d\phi) \\
&= r^2 dr\ \sin\theta d\theta\ d\phi
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Prob}(r,\theta,\phi) &= \int \left(\Psi(r,\theta,\phi)\right)^2\ dV \\
&= \int \left(\Psi(r,\theta,\phi)\right)^2\ r^2 dr \sin\theta d\theta\, d\phi
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Prob}(r,\theta,\phi) &= \int \left(R(r)\Theta(\theta)\Phi(\phi)\right)^2\ r^2 dr \sin\theta d\theta\, d\phi \\
&= \int \left(R^2(r) r^2 dr\right)\ \left(\Theta^2(\theta)\sin\theta d\theta\right)\ \left(\Phi^2(\phi) d\phi\right) \\
&= \int R^2(r) r^2 dr \int \Theta^2(\theta)\sin\theta d\theta \int \Phi^2(\phi) d\phi
\end{aligned}
$$

$$
\int 1\ \sin\theta d\theta
$$

$$
\int 1\, d\phi
$$

$$
R(r) = Q e^{-r/a}
$$

$$
\mathrm{Prob}(r_1 < r < r_2) = 4\pi \int_{r_1}^{r_2} R^2(r) r^2 dr
$$

$$
\mathrm{Prob}(r_1 < r < r_2) = 4\pi\, (Q^2 \frac{a^3}{8}) \int_{2r_1/a}^{2r_2/a} u^2 e^{-u} du
$$

$$\mathrm{Prob}(r_1 < r < r_2) \;=\; 4\pi\,(Q^2\frac{a^3}{8})\left[-e^{-u}(u^2+2u+2)\right]_{2r_1/a}^{2r_2/a}$$

$$< r > \quad = \quad \frac{\int \Psi^2\, r\, dV}{\int \Psi^2\, dV}$$