

- <sup>7</sup> W. Pauli, *Theory of Relativity* (Pergamon Press, London, 1958), pp. 19, 151.

$$(3) \quad v_s = v_s(1+2\Phi/c_s)^{1/2} \approx v_s(1+\Phi/c_s).$$

Thus, the observer in  $K$  will come to the conclusion that his clock is slowed down by the gravitational potential. The frequency  $\nu_A$  measured in his frame of reference is given to a first approximation by

$$(7) \quad \Phi = -\frac{z}{4} R^a \omega_a.$$

where  $E_A$  and  $E_S$  are the characteristic energies of the absorber and the source. However, when the experiment is analyzed in a reference frame  $K$  attached to the accelerated absorber, the problem could be treated by the principle of equivalence and the general theory of relativity. The centrifugal force acting on the absorber is then interpreted as a gravitational force with respect to the center of rotation.

$$(E_a - E_s)/E_s = (1 - \beta^2)^{1/2} - 1 \approx -\frac{1}{2}\beta^2 = -R_a \omega_a^2 / 2c_s^2, \quad (1)$$

When the experiment is analyzed in the inertial frame of the source, the result follows from the time dilatation in the special theory of relativity.<sup>7</sup> Since the relative velocity  $v = c_2 - c_1$  of the source and absorber is always in a direction perpendicular to the line joining them, there exists a transverse Doppler effect giving in first approximation a fractional energy change,

## INTRODUCTION

Using an ultracentrifuge rotor, the shift of the 144-keV Mossbauer absorption line of  $\text{Fe}^{67}$  in a rotating system was measured as a function of the angular velocity  $\omega$ . An  $\text{Fe}^{67}$  absorber was placed at a radius of 3 cm from the axis of the rotor. A  $\text{Co}^{67}$  source was mounted on a piezoelectric transducer at the center of the rotor. By applying a triangular varying voltage to the transducer, the source could be moved relative to the absorber. This arrangement makes possible the observation of the entire resonance line at various values of  $\omega$ . The measured transverse Doppler shift agrees within an experimental error of 1.1% with the predictions of the theory of relativity. Possible sources of systematic errors are discussed.

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Measurement of the Transverse Doppler Effect in an Accelerated System\*

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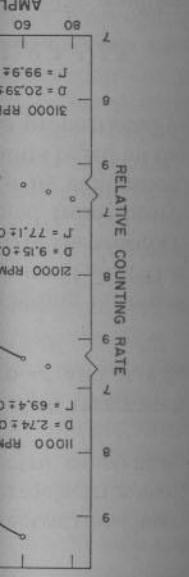
*All journal of experimental and theoretical physics established by E. L. Nichols in 1893*

# PHYSICAL REVIEW

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the scales only a and counter were meted with a 10 g balance times were summed the photo also by the last three arbitrary chosen counting rates. Figure 3 shows a histogram of the measured count rate for the last three dimensions by the last three dimensions.

Fig. 3. Typical results of a regular wave in propulsions to a motion of the ship plotted curves of the ship's speed versus the angle of attack.



increases this low portion of counters were amplitude selected the 14.4 pulse put out of the pulse two scalers. And and B were gated in the following way voltage applied to 20 to 80% and increasing from 80 additional gating

EXTRIMENTAL ARRANGEMENT

$$\cdot E_a - E_s)/E_s \approx -Ra^2\omega^2/2c. \quad (4)$$

The traditional energy shift is, as before,

The over-all experimental arrangement is shown in Fig. 2. The potocell and the light source are actually mounted in the block diagram of Fig. 2. The rotor was suspended by a flexible steel shaft, and was driven over this shaft by a simple ultracentrifuge motor. This suspension made the rotor self-balancing. The speed of the rotor was regulated manually to 0.1%. The rotor was suspended from a magnetic member to minimize friction heating. Physical protection was provided by a steel sleeve riding around the vacuum chamber. The sleeve was divided into 1000.0±0.5 cps amplitude to an amplifier of 100 V and kept constant to 0.2%. The voltage was then divided by 40 to induce a free 100 CPS pulses applied to the transducer. The stationary counterabsorber during about 2% of the time. In order to beyond the rotor saw the diametrical hole with the transducer to the transducer. The stationary counterabsorber during about 2% of the time.

With an improved technique it was tried to increase the precision of this fundamental experiment in reliability. Using an ultracentrifuge rotor the shift of the 14.4-KEV Moesbauer absorption line of  $\text{Fe}^{67}$  was measured as a function of  $\omega$ . A  $\text{Co}^{67}$  source was placed on a piezoelectric transducer at a radius  $R_a$ . By applying a transverse varying voltage to the transducer, the source could be moved relative to the absorber. This arrangement makes possible the observation of the entire resonance line at various values of  $\omega$ . Thus, in contrast to earlier measurements, the variation is independent of the line shape. The motion of the source caused by the transducer was calibrated in an additional experiment.

Fig. 2. Schematic diagram of the experimental arrangement. The potocell and the light source are actually mounted in the block diagram of Fig. 2. The rotor was suspended by a flexible steel shaft, and was driven over this shaft by a simple ultracentrifuge motor. This suspension made the rotor self-balancing. The speed of the rotor was regulated manually to 0.1%. The rotor was suspended from a magnetic member to minimize friction heating. Physical protection was provided by a steel sleeve riding around the vacuum chamber. The sleeve was divided into 1000.0±0.5 CPS pulses applied to an amplifier of 100 V and kept constant to 0.2%. The voltage was then divided by 40 to induce a free 100 CPS pulses applied to the transducer. The stationary counterabsorber during about 2% of the time.

Fig. 1. Scale drawing of the longer side of the  $3 \times 8$  mm source actually turned  $90^\circ$  so that the longer side of the  $3 \times 8$  mm source was parallel to the axis of rotation. The transducer shown was

M. T. Compton (John Wiley & Sons, Inc., New York, 1962).  
H. J. Hay, J. P. Schiffer, T. D. Cranshaw, and P. A. Egelstaff, "Proceedings of the Second Conference on the Mossbauer Effect," edited by A. Schoen and D. Phys., Rev. Letters, 4, 165 (1960); in *Proceedings of the Second Conference on the Mossbauer Effect*, John Wiley & Sons, Inc., New York, 1962).

The experiment was first done by Hay, Schifter, Cranshaw, and Egelsztag.<sup>8</sup> They observed the increase of the  $\gamma$ -ray transmission through the absorber as the velocity of the absorber increased, indicating a shift in the characteristic energy of the absorber due to the absorption of the  $\gamma$ -ray transmission through the absorber as the absorber moved toward lower frequency. Assuming the experimentally known line shape for the absorber at rest stayed constant, the magnitude of the frequency shift was determined. They found that within the experimental errors the effect is in agreement with the expectation.

different modes of expressing the same fact, namely that the clock which experiences acceleration is retarded compared to the one which does not.

We thus see that the transverse Doppler effect and the

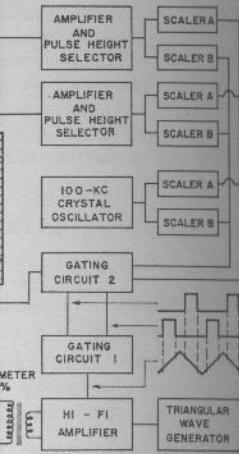
$$(\bar{E}_A - \bar{E}_S)/\bar{E}_S = R_{A^2, S^2}/R_{A^2}$$

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## ARRANGEMENT

ion of the rotor. It was aluminum alloy and had a source and absorber were diam which was drilled center. The absorber, a 0.25-mm Fe<sup>57</sup> was placed inside a was mounted at a radius of 10 mC Co<sup>57</sup>, plated on and glued to an isolating piece of face of a piezoelectric rotor. For the transducer a width and 1-mm thick ceramic, was used.<sup>9</sup> A to the transducer by an dipping into an oil-



The experimental arrangement is actually turned 90° with proportional counters. The two used to activate the photocell, the operation of the scalers.

arrangement is shown. The rotor was suspended was driven over this shaft motor. This suspension ring. The speed of the rotor was 1%. The rotor was spun to minimize frictional loss. It was provided by a 6 in. vacuum chamber. The tri-tutt-Packard function generator was amplified to an amplitude of 0.2%. The voltage was non-free 100 Ω resistors and capacitors. The stationary counter has a diametrical hole with the time. In order to

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increase this low duty cycle, two krypton-filled proportional counters were used.

The pulses of each of the two proportional counters were amplified and fed to a pulse-height analyzer selecting the 14.4-keV Mössbauer radiation. The output of the pulse-height selector was connected with two scalers A and B as indicated in Fig. 2. Scalers A and B were gated according to the motion of the source in the following way: Scalers A were counting when the voltage applied to the transducer was increasing from 0 to 80% and scalers B when the voltage was decreasing from 80 to 20%. To reduce the background additional gating was provided by a photocell so that

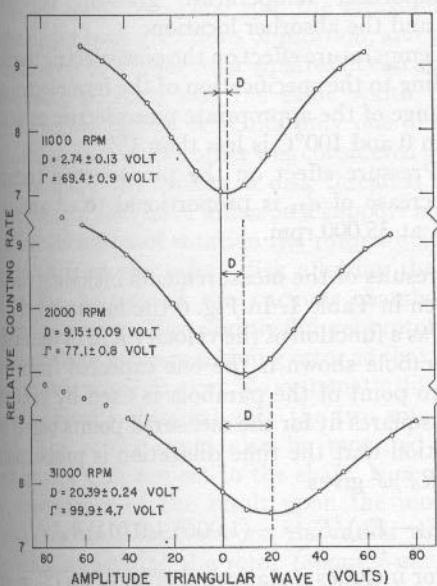


Fig. 3. Typical resonance curves. The amplitude of the triangular wave is proportional to the linear velocity of the source with respect to the absorber. The left (right) side of the plot corresponds to a motion of the source toward (away from) the absorber. The plotted curves are the fitted Lorentz curves normalized to 10 at  $v = \infty$ .  $\Gamma$  is the full width of the resonance line. With increasing speed of the rotor a considerable broadening of the resonance line was observed. The statistical errors of the points are smaller than the circles.

the scalers only accepted pulses when source, absorber and counter were in a straight line. Two scalers connected with a 100 kc/sec crystal oscillator and gated also by the photocell and the triangular wave determined the gating times of scalers A and B. These gating times were used for the normalization of the counting rates. The measurements were done under arbitrarily chosen speeds of the rotor, but so that no resonance with the 1 kc/sec triangular wave occurred. Figure 3 shows an example of the measured resonance line at three different velocities of the rotor. The measured counting rates were fitted to a Lorentz curve by the least squares method, normalized so that the counting rate for  $v = \infty$  was 10.

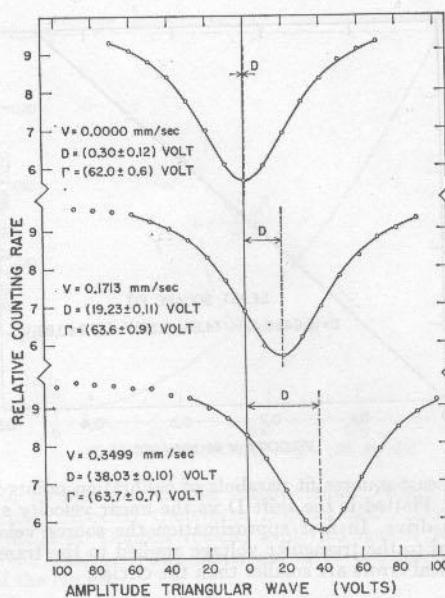


Fig. 4. Typical resonance curves measured with the linear drive. The source was moving with the indicated velocity  $v$  toward the absorber causing a linear Doppler shift  $D$ . Amplitude of the triangular voltage at the left side of the figure means the superimposed velocity of the source caused by the transducer has the same direction as  $v$ ; at the right, the opposite direction. The absorption line measured at  $v=0$  shows the existence of a small chemical shift of the resonance line. The plotted curves are the Lorentzians obtained by the least-squares fit method.

The motion of the source mounted on the transducer must be calibrated as a function of the amplitude of the triangular voltage applied to the piezoelectric transducer. This was performed in the following way: The source mounted on the transducer was moved toward the absorber with a constant velocity  $v$  by means of a mechanical linear drive. The mechanical drive consisted of a micrometer screw driven by a synchronous motor. The average velocity was constant to 0.1% and could be determined by measuring the time for 24 revolutions of the micrometer (= 12 mm) with a photocell. The linear shift caused by moving the source toward and away from the absorber was measured as in the centrifuge experiment by applying different triangular voltages to the transducer. This method of calibration has the advantage over the use of the piezoelectric constant that it was not necessary to know the absolute amplitude and distortion of the triangular wave and the exact dimensions of the transducer. Also by using the same source and absorber as in the centrifuge experiment a possible chemical shift of the resonance line would cancel. Figure 4 shows an example of these measurements which were done every few days. The position  $D$  of the resonance line in terms of the triangular wave amplitude was determined as in the centrifuge experiment by fitting the experimental points to a Lorentz curve by the least-squares method. In Fig. 5,  $D$  is plotted as a function of the velocity of the linear drive. The points were fitted by the method



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$$0.11) R_A^2 \omega^2 / 2c^2.$$

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a parabola (Fig. 5). No  
able.

$$D / (R_A^2 - R_S^2) \omega^2  
(10^{-9} \text{ sec/m})$$

$$\begin{aligned} &-1.7 \pm 2.1 \\ &+1.803 \pm 0.127 \\ &+1.705 \pm 0.029 \\ &+1.703 \pm 0.026 \\ &+1.653 \pm 0.025 \\ &+1.666 \pm 0.020 \\ &+1.679 \pm 0.013 \\ &+1.668 \end{aligned}$$

As Fig. 3 shows, a considerable broadening of the resonance line with increased velocity was found. This may be explained by vibrations in the rotor. In the evaluation it is assumed that the broadening of the resonance line has no influence on its position. This is true when the rotor vibrations are random with respect to the phase of the rotation of the rotor and also with respect to the phase of the transducer vibrations. An example of a phase related rotor vibration would be the forced vibration caused by a faulty bearing of the shaft connecting the rotor with the driving unit of the centrifuge. A forced phase related vibration may give a different broadening and a different shift for the two directions of rotation and different results with different rotors. At the speed of 25 000 rpm the measurements were done under both directions of rotation, and within a statistical error of 3% the least-squares fit calculations gave the same position and the same width of the resonance line. At higher velocities only one direction of rotation was used, as the other was considered unsafe.

If the drive shaft had broken, a disk threaded to the top of the rotor would have fallen on a support beneath it. With one direction of rotation the rotor would then be unscrewed from the safety disk allowing the rotor to fall to the bottom of the vacuum chamber and destroy the apparatus. Two slightly different rotors were used. One of them had a machining error so that it had a statical unbalance of 20 g cm. No systematic difference between the results obtained with the two rotors was observed. The rotors could also be mounted under different angles with respect to the shaft. No evidence of any dependence of the result upon the mounting angle was observed. Furthermore, the rather thin and flexible shaft suspending the rotor (about 2-mm diam and 10 cm long) makes the transmission of a forced vibration which would be in phase with the rotation very unlikely. The only reasonable explanation of the vibrations is that they are the almost undamped characteristic vibrations of the rotor. An amplitude of these elastic vibrations of 10–15 Å would be enough to cause the broadening observed at 31 000 rpm. The discrete values of frequencies of these vibrations makes a phase relation with the rotation improbable. These characteristic vibrations could be induced by the occasional accelerating and braking actions of the motor and its gears due to slight changes in the motor speed.

In the later phases of the experiment the resonance line became irreproducibly nonsymmetric by a few

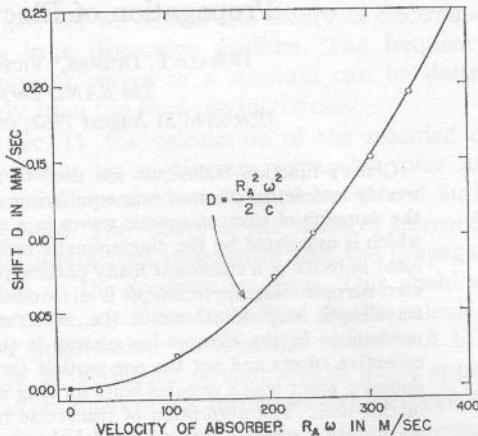


FIG. 6. Comparison of the experimental points with the theoretically expected transverse Doppler shift. The shift in units of the linear Doppler velocity is plotted against the velocity  $R_A \omega$  of the absorber. The statistical error corresponds to the radius of the circles.

percent. Since the effect was observed in the centrifuge and the calibration experiment it cannot be explained by the above discussed rotor vibrations. This effect may be an instability in the shape of the triangular wave or the gating circuits, or a change in the mounting or the linearity of the piezoelectric transducer, but the exact cause could not be located. None of these irreproducible measurements are included in the results. The results of measurements reported earlier<sup>12</sup> are also omitted, because the rotor used for those measurements exploded before any check and recalibration could be performed.

The result given here shows that the experimentally measured second order Doppler effect agrees within an error of 1.1% with the predictions of the theory of relativity, and is to our knowledge the most accurate determination of the transverse Doppler effect up to date.

#### ACKNOWLEDGMENTS

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<sup>12</sup> W. Kündig, Bull. Am. Phys. Soc. 7, 350 (1962).