UNIVERSITY PHYSICS II
1017-312

AND

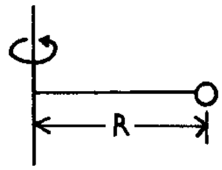
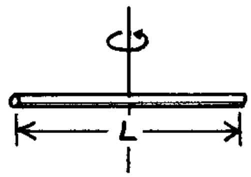
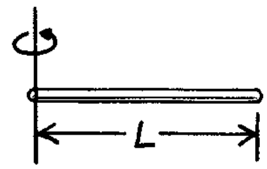
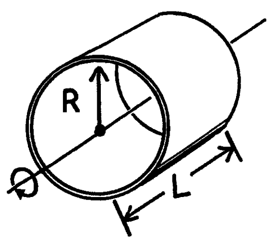
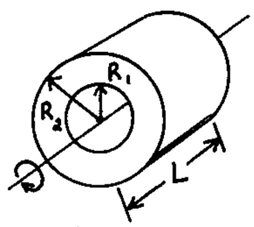
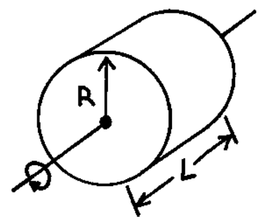
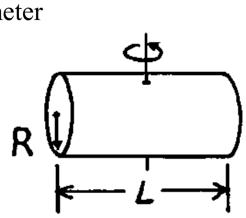
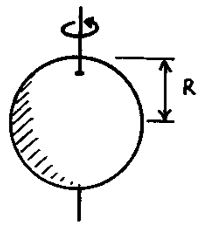
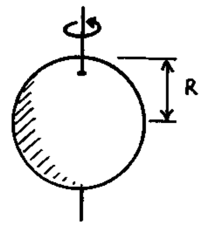
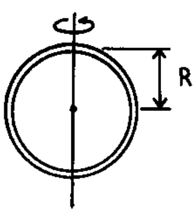
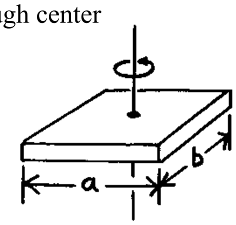
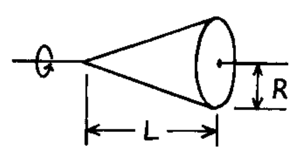
UNIVERSITY PHYSICS IIA
1017-389

Activities Manual

Department of Physics
Rochester Institute of Technology

Effective September 2011

Table of Selected Moments of Inertia

Point mass at a radius R  $I = MR^2$	Thin rod about axis through center perpendicular to length  $I = \frac{1}{12}ML^2$	Thin rod about axis through end perpendicular to length  $I = \frac{1}{3}ML^2$
Thin-walled cylinder about central axis  $I = MR^2$	Thick-walled cylinder about central axis  $I = \frac{1}{2}M(R_1^2 + R_2^2)$	Solid cylinder about central axis  $I = \frac{1}{2}MR^2$
Solid cylinder about central diameter  $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$	Solid sphere about center  $I = \frac{2}{5}MR^2$	Thin hollow sphere about center  $I = \frac{2}{3}MR^2$
Thin ring about diameter  $I = \frac{1}{2}MR^2$	Slab about perpendicular axis through center  $I = \frac{1}{12}M(a^2 + b^2)$	Cone about central axis  $I = \frac{3}{10}MR^2$

Note: All formulas shown assume objects of uniform mass density.

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Group Members: ______________________

______________________ Group: ________

Rotation – Summary Tables

Equations for Rotation: Fill in the charts below as we proceed through the two chapters on rotation. Keep this as a handy reference and do not turn it in.

A. Kinematics

Variables and units

Linear Variable	Rotational Variable (symbol)	Units for Rotational Variable	Scalar or Vector?
Time, t			
Position, x			
Velocity, v			
Acceleration, a			

Equations for constant acceleration

1D Linear Motion	Rotation about a Fixed Axis
$v = v_0 + a\,t$	
$x = x_0 + v_0\,t + \frac{1}{2}a\,t^2$	
$x = x_0 + \frac{1}{2}(v_0 + v)t$	
$v^2 = v_0^2 + 2\,a(x - x_0)$	

Relation between linear and angular quantities

Quantities	Relation (Scalar form; relating magnitudes)
θ, r, s	$s =$
ω, r, $v_{tangent}$	$v_t =$
α, r, $a_{tangent}$	$a_t =$
ω, r, $a_{radial\ (centripetal)}$	$a_r =$

B. Energy, Work, Power

Linear Variable	Angular Variable	Units for Angular Variable	Scalar or Vector?
Kinetic Energy, K			
Work, W			
Power, P			

Linear Equation	Rotational Equation
$K = ½\, mv^2$	
$W = \int \vec{F} \cdot d\vec{r}$	
$P = \vec{F} \cdot \vec{v}$	

C. Dynamics and Momentum

Linear Quantities	Angular Quantities	Units for Angular Variable	Scalar or Vector?
Mass, m			
Force, $\vec{F}$			
Linear Momentum, $\vec{p}$			

Definitions relating linear and angular quantities

Moment of Inertia, I	
Torque, $\vec{\tau}$	
Angular Momentum, $\vec{L}$	

Formula true for symmetric rotations of rigid objects: $\vec{L} = I\,\vec{\omega}$

Newton's Second Law: Rotational form

Linear Equation	Rotational Equation
$\vec{F}_{net} = \sum \vec{F} = m\,\vec{a}$	
$\sum \vec{F} = \frac{d\vec{p}}{dt}$	

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______________________ Group: __________

Circular Reasoning

Review: Much of what we do in University Physics II builds upon University Physics I. We're going to change the variables from linear to rotational, but the basic game will be the same.

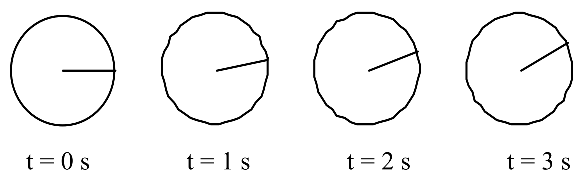
(a) An object is moving to the left and is slowing down. What is the direction of its acceleration?

(b) If acceleration is constant, write down as many of the one-dimensional kinematics equations as you can recall.

(c) In University Physics I you used Newton's Second Law. Write it in symbols and give the meaning of the symbols in words.

Describing Rotation

1. Here are snapshots of a rotating wheel at one second intervals. What is the appropriate variable with which to describe the rotation? Estimate the rotational position at $t = 3$ s relative to $t = 0$.



2. (a) Estimate the angular speed of the object pictured above. Express this angular speed in all three units: revolutions/second, °/sec, and rad/sec. What is the angular speed in rpm?

(b) What additional information must you add to give the **angular velocity** rather than the angular speed?

3. Angular Velocity and Acceleration

The symbol for **angular velocity** is the lower case Greek letter omega $\vec{\omega}$.

Vector Direction: Rather than saying clockwise (CW) or counterclockwise (CCW), we can define a unique vector direction of $\vec{\omega}$ as along the axis of rotation (axle) of the rotating object. The axle points both ways so we have a rule to decide which way is correct. We use the right-hand rule for angular velocity: Wrap the fingers of your right hand around the axis with the thumb along the axis. Orient your hand so the fingers curl around the object in the direction it rotates. The thumb gives the vector direction of angular velocity.

The symbol for **angular acceleration** is the lower-case Greek letter alpha: $\vec{\alpha} = \dfrac{d\vec{\omega}}{dt}$. The angular acceleration $\vec{\alpha}$ points in the same direction as the angular velocity $\vec{\omega}$ if the object is speeding up; $\vec{\alpha}$ points opposite to the angular velocity $\vec{\omega}$ if the object is slowing down.

For an object rotating in the *xy* plane, the angular velocity lies along the *z*-axis. The *z*-components of the angular velocity and angular acceleration are given by

$$\omega_z = \frac{d\theta}{dt} \qquad\qquad \alpha_z = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2}$$

a) The angular position θ as a function of time t of a reference line on a rotating disk is given by

$$\theta(t) = (\, 0.25\, rad/s^2\,)\, t^2 + (\, -0.85\, rad/s\,)\, t + (\, -2.3\, rad\,)$$

Calculate the angular acceleration as a function of time $\alpha(t)$. Is the angular acceleration constant?

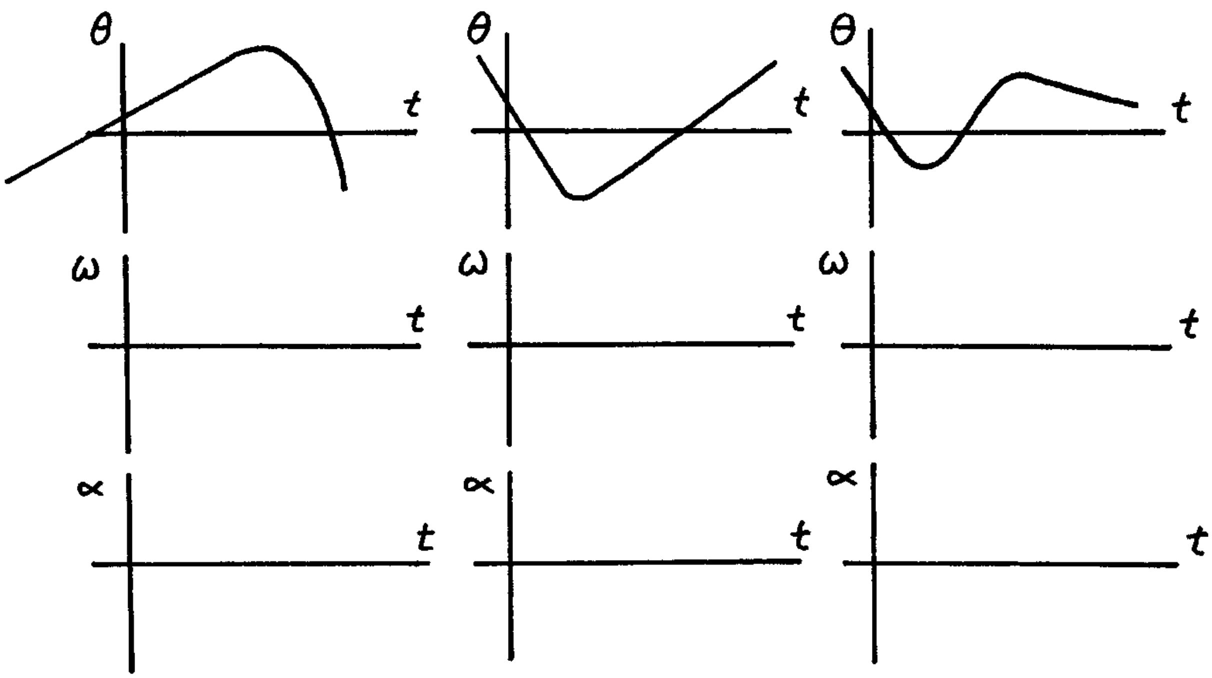
b) The angular velocity ω as a function of time t of a point on a rotating wheel is given by

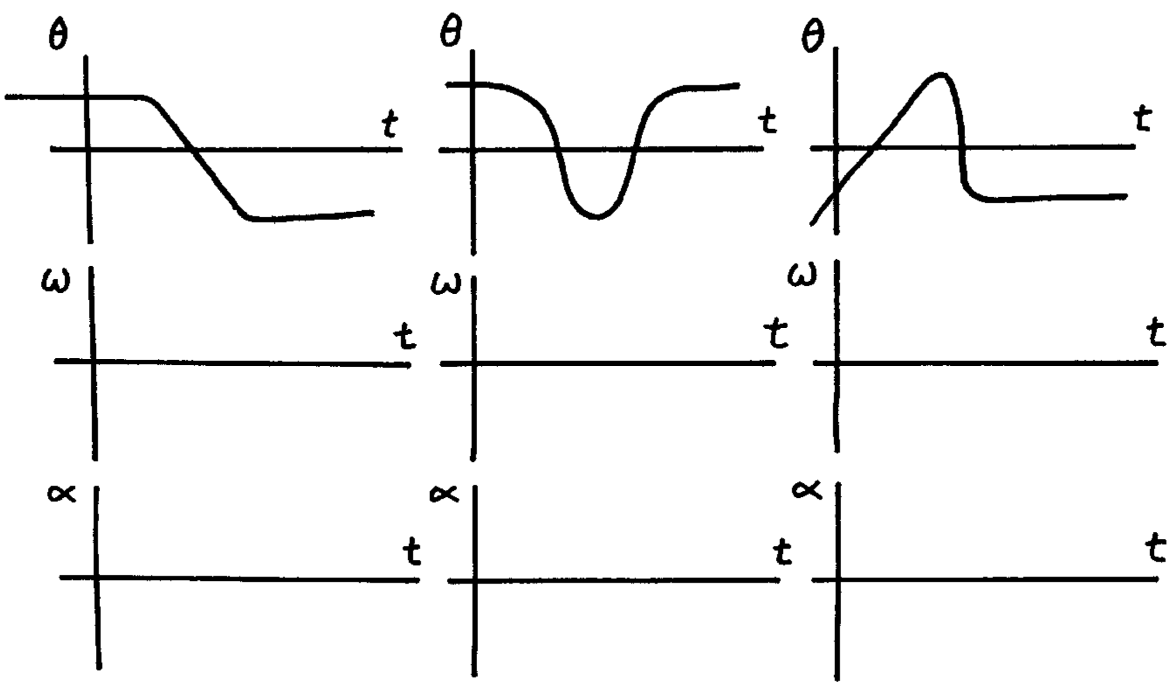
$$\omega(t) = -(6.18\, rad/s\,)\, sin\, [(3.00\, rad/s)\, t\,]$$

i) Calculate the angular acceleration as a function of time $\alpha(t)$. Is the angular acceleration constant?

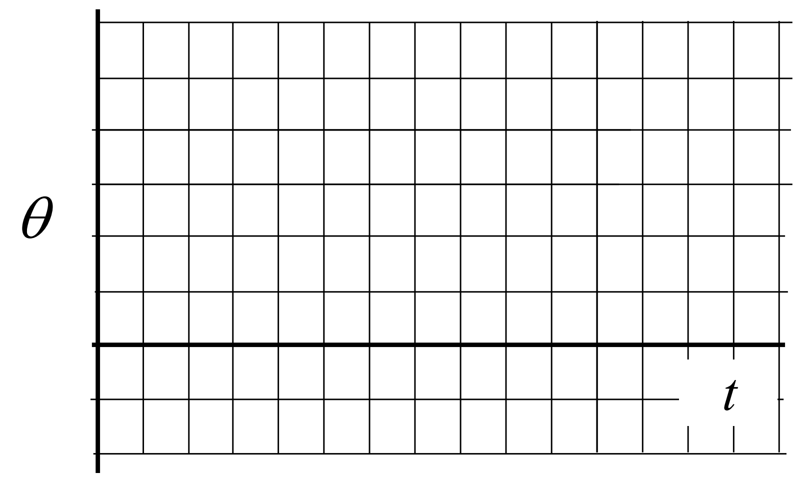
ii) The point is located at angular position +2.09 rad at time $t = 0$. Calculate the angular position as a function of time.

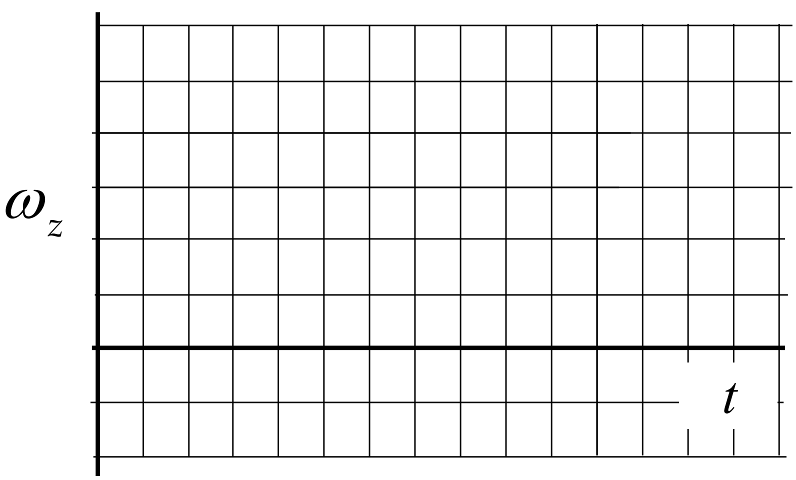
c) Below are three sketches of angle versus time. Sketch the corresponding graphs of angular velocity and angular acceleration versus time. Each group member should do a different column and compare answers.

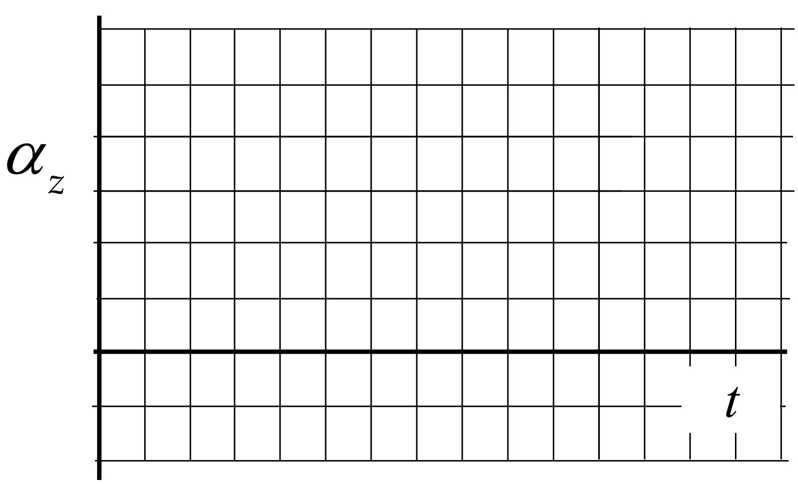




d) A wheel rotates in the *xy* plane. At time $t = 0$, the wheel has an angular velocity of 20 rad/s, CW ($-\hat{k}$ direction). From $t = 0$ to $t = 6.0$ s, the wheel undergoes a uniform acceleration of 10 rad/s^2, CCW. At $t = 6.0$ s, the wheel stops abruptly and remains at rest for 4.0 s. At $t = 10$ s, the wheel begins to rotate at a constant angular velocity of 10 rad/s, CW. On the grids below, CAREFULLY sketch the θ vs. t, ω_z vs. t, and α_z vs. t graphs for the wheel's motion. The time axes are identical for all three grids. Label all the axes with numerical values.

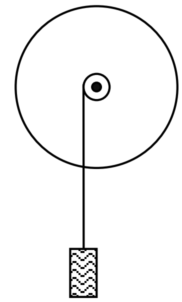






4. Consider the following demonstration: a bicycle wheel has a hub around which a string is wrapped. The other end of the string is connected to a mass. The wheel is given an initial spin so that the mass **initially rises**. A coordinate system typically has right = $+x$, up = $+y$, and out-of-the paper = $+z$.

Solid dot is the axle around which the wheel rotates



Answer with unit vectors whenever possible.

(a) What is the initial direction of the velocity of the mass?

(b) What is the direction of the acceleration of the mass?

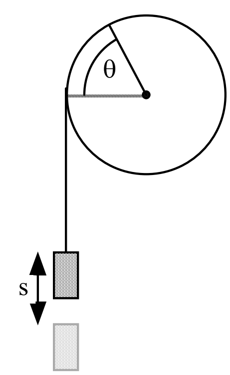
(c) Is the wheel's initial angular velocity CW or CCW, and what is its vector direction?

(d) Is the wheel's angular acceleration CW or CCW, and what is its vector direction?

(e) Your friend stands on the other side of the wheel from you, but uses the same coordinate system. How would his answers differ from yours for the four parts above?

5. Relation between Angular and Linear quantities.

(a) A wheel of radius R has a string wrapped around the rim and connected to a mass. The string does not slip or stretch. When the wheel rotates through an angle θ, by how much does the mass rise? Why are radians a better choice than using degrees here?



(b) A point along the wheel will also move a distance s when the mass moves. What's the technical name for this distance when it wraps around a circle like this?

(c) When a rigid wheel rotates, any point on the wheel has a velocity $\vec{v}$ that is tangent to the wheel. What is the relation between the (rotational) angular speed ω and the (linear) tangential speed v_t for a point out at some constant radius R?

Hints:
Step 1: Draw a small sketch of this so you understand the words here.
Step 2: Write the arc length relation and differentiate both sides with respect to time.
Step 3: Check that the units make sense.

(d) Repeat the same procedure to write a similar simple relationship between the angular acceleration α and the tangential acceleration a_t.

(e) If you've got an rotating object, possibly not even a circular one, which set of variables, $\{\theta, \omega, \alpha\}$ or $\{s, v, a\}$, would it make more sense to use, and why?

(f) Can the magnitude of a velocity change without a change in the direction of the velocity? (If yes, give an example and draw a sketch of the case).

(g) Can the direction of a velocity change without a change in the magnitude of the velocity? (If yes, give an example and draw a sketch of the case).

(h) Along the line of the previous questions, what is the relation between the angular speed ω and the radial (centripetal) component of the acceleration a_r (from University Physics I)? Draw a sketch of this case, along with the radial component of the acceleration.

6. A wheel rotates initially 0.250 rev/sec clockwise. After 1.50 sec it has rotated to an angle 65.0° clockwise relative to the initial orientation.

a) Find the angular acceleration, assuming that it is constant.

b) Find the time t at which $\omega = 0$.

c) Find the angle θ at which $\omega = 0$.

d) Sketch graphs of the variables θ, ω, and α versus time t in the interval t = 0 to 1.50 sec.

Your Name (**Print**): ______________________________ Date: __________

Group Members: ______________________________

______________________________ Group: __________

The Right Hand Rules

This is a warm-up exercise to get some practice laying down a coordinate system, sticking to it, and using the right-hand rule for rotations. Start by drawing a 3-d coordinate system in the box below and placing one of the sheets on the table as a reference for everyone in your group.

Coordinate system used →

Now, each student hold your nametags in some starting orientation that's the same for everyone in your team, and sketch what the tag looks like from directly above in the 'before' box below. (Everyone's sketch for the first box should look the same).

Choose one of the following sequences, a different one for each team member, and sketch what the end result looks like from directly above in the 'after' box. Note that they are jumbled up in terms of the order that each one is done, just to make sure each person is thinking through the right-hand rule for themselves.

A) $\pi/2$ rad about x, followed by $\pi/2$ rad about y, followed by $-\pi/2$ rad about z.
B) $-\pi/2$ rad about z, followed by $\pi/2$ rad about x, followed by $\pi/2$ rad about y.
C) $\pi/2$ rad about x, followed by $-\pi/2$ rad about z, followed by $\pi/2$ rad about y.

Before Rotations: After Rotations:

Do not turn to page 2 until everyone has completed their sketch. Do your team's 'after' rotations all look that same?

No, you can't blame it on your lab partners, this time…….

When every team member has carried out their assigned rotations correctly, you will notice that the final positions are not the same, even though the only difference is the order in which they were carried out. The technical phrase for this is that "rotations do not necessarily commute".

The 'not necessarily' part is necessary because it may be possible that some rotations commute, but in general they do not. Repeat the same exercise using π rad in place of π/2 rad and see what happens:

Before: After:

This business was never a problem in University Physics I because we were only concerned with moving non-rotating things around, and the order doesn't matter in moving things from one position to another. Technically we say that 'translations commute'.

Physicists know exactly how to handle this extra complication of rotations not commuting, but it requires the use of matrix mathematics and will be covered in subsequent courses. This is why almost all of the examples in the text are rotations about a single axis that does not change direction. Sometimes this leaves students with the false impression that rotations are scalar operations and not operations on vectors. It's important to be aware that **rotations are operations on vectors**, it's just that we only concentrate on the one-dimensional rotations (rotations about a single fixed axis) to start with. Fortunately, most physical examples can be modeled using rotations about a single axis.

Your Name (**Print**): ______________________________ Date: __________
Group Members: ______________________________
______________________________ Group: __________

Let's Play 'What's Wrong With This Number?'
(or 'How not to drive your professor nuts with the lab reports')

Most of the marks lost on lab reports are simply due to not writing the final answer in the technically correct form. Always keep extra digits during intermediate calculations, but for recorded measurements or the final answer that you put a box around in the end, always follow these three simple rules;

- ✓ **Units on the number** (unless it happens to be a unitless ratio).
- ✓ **1 significant figure on errors** (sometimes 2 sig. figs. if it starts with a "1" or "2").
- ✓ **Digit placement matching** on number and error, no exceptions.

1) How many sig. figs. are the following numbers?

__________ 24.7
__________ 0.00247
__________ 0.003
__________ 2000
__________ 2.00×10^3

2) Each of the following measurements of a length is screwed up somehow, and would cause anyone in a technical trade to shudder. State what is wrong with each number and write a corrected version. If the number cannot be corrected due to lack of information, state that and explain why.

a) (24.7 ± 0.367) cm

b) (95.8 ± 2) mm

c) (24.7 ± 0.007) cm

d) $(1/2 \pm 1/32)$ mm

e) 4.35 cm ± 1.2 mm

f) (5.345 ± 0.004)

Pay attention to units. You will always loose marks on labs and exams if you don't have the proper units on your numbers.

- July 23, 1983. Air Canada flight 143 ran out of fuel in mid-flight at 26,000 ft. Amazingly, the pilots were able to glider land the plane and nobody was seriously hurt. The whole thing was traced back to a refueller who, when asked by a mechanic what the density of the fuel being loaded was, said "That's easy, 1.77". He should have said "1.77 lbs per liter" because the mechanic assumed he meant "1.77 kg per liter" with everything in the Metric System. He asked this because there was a fuel gauge problem and they were calculating by hand how much to add to top off the tanks. Of course, the calculation was triple-checked by three people, but that was useless because there were no units to begin with. After this incident, all aircraft around the world use only the Metric System.

- Sept 23, 1999. NASA loses a $125 million Mars orbiter which exploded to bits in the Martian atmosphere. This somewhat expensive fireworks show was all because a subcontractor used Imperial pounds-force instead of Metric newtons in the coding of a software file. This is a common problem with computers because programmers seldom emphasize what units they are using in the actual code.

This is serious business. There are <u>hundreds</u> of other examples like this, some resulting in lost lives. It doesn't matter what technical field you enter; engineering, computer programming, pharmaceuticals, any field, always put the correct units on your numbers!

Your Name (**Print**): ______________________________ Date: __________
Group Members: ______________________________
______________________________ Group: __________

Physical Pendulum - Period

This activity is designed to reinforce the experimental skills you learned in your previous physics class, specifically, measurement errors, error propagation and graphical analysis. Review Part I of Dr. Lindberg's "Uncertainties, Graphing, and the Vernier Caliper" manual starting at http://www.rit.edu/cos/uphysics/uncertainties/Uncertaintiespart2.html. Review Part II of the manual regarding graphing techniques starting at http://www.rit.edu/cos/uphysics/graphing/graphingpart1.html.

Each group will hand in one report along with one hand-drawn graph at the end of class.

Review: Formulae for straight line slopes

1. In University Physics I you may have taken data to examine the drag force acting on one or more falling coffee filter(s). The magnitude of the drag force D (equal to mg when the filter reaches terminal speed) is related to the time t to fall a distance L

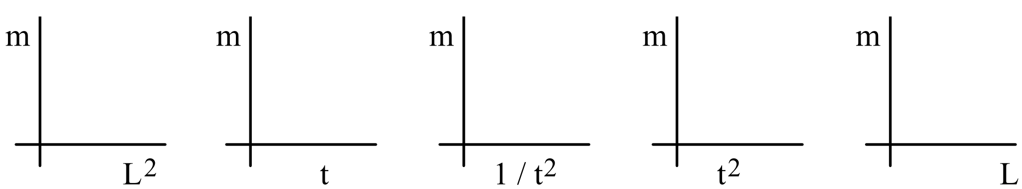
$$D = mg = \frac{1}{2} C \rho A \frac{L^2}{t^2} \qquad (1)$$

where m is the mass that is falling, ρ is the density of air, A is the cross-sectional area of the falling object, and C is the drag coefficient (which we assume is constant). There are many ways in which this equation can be used. For example, you can measure the distance L that a given filter falls in a fixed time (for example, t = 60 seconds) with different amounts of mass m attached to it. Or, you can measure the time t is takes for a given filter to fall a fixed distance (for example, L= 10 m) with different amounts of mass m attached to it.

a) Circle the symbols listed below that represent quantities that are variables in the equation above.

m ***g*** ***C*** ***ρ*** ***A*** ***L*** ***t***

b) On each set of axes below, sketch the graph that will result from plotting the quantities indicated.



c) For each graph in b), **if it is a straight line**, write the equation for the slope:

slope = ________ ________ ________ ________ ________

2. A physical pendulum is a rigid object constrained to rotate around an equilibrium position. The period T is the time required for the pendulum to complete one full oscillation. For a thin ring of diameter d, the period is

$$T = 2\pi\sqrt{\frac{d}{g}} \qquad (2)$$

where g is the magnitude of the gravitational acceleration. You will measure the period for five rings with different diameters and graphically verify this relationship. Finally, you will determine a value for g, with an uncertainty.

a) Circle the symbols listed below that represent quantities that are variables in Equation (2).

T   **2**   ***π***   ***d***   ***g***

b) Decide what variable you will plot on which axis to produce a **<u>linear graph</u>**:

we will plot _______________ versus ______________.

[NOTE: A graph of *A versus B* means *A* is plotted on the vertical axis and *B* is plotted on the horizontal axis.]

c) Rewrite Equation (2) to express it in the form $y = mx + b$.

d) Write the symbols from your equation in c) that represent the following:

Plotted on vertical axis ___________   Plotted on horizontal axis ___________

Slope of the graph ___________   Vertical intercept of the graph ___________

e) Now **<u>label</u>** the axes on the grid below with appropriate symbols and SI units, and **<u>sketch</u>** what you expect the graph to look like.

Measurements

Record all your measurements in neat tabular form. Each column heading should include the units of the numbers in that column. Remember that uncertainties have units, that uncertainties should have only one or two significant digits, and that a quantity should always be rounded off according to its uncertainty. *<u>This is the main area where people tend to loose marks on labs.</u>*

Support a ring from a knife edge and set it into <u>small</u> amplitude oscillation (use a maximum angle that is less than ~20^o). Using a stopwatch, determine the period of oscillation by finding the time to complete something like 10 or more complete oscillations. The more data you have, the smaller your uncertainties will be. You'll have to gauge what a good number for each size ring is. (And be careful how you count, it's very common to make a ± 1 count error with this). Make something like 5 measurements of this time (partners… take turns) in order to find the mean period and the uncertainty in that quantity. Determine the inside and outside diameters of the ring and use an average value in order to treat it as a <u>thin</u> ring.

Repeat those measurements for a total of at <u>least</u> 5 rings of different diameter. Just carry one at a time to your table to allow others to choose varying sizes too.

DATA TABLE(S):

Analysis

Construct (**by hand)** the graph you chose in part 2e). Data points should have uncertainty bars on them for both axes if they're large enough to show up on the graph. Be sure to label the axes with names and units, and give the graph a descriptive title.

3. Does your graph produce a linear relationship? What, if anything, does this prove?

4. Determine the slope. Choose two widely-spaced points on the line that are *not* data points (you don't want to be biased here) and use them to calculate the slope of the line. Indicate those points on the graph and show the details of the calculation below.

5. Determine the uncertainty in the slope by calculating the minimum and maximum slope you might have tolerated in your choice and use those to estimate a single uncertainty on your slope. Show the details of the calculation below.

Write the slope in the form $slope \pm \Delta slope =$ ___________________________________

6. Using the slope and its uncertainty, determine a value for g and its uncertainty. Show all work below.

Write g in the form $g \pm \Delta g =$ ___________________________________

7. Is the value of *g* equal to the expected value? If they don't agree, that's technically ok, just state that's what your result was, but try to offer reasons about what could have happened to account for the discrepancy. And never use the term 'human error'. It is technically meaningless. Any mistakes should be traced back to their experimental origin and corrected.

8. From your graph, **directly read** the vertical intercept and its uncertainty.

9. Is the vertical intercept equal to the value you predicted in 2d), within experimental uncertainty?

10. In the space below, write an **abstract** of this lab report. The word 'abstract' means a **concise** scientific paragraph or two that cuts to the core of a report in very few words, and you'll see them at the beginning of all real-life technical reports. Talk among your group about how to word it as concisely as possible. Start with a one-sentence overview of the whole thing. Then write a sentence about the mathematical or theoretical background. Describe in one or two sentences how you made the measurements and analyzed them. In another sentence, give your final result with its uncertainties. State whether or not your result agrees with the mathematical value of *g* within the uncertainties. If it does not, add another sentence explaining which uncertainties may have been underestimated, or possible suggestions about what might have gone wrong (this is also done in the real world).

    Remember to be concise. Pretend this is like a classified ad where you're getting charged for every letter used. It should be one paragraph only and not even close to running off the end of this page.

    (In a real report, this would go at the very beginning of the scientific paper, but we're just getting some practice here)

    ABSTRACT:

Your Name (**Print**): ______________________ Date: __________
Group Members: ______________________
______________________ Group: __________

Torque

1. Five forces, labeled A through E, act on a rigid object. The object is constrained to rotate about an axis perpendicular to the page and passing through the circled dot shown on the diagram. For each force, A through E, state whether the torque caused by the force is positive, negative or zero (take $+\hat{k}$ to be out of the paper).

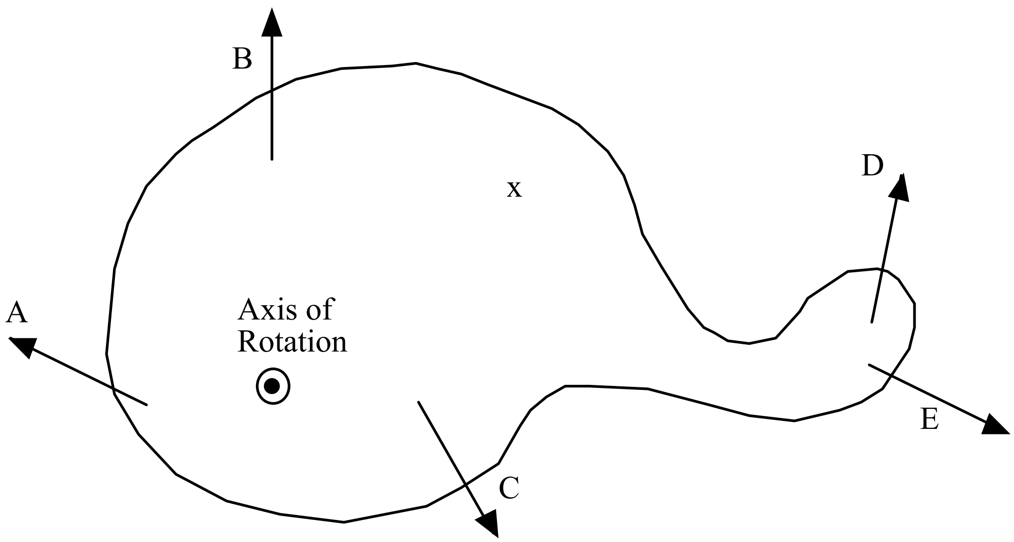
A ___________

B ___________

C ___________

D ___________

E ___________



Assume that all forces are of the same magnitude. Rank the forces A through E in terms of the **magnitude** of the torque that they will produce, largest to smallest.

Largest ____ Smallest

Consider a different axis of rotation located at the point marked "x." Again tell whether the torques are positive, negative or zero (and again take $+\hat{k}$ to be out of the paper).

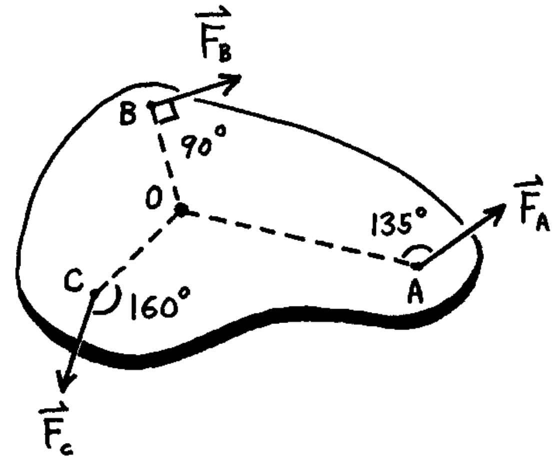
A ________ B _______ C _______ D _______ E _______

(Note here that the torque can be evaluated at any point, not just an axis of rotation).

2. The body shown in the diagram is pivoted at O. Three forces act on it:

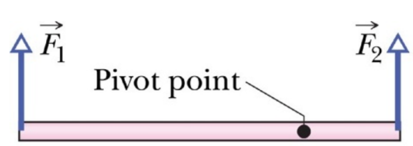
$F_A = 10.0$ N at point A, 8.00 m from O;
$F_B = 9.00$ N at point B, 4.00 m from O;
$F_C = 12.0$ N at point C, 5.00 m from O.

What is the net torque about O? Choose $+\hat{k}$ to be out of the paper.



3. Shown below is an overhead view of a horizontal bar that can pivot. Two horizontal forces act on the bar, and the bar is stationary. If the angle between $\vec{F}_2$ and the bar is now decreased from 90° and the bar is still not to turn, should F_2 be made larger, made smaller, or left the same?

Use correct physics principles to explain why.



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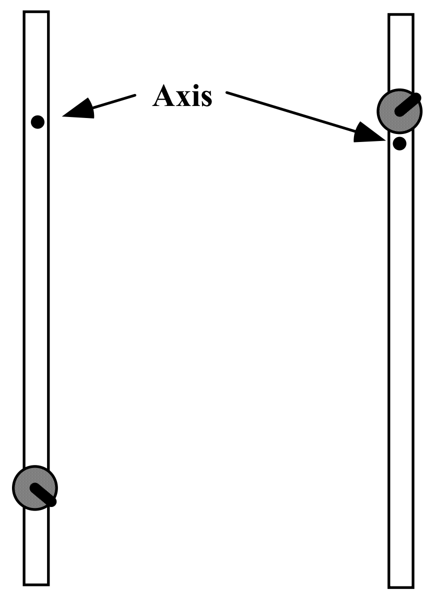
Moment of Inertia (a.k.a. Rotational Inertia) and the Parallel Axis Theorem

311 Review: Consider Newton's second law. It can be thought of as a way to define mass: mass is the proportionality constant that relates the net force acting on an object and the acceleration of that object. Suppose you hit two different masses with the same net force, which one will move the most, the one with the **larger** mass or the one with the **smaller** mass? _________________

Tape some 50 or 100 gram masses to a meter stick, less than 20 cm from the end. Then hold it at one "axis," 20 cm from the end, and rotate it. Now hold it at the other "axis," 20 cm from the other end. Try to rotate it by applying the force at the same distance from the axis with the same force. Gently swing it back and forth like a sword to get the feel of it. Does it resist being swung (angular acceleration) more or less now compared to the first case?



Everyone should try this, and everyone should watch while their partners try it. Take care to try to move it with the same force or 'twist' (torque) in both cases.

The equivalent of Newton's second law for rotation written out in words is:

Net Torque = Moment of Inertia * Angular Acceleration.

This can be thought of as a way to define moment of inertia: moment of inertia is the proportionality constant that relates the

________________ acting on an object to the ________________________________ of that object.

If the moment of inertia is the proportionality constant relating torque [N·m] and angular acceleration, what must its units be in terms of kg, meter, and sec?

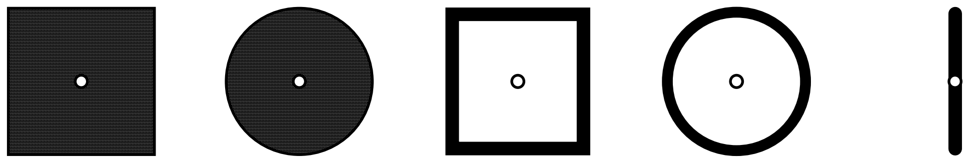
Dimensional Analysis: (You can tell a lot about something just by looking at the units).

In which case was the moment of inertia larger, when the masses were close to the axis or when they were far from the axis of rotation?

If you attached more mass to the meter stick, would you expect the total moment of inertia, I, to increase or decrease? If you doubled the mass, how would I change? (ignoring the mass of the stick)

If you moved the masses **farther from** the **axis** of rotation, would you expect its moment of inertia, I, to **increase** or **decrease**? If you doubled the distance, how would I change?

Brain-Buster: Diagrams of 5 objects (a solid square, a solid disk, a hollow square, a hoop, and a thin rod) are shown below. The center is the axis of rotation (out of the page) around which they rotate. The objects all have the <u>same total mass</u>, and the vertical dimension is the same for all of them.



______ ______ ______ ______ ______

Without looking at any formulas from the table in the book just yet, rank the objects from the smallest [= 1] to the largest [= 5] moment of inertia. Explain how you made your ranking.

Given a uniform solid cylinder and a uniform solid sphere, both of the same radius and mass, which one has the smaller moment of inertia? Assume the rotation axis passes through the center of mass and is along the central axis for the cylinder.

Answer the previous question assuming the cylinder and sphere are hollow.

Something to be careful of: If the moment of inertia *I* depends in general upon the axis of rotation through any given object, is the moment of inertia a vector or a scalar?

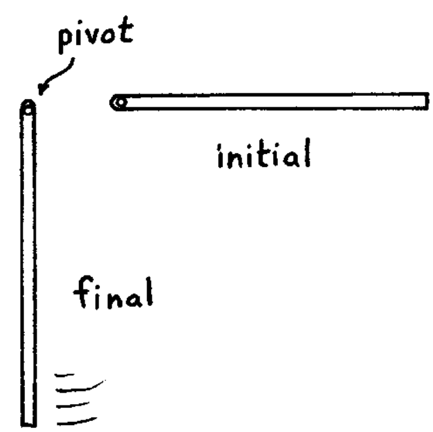
The text has a table of the moments of inertia of various simple objects provided the objects are rotated about their centers of mass. We can find the moment of inertia about points other than the center of mass of an object using something called the Parallel Axis Theorem. We will revisit this later, but it is simple to state and start using now. The Parallel Axis theorem states;

$$I_p = I_{cm} + Md^2$$

where d is the distance from the center of mass to the axis of rotation, and I_{cm} is the known center of mass (from the tables) about an axis parallel to the axis of rotation.

Q1) Use the parallel axis theorem and a formula from the textbook to find a formula for the moment of inertia I about one end for a uniform rod of length L and mass m.

Q2) (Challenging) Consider holding a meter stick horizontally. It is pivoted at one end and released from rest. What will be the angular speed of the meter stick when it is momentarily vertical?



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Vector Cross Product Practice

Evaluate the following cross products. There are three common ways to calculate these, all of which are equivalent and basically do the same thing;

a) Using the RH rule on the unit vectors.
b) Putting the components into the cyclical formula.
c) Using the determinant method.

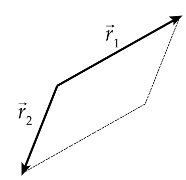
Have each person in your group use a different one of the above techniques for the first question, and then rotate your choice of techniques for the next ones.

1) $(2\text{ m})\,\hat{i}\ \text{x}\ (3\text{ N})\,\hat{j}\ =$

2) $[\,(2\text{ m})\,\hat{i} + (3\text{ m})\,\hat{j} - (2\text{ m})\,\hat{k}\,]\ \text{x}\ [\,(3\text{ N})\,\hat{i} - (4\text{ N})\,\hat{k}\,]\ =$

3) $[\,(7\text{ m})\,\hat{i} + (4\text{ m})\,\hat{j}\,]\ \text{x}\ [\,(-2\text{ m})\,\hat{i} - (5\text{ m})\,\hat{j}\,]\ =$

The magnitude of this last vector product is the area of the parallelogram formed by the vectors $\vec{r}_1$ and $\vec{r}_2$ $(A = |\vec{r}_1 \text{ x } \vec{r}_2|)$.



Main points to remember about the cross product

These are things that people very commonly make mistakes with on exams, so make sure you're clear about each one of these:

- ✓ Just like the dot product, the <u>cross product has units</u>. The units are the usual product of the units of each of the vectors in the cross product.

- ✓ Unlike the dot product, the <u>order of the vectors matters</u> in the cross product. Reverse the order and there's a sign flip in the result. Just think about the RH rule and you'll see that this is the case.

- ✓ The result of the cross product between two vectors is another <u>vector</u>, so don't just write a number down. Typically there will be three components, so show the proper unit vectors for each direction. (If a component happens to be zero, you can just leave it out, but all non-zero components must have the appropriate unit vectors next to them).

Your Name (Print): ______________________________ Date: __________
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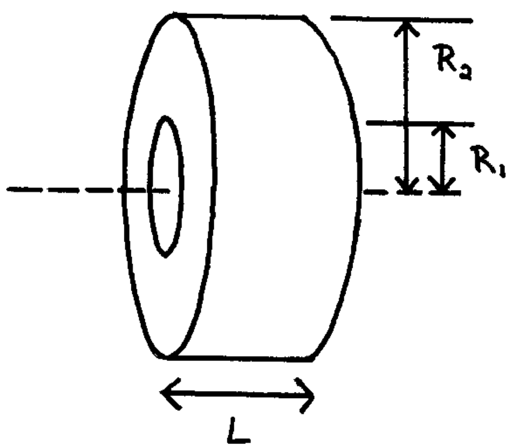
Uncertainty Exercise

Consider the following object shown below, with the given dimensions and mass.



M = (1.0672 ± 0.0002) kg
R_1 = (2.503 ± 0.001) cm
R_2 = (4.021 ± 0.001) cm
L = (2.627 ± 0.001) cm

1) Without looking at the text tables, first derive the formula for the moment of inertia of the object about its cylindrical axis.

2) Does your answer make sense in the limits? For example, what form does the equation take when $R_1 \rightarrow 0$, or when $R_1 \rightarrow R_2$. Do these make sense in terms of other figures in the text table?

3) Now use this formula to calculate the moment of inertia of the object, along with the corresponding uncertainty on the final answer. Write the answer in the form ($I \pm \Delta I$).

Your Name (**Print**): ________________________ Date: __________

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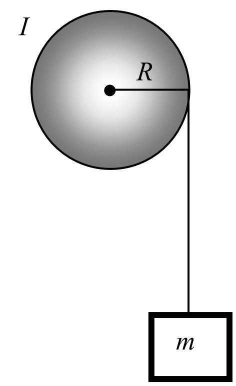
Rotational Inertia - PreLab Exercise

This problem is similar to a configuration arising in an upcoming lab, so it's very important to go through this in detail and get the signs correct here.

Mass, String and Wheel



A wheel of radius R with moment of inertia I is mounted on a frictionless horizontal axle. You attach a light string to the wheel and wrap the string around the wheel. On the free end of the string there is a block of mass m attached. While you were wrapping the string, you held the wheel so it would not turn. At time $t = 0$ you release the wheel so the block begins falling, causing the wheel to turn.

a) Lay down a coordinate system on the side of the diagram, and stick to it in all of the following steps.

b) Draw the force diagram for the descending mass m in the space below. Let T represent the tension in the string.

c) Write down Newton's Second Law for the block in terms of T, m, g, and its linear acceleration a.

d) Draw the force diagram for the rotating wheel.

e) Write down Newton's Second Law in rotational form for the wheel in terms of T, I, R, and its angular acceleration α. Leave I as an unknown, because we want to leave it completely general here and not assume the wheel is a thin disk (in fact, in the upcoming lab, it's not).

f) What is the relationship between the linear acceleration of the mass and the angular acceleration of the wheel? Be careful about signs! Assume the string does not slip across the wheel and that it does not stretch.

g) Combining your results, derive an expression for moment of inertia of the wheel in terms of its angular acceleration, its radius and the mass of the block. (This is essentially the configuration you will be presented with in the lab).

h) Check that your answer 'makes sense in the limits'. What's meant by this is that you should check your equation to make sure the answer is what you'd think it would be if, for example, the moment of inertia of the wheel were to be negligible or extremely large, or the mass of the weight were to drop to zero, etc. Engineers and physicists will often do this as a check to make sure there are no sign errors. This business of checking things 'in the limits' doesn't guarantee an answer is correct, but it does reveal very fast if an answer is wrong.

Your Name (**Print**): ______________________ Date: __________
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______________________ Group: __________

Rotational Inertia

The goal here is an **accurate** determination of the moment of inertia of an annular ring and the moment of inertia of the two brass masses flush at the end of a rod (including the attachment screws, but **not** including the rod). This will be done by checking it two ways; (1) direct dynamical measurement (experimental) and (2) using the moment tables in the text (theoretical or static). **Both of these will have an associated uncertainty**, and there will be some overlap if they are in agreement.

The Rotary Motion Sensor (RMS) has an axle with a triple pulley. A string is wrapped around the RMS 3-step pulley, passed over a "massless" super pulley and attached to a hanging mass that will cause the system to accelerate. You will start this system from rest and measure the angular acceleration of the axle. From your measurements you should be able to do two things: (1) determine the rotational inertia of the RMS pulley/axle and of the object attached to it from these dynamical measurements; and (2) compare this result to the rotational inertia calculated theoretically from the mass and dimensions of the object.

Equipment list

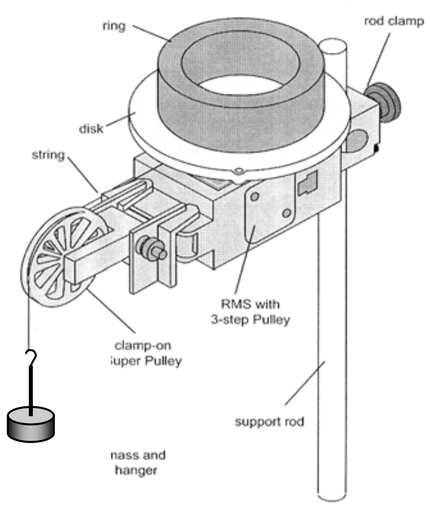

Figure 1. RMS with base plate and ring.

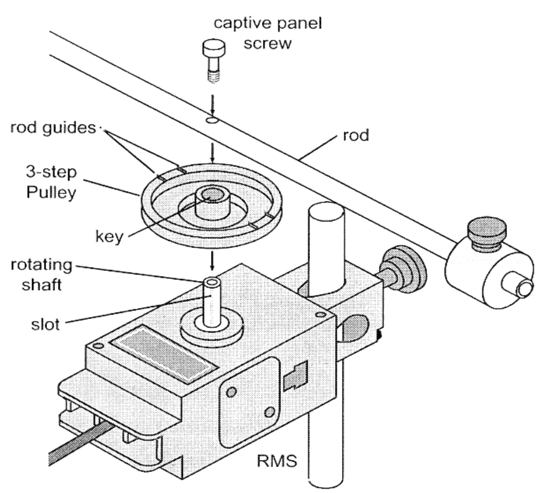

Figure 2. RMS with rod.

Rotary Motion Sensor with aluminum base plate, black ring, rod and masses for the rod.
Team Physics Lab Computer with LoggerPro software and LabPro interface
Super Pulley (which is so light we assume it's massless).
Hanging Mass (use a brass 50 gram mass hanger, the plastic hangers break)
Support rod and table clamp
Physics thread (which is so light we assume it's massless).

Do not lose the screws! This happens in every lab. If you don't remove them from the rod, then you won't lose them.

Setup Details

Using the drawings as a guide, attach the RMS to a support rod held in a table clamp on your table. Carefully clamp a Super Pulley on the end of the RMS as shown. The Super Pulley should be oriented so its top is level with the largest pulley on the 3-step pulley.

Attach the RMS cable through the short adapter cable into the DIG/SONIC 1 port of the LabPro box. Plug in the power to the LabPro box **before** connecting it to the laptop with the USB cable. After you log onto the computer, go to Student Shares (on Desktop or My Computer)… 3xxUniversity Physics Students… 312_389 University Physics II… 1LoggerPro. In that folder you will find a file named Moment_of_Inertia.xmbl. Drag that icon onto the LoggerPro icon that is on the desktop. This will launch LoggerPro using the Moment_of_Inertia file. In LoggerPro, when you click the Collect button, you should start getting a graph of angle versus time for the RMS. Try it (start the RMS wheel spinning while making a graph). When you click Collect again, it should erase the old graph and start a new one.

Measurements I – Black-Anodized Annular Ring

1. Cut a piece of thread slightly longer than the distance from the RMS to the floor. Tie a small knot in one end and wrap the thread around the ***largest*** pulley in the 3-stage pulley. **ALWAYS wrap the thread so the pulley rotates in the positive sense when the suspended mass falls to the floor.** Use the little groove (**not the hole**) on the rim of the pulley to hold the knot so the thread will come off easily once the mass hits the floor. Put the 50 g brass mass hanger on the free end of the thread. Without letting go of the 3-stage pulley, remove the screw that holds it on the axle and attach the aluminum base plate. There is a square hole on one side of the aluminum plate that fits over the end of the 3-stage pulley. Put the base plate on the axle and replace the screw. (Do not place the black ring on for the first part. We first need a measurement of the background I to subtract off).

   Always hit *Control '0'* prior to starting to Collect. This resets the RMS to zero radians so that you're not racking up thousands of radians each time it rotates.

2. Make sure the string is carefully wrapped around the largest pulley and the Super Pulley is free to spin without rubbing anything. Click the Collect button. ***After the graph begins***, release the 3-stage pulley so the mass hanger falls. If either of the plots appears choppy, go to the 'Experiment… Data Collection' menu and try fiddling with the sampling rate (100 → 20 seems to work), or maybe try changing the hanger mass. **Feel free to alter the 50 g mass you've used if you think it might be too heavy**, but make sure to weigh (as a double-check) and record any new choices. When the graphs have finished, determine which portion of each graph corresponds to the time interval during which the mass is descending. Use these portions (and only these portions) to determine the angular acceleration from the graphs. It might be a good idea to make sure different methods are in agreement and choose one which gives the least uncertainty. Print one set of graphs for each group member. Make sure the curve fitting parameters (including uncertainty) are visible on the printed graph.

3. **Repeat step 2 several times.** The numbers will tend to jump around a little bit, and you should use this variation in the results to come up with an estimated uncertainty for your experimental angular acceleration.

4. Now place the black ring on the aluminum base plate. Note that there are two studs on the ring that fit into holes in the base plate. Repeat steps 2 and 3 with the ring on the base plate, still using the 50 g mass hanger on the string.

Measurements II – Other Data Required

5. To complete your static calculation of the moment of inertia of the annular ring you must decide which lengths, distances, masses, diameters, etc. you need to measure, and then measure them. Other than the diameter of the pulley that the string was wrapped around, you will NOT need to measure the aluminum base nor anything inside the RMS itself to calculate the dynamic measurement of the rotational inertia (think about this).
   Record the measurements in a neat, easily read format. Remember to include **units** and **uncertainties.** As a general rule, always have another lab partner verify your measurements independently.

Analysis

6. Use the formula derived in the Pre-lab Exercise to calculate the rotational inertia of the 3-stage pulley plus aluminum base plate ($I_{puley+plate}$) from the measured angular acceleration determined from the graphs. Also calculate the uncertainty in this rotational inertia.

7. Use the formula derived in the Pre-lab Exercise to calculate the rotational inertia of the pulley + plate + ring system ($I_{puley+plate+ring}$) in terms of the angular acceleration determined from the graphs. Also calculate the uncertainty in this rotational inertia.

8. Using the above two results, calculate the moment of inertia of the ring (and its uncertainty). Hint: If your pet porcupine won't stand still on the bathroom scale, how do you weigh it?

9. Use a moment of inertia formula from the textbook and your various length/mass measurements to determine the ***static*** moment of inertia of the annular ring. Determine the uncertainty in this value.

10. Compare your ***dynamic*** result from step 8 with the ***static*** result from step 9. Are the two values equal within experimental error? Discuss reasons for any discrepancy, as well as general sources of error.

Measurements III – Two Brass Masses (ask your instructor whether this part is required)

11. Remove the black ring and aluminum base plate. Put the screw in a safe place so it doesn't get lost! As shown in Figure 2, turn the 3-stage pulley over and mount the rod on it using its built-in screw. Note that there is a tab in the pulley and a slot on the axel for it. **Don't jam it on**. You will need to readjust the Super Pulley so its top is level with the largest part of the 3-stage pulley.

12. Repeat steps 2 and 3 with the rod. **Feel free to alter the 50g mass you've used if you think it might be too heavy**, but make sure to weigh (as a double-check) and record any new choices. This will allow you to determine $I_{pulley+rod}$.

13. Put the two masses back on the ends of the rod, flush with the very end of the rod. Repeat steps 2 and 3 with the rod and two masses in this configuration. This will allow you to determine $I_{pulley+rod+masses}$.

14. Using the above two results, calculate the moment of inertia of the two masses (and its uncertainty).

15. Calculate the ***theoretical*** value of the moment of inertia of the two masses (and its uncertainty). First treat the two masses on the rod as point masses and see if this is good enough for agreement with experiment, but for a more accurate approximation, also try treating them as solid cylinders (ignoring the bore hole through the axis) shifted out by the parallel axis theorem and see if this provides better agreement with the experimental result.

Your instructor will give you the details of the due date and what must be included in the report.

Your Name (**Print**): ______________________________ Date: __________
Group Members: ______________________________
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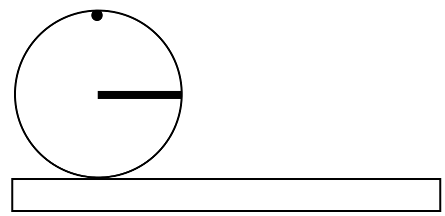
Rolling and Ramps



1) **Warm-up:** Consider a rolling wheel. What is the instantaneous linear velocity of a point at the bottom of the wheel in contact with the ground?

What is the relation between the velocity of the wheel as a whole (essentially the linear velocity of the axle) and the wheel's angular velocity?

What is the instantaneous linear velocity of a point at the top of the wheel, relative to the ground, in terms of the angular velocity?

2) A hollow 45.8 g cylinder of radius R = 2.25 cm rolls down a 27.3° ramp starting from a height of h = 1.50 m. The cylinder does not slip or slide as it rolls down. What is the linear speed v of the cylinder at the bottom of the ramp?

3) Which object would win a rolling race to the bottom of the ramp; the hollow cylinder above or a solid cylinder of exactly the same mass and radius?

First, each person make an intuitive prediction individually and circle one of the following;

Hollow cylinder wins the race **Solid cylinder wins the race**

It's a tie between them **My lab partner is generally in charge of these things**

Now use the awesome power of physics to predict mathematically which one will be the case. Discuss this with your team so that your whole group is in complete agreement about which one it will be.

Brain-buster: If the two cylinders above are rolled up the ramp, both starting with the same initial velocity, which one will roll to the highest point up the ramp?

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Rotational Inertia – Thin Rod
(Sum of Segments)

A thin rod has a length *L* and a total mass *M* which is spread uniformly along the rod. Find the rotational inertia of this rod about one of its ends using the following approximate method.

a) Break the rod into *N* short segments each of the same length (each table will be assigned two values of *N* by the instructor).
b) Find the mass m_i of each segment.
c) Find the distance r_i from the axis of rotation to the center of mass of each segment (midpoint of the segment).
d) Approximate the rod as *N* point masses at these midpoints and determine the rotational inertia of these point masses about the end of the rod. Express your answer as αML^2 where α is a number. Write α to four decimal places.
e) Place your results for α in the table below and include the results from other tables.
f) What is the limiting value of α as the number of segments increases (as the length of each segment decreases)? Is this consistent with the value expected using the tabulated value of I_{com} and the parallel-axis theorem?

N	***α***	***N***	***α***
1		8	
2		9	
3		10	
4		11	
5		12	
6		13	
7		14	

g) If you finish your calculation before all of the groups have reported their results, redo the calculation for *N* segments where *N* is an unspecified integer. With this result, take the limit as *N* approaches infinity and write an expression for the rotational inertia of the rod when it is divided into *N* segments (then check and make sure your classmates have the correct values in the table above!)

Note: $\sum_{k=1}^{N}(2K-1)^2 = \frac{1}{3}N(4N^2-1)$

Your Name (**Print**): ______________________________ Date: ___________
Group Members: ______________________________
______________________________ Group: ___________

Rotational Inertia – Integration
(One Dimensional)

The rotational inertia (or moment of inertia) for a continuous mass distribution is defined as

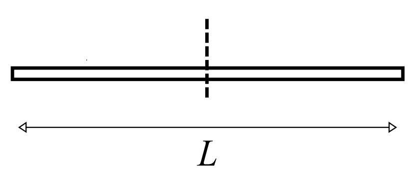
$$I = \int_{\substack{\text{rigid}\\\text{body}}} dI = \int_{\substack{\text{rigid}\\\text{body}}} r^2 dm$$

where r is the perpendicular distance of the mass dm from the axis of rotation.

The method to use is:

- draw a diagram showing the object and the axis of rotation
- choose a coordinate system that you think will be best to use
- pick a "slice" (or piece), dm, a perpendicular distance r from the axis of rotation; the piece should not be at the middle or the end of the object; show the distance r on your diagram
- write an expression for dm in terms of a small element of whatever coordinate(s) you have chosen
  $dm = \lambda\, dx$ for linear objects, where λ is the mass per length and dx is the length of dm;
- substitute for dI in the integral definition
- do the integral.

1. Calculate I_{cm} for a long thin rod (for example, a meter stick) about an axis perpendicular to the rod, passing through its center of mass. Assume the mass is distributed uniformly along the rod. Express this moment of inertia as a function of its length L and mass M. As a check, compare your result to the table in the book.



2. Now calculate the moment of inertia for the same thin rod about an axis passing through the end of the rod. The new axis is parallel to the first. Do the integral and check your answer using the parallel axis theorem. ***Use correct physics principles*** to explain why the rotational inertia is greater when the axis passes through one end rather than through the center.

3. Consider the rotational inertia integral given by

$$I = \int_{R-\frac{L}{2}}^{R+\frac{L}{2}} \lambda x^2 \, dx$$

where λ is a constant.

a) What physical situation does this correspond? Draw the picture, clearly label the object, the rotation axis, and all relevant distances/dimensions.
b) What physical mass distribution does the result correspond to in the limiting case of $R >> L$?
c) Evaluate the integral and compare it to the result from the parallel-axis theorem.

4. Now consider a rod of total mass M and length L with a non-uniform mass distribution; specifically, let the mass/length be given by

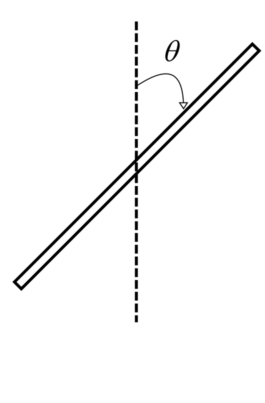
$$\lambda(x) = (2.00\ kg/m) \ + \ (1.00\ kg/m)\frac{x^2}{3L^2}$$

The ends of the rod are at $x = -L/2$ and $x = +L/2$.

a) Before you start calculating, what does this variable mass density mean in a physical sense? How might you construct such a rod?
b) Derive an expression for the total mass of the rod. What is the numerical value of the total mass of the rod if the rod is 1.50 m long?
c) Determine the moment of inertia of the rod about an axis passing through its center at $x = 0$. Write the answer in terms of some constant times ML^2.
d) How does this result compare to that of a uniform rod pivoted about its center? Is this result reasonable?

Challenging Question:

Calculate I_{cm} for a long thin rod of mass M and length L oriented at an angle θ with respect to a vertical axis of rotation?
(Hint: you can compute this directly, just be careful that r is the perpendicular distance from dm to the axis of rotation, or you could use the conceptual trick of 'squishing it flat' along the axis of rotation, which doesn't change I about that axis).



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Rotational Inertia – Integration
(2D and 3D)

The rotational inertia (or moment of inertia) for a continuous mass distribution is defined as

$$I = \int_{\substack{\text{rigid}\\\text{body}}} dI = \int_{\substack{\text{rigid}\\\text{body}}} r^2\, dm$$

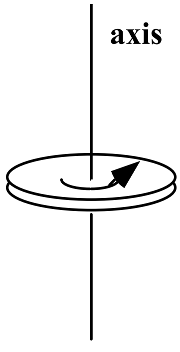
where r is the perpendicular distance of the mass dm from the axis of rotation.

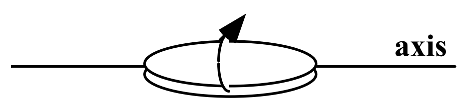
The method to use is:

- draw a diagram showing the object and the axis of rotation
- choose a coordinate system that you think will be best to use
- pick a "slice" (or piece), dm, a perpendicular distance r from the axis of rotation; the piece should not be at the middle or the end of the object; show the distance r on your diagram
- write an expression for dm in terms of a small element of whatever coordinate(s) you have chosen
  $dm = \sigma\, dA$ for 2-dimensional objects where σ is the mass/area and dA is the area of dm; dA will have to be expressed in terms of your spatial coordinates, for example, in Cartesian coordinates $dA = dx \cdot dy$)
  $dm = \rho\, dV$ for 3D objects where ρ is the mass/volume and dV is the volume of dm; dV will have to be expressed in terms of your spatial coordinates, for example, in Cartesian coordinates $dV = dx \cdot dy \cdot dz$)
- substitute for dI in the integral definition
- do the integral.

1. Calculate I_{cm} for a penny. Treat the penny as a flat, circular disk of mass M and radius R with a uniform mass distribution.

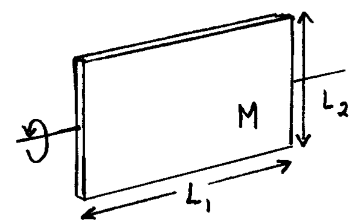
   a) Choose the axis to be perpendicular to the plane of the penny.
   b) Choose the axis to be in the plane of the penny.
   c) For both cases, also calculate the rotational inertia about a parallel axis passing through the edge of the penny.



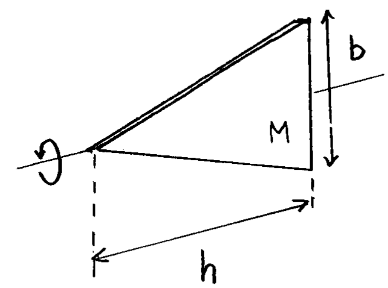


HINT: Cut the disk up into small segments for which all of the mass is at the same distance from the axis. Then decide over what variable you have to integrate.

2. Calculate I_{cm} for a thin rectangular plate about an axis parallel to the plane of the plate and passing through the center of mass. The plate has edge lengths L_1 and L_2, a uniform mass/area σ and a total mass M. Express your answer in terms of M, L_1, L_2 and constants. After you have completed your calculation, compare your answer to that in the book. There are a couple of ways to make the slices, so if you finish early, try making the cuts in the other direction to make sure you get the same answer.

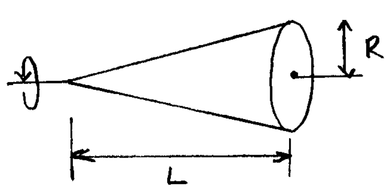


3. Determine the moment of inertia of a thin triangular slab as shown below. Be careful with the limits of integration. There are still a couple of ways to make the slices, but the answer should be the same either way.



<u>Challenge Question</u>:

Find the moment of inertia I_{cm} for a solid circular cone of mass M, length L and radius R at the base. Choose the axis perpendicular to the base, along the cone's axis. The cone has a uniform mass density $\rho = M/V$).



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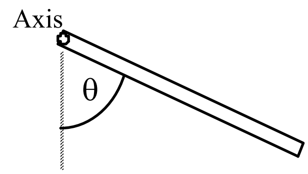
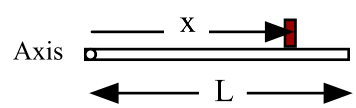
Swinging Rods

1. A thin rod, mass M and length L, is pivoted on a fixed axis at one end and falls due to its own weight. Find an equation for the angular acceleration of the rod when it is at the angle θ, in terms of M, L, g, and θ.



2. The same thin rod is used, and it is horizontal ($\theta = 90°$) at the time we are considering it. There is a mass M equal in size to the mass of the stick at a distance x from the axis along the rod.



   a) **Predict/guess**: For what value of x will the angular acceleration be the largest?
      - o at $x = 0$
      - o between 0 and L but closer to 0
      - o at $x = L/2$
      - o between 0 and L but closer to L
      - o at $x = L$
      - o The angular acceleration is the same in all cases

b) Find the angular acceleration in terms of M, L, g and x.

c) If $L = 1.0$ m and $M = 3.0$ kg, find the angular acceleration for the value of x given below. Each team try a different value of x, to cover all the bases quickly, then we'll put the results on the board to see the general trend.

Table **A**, Team 1: 0.1m and Team 2: 1.0m,
Table **B**, Team 1: 0.2m and Team 2: 0.9m,
Table **C**, Team 1: 0.3m and Team 2: 0.8m,
Table **D**, Team 1: 0.4m and Team 2: 0.6m,
Table **E**, Team 1: 0.5m and Team 2: 0.7m,
Table **F**, Team 1: 0.35m and Team 2: 0.65m
Table **G**, Team 1: 0.25m and Team 2: 0.75m

d) **Brain-Buster:** Determine analytically the value of x that will provide the maximum angular acceleration.

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Angular Momentum Introduction

1) Under what conditions is mechanical energy conserved?

2) Write down an equation that expresses Conservation of Linear Momentum. Under what conditions is it true?

By analogy, write down an equation that expresses Conservation of Angular Momentum. Under what conditions is it true?

3) Stand on a rotating platform with arms outstretched. Begin rotating and move your arms close to your body. What do you observe? Try it with the weights, but be careful; the effect is much stronger because most of the moment of inertia is in the weights extended out a distance from the axis of rotation.

Explain mathematically why this happens?

Take two time measurements to estimate the ratio of the rotational inertia with hands out to the rotational inertia with hands in. Record the necessary measurements in the space below.

$$\frac{I_{\text{hands out}}}{I_{\text{hands in}}} =$$

4) Standing on a stationary platform, move a weight back and forth in a straight line. First try having the line intersect the center of your axis of rotation, and then also missing the axis by half a meter or so. Does your body rotate to compensate? Would this mean objects moving in a straight line have angular momentum? Does angular momentum depend on the point you're using as your reference axis?

5) Stand on a stationary platform holding a rapidly-rotating bicycle wheel. Have one of your esteemed lab partners help you to get the wheel rotating while you hold it. Try to twist the rotation vector (right-hand rule) from horizontal to vertical. What do you observe?

Why does this happen? Draw arrows to represent the angular momentum of you and of the bicycle wheel before and after you move it. If a vector is zero, represent it by "0".

Try to flip the wheel axis from vertical upward all the way to vertical downward, again using the RH rule. What do you observe? Is the effect bigger or smaller than what you observed in the first part above? Why? Draw arrows to represent the angular momentum of you and of the bicycle wheel before and after you move it.

6) **Brain-Buster:** Compare a spinning gyroscope to a non-spinning gyroscope. If the axle is pointing toward you and you give its end a slight push to your right, observe what happens in both cases below.

When you push to the right, which way does the non-spinning one move? (This is trivial, not a trick question)

Now start the gyroscope spinning. When you push to the right, which way does the spinning one want to move? (The 'why' of this is a little tricky. Make a note in the form of a diagram which way you feel the force, and note which way the wheel is rotating on the diagram).

To understand the ‘why’ of this, start by writing down the rotational analogy of Newton’s Second Law written in terms of angular momentum.

We know that if you push on an object that changes its linear momentum, you ‘feel’ a force pushing back on you. Draw an analogy to the rotational version of this to see that if you ‘twist’ the gyroscope and change its angular momentum, you ‘feel’ a torque pushing back on you according to the rotational form of the same equation. Use the RH rule to understand the directions here. Draw a diagram of what’s going on here so you understand the directions:

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Angular Momentum Problems

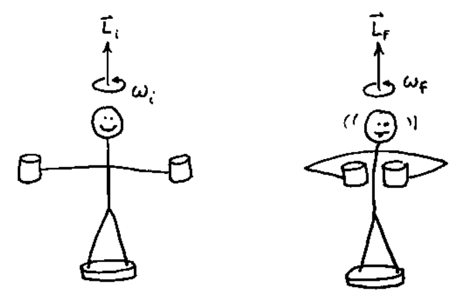
1) **Conceptual Warm-up:** Imagine you're standing on one of the rotating platforms as in a previous activity, holding the weights with your arms outstretched. You suddenly drop the weights without changing the position of your arms in any way. Your angular velocity will

a) increase b) remain the same c) decrease

d) suddenly stop e) suddenly reverse direction

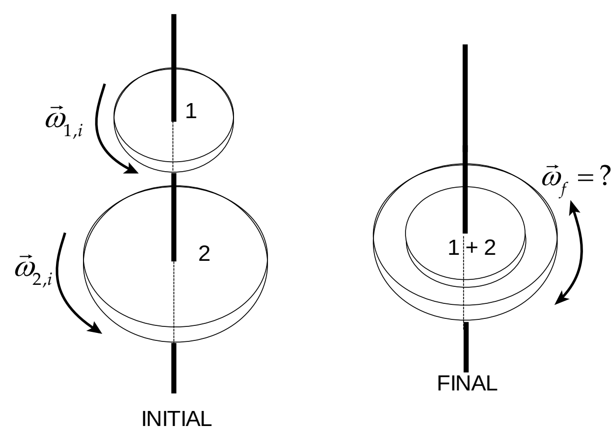
Explain the correct answer using correct physics principles.

2) A student stands on a platform that is rotating without friction with an angular speed of 1.20 rev/s. His arms are outstretched and he holds a weight in each hand. The rotational inertia of the system consisting of the student, weights, and platform about the central vertical axis of the platform is 6.00 kg·m^2. If by moving the weights the student decreases the rotational inertia of the system to 2.00 kg·m^2, what are



a) the resulting angular speed of the platform and
b) the ratio of the new kinetic energy of the system to the original kinetic energy?
c) If the kinetic energy decreased, where did it go? If the kinetic energy increased, where did it come from?

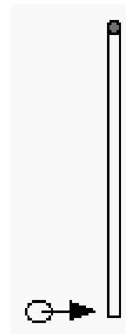
3) Two disks are mounted on low-friction bearings on the same axle and can be brought together so they couple and rotate as one unit. The first disk, with rotational inertia 3.30 kg·m^2 about its central axis, is set spinning counterclockwise at 450 rpm (rev/min). The second disk with rotational inertia 6.60 kg·m^2 about its central axis is set spinning counterclockwise at 900 rpm. They then couple together.



a) What is their angular speed after coupling? If instead the second disk is set spinning clockwise at 900 rpm, what are their
b) angular speed and
c) direction of rotation after they couple together?

4) A solid disk is mounted so it can rotate without friction about an axis through its center. Initially it rotates with some angular velocity ω. A second circular object, initially not rotating, is dropped concentrically onto the disk. The second object has the same mass and radius as the disk. When the two come to a common rotation, the final angular velocity is 1/3 of the original. What is the shape of the second object?

5) A thin rod of mass 9.00 kg and length d = 0.800 m is pivoted at one end and can rotate without friction along a horizontal surface. The diagram is a top view. Initially it is rotating clockwise at 1.20 rad/s. A piece of putty of mass 50.0 grams slides without friction toward the end of the rod as shown with a speed of 60.0 m/s. The putty collides and sticks to the rod at the location illustrated. What is the final angular velocity?



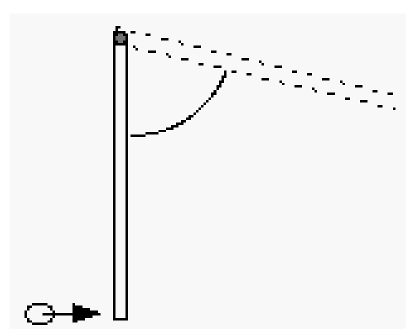
6) Two 2.00 kg masses are out at a radius of 0.500 m and rotating at a rate of 1.00 rev/s. They are moved in to a new radius of 0.250 m.

a) What is the final angular velocity (in rev/s)?

b) What is the ratio of the kinetic energy before and after?

Brain-Buster:

7) The same rod as in #5 is hung vertically, and now it is initially at rest. The same piece of putty is shot at the bottom end with some speed, hits the rod and sticks to it, and now the rod plus putty swings up. When the rod and putty momentarily come to rest, the rod makes an angle of 76.0° with the vertical. What is the speed of the putty?



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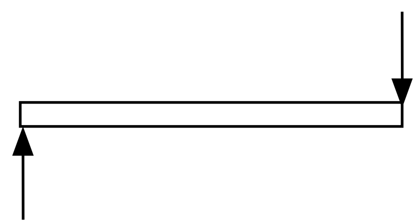
Forces on Extended Bodies (Statics)

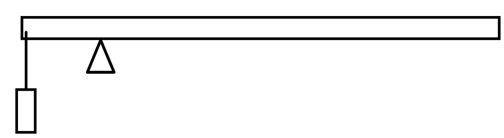
In University Physics I we concentrated on forces that can be considered to act at the center of mass of an object, and we basically treated the object as a point. Now we consider that forces on an extended object (not just an idealized point) might also rotate the object.

1. ***Equilibrium?*** A stick is shown that has only two forces acting on it, of equal magnitude and in the directions shown. (We have temporarily turned gravity off for simplicity.) Is the stick in equilibrium or not? Explain your answer.



2. ***Free Body diagram for an extended object.***

Gravity is back on again. Take a meter stick and place or attach a mass on the left end. Balance it horizontally either with your finger or on a ruler edge.



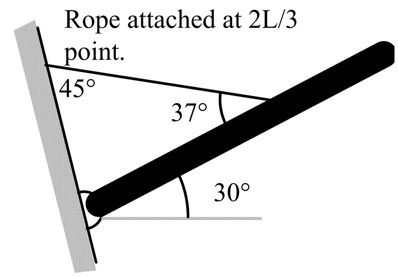
Describe in words what you would look for to know that the stick is in static equilibrium.

Draw a free body diagram for the meter stick, indicating **all** forces that act on the stick. Use the value of the suspended mass and its position, along with the balance position to calculate the mass of the meter stick and its uncertainty. Assume the meter sick has a uniform distribution of mass. Be sure to write all relevant data below.

3. ***Engineering Application***

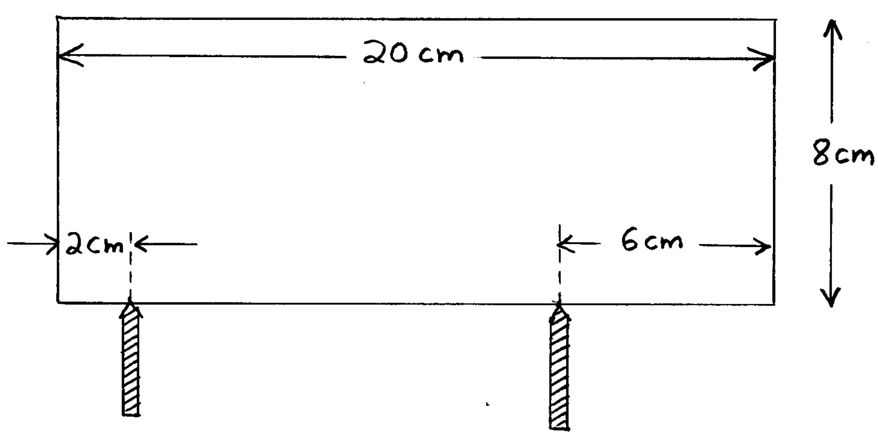
A beam of length *L* is attached to a wall with a hinge, and supported by a rope as shown. The beam is uniform with a **weight** of 500 N. Find:

(a) The tension in the rope
(b) The horizontal and vertical components of the hinge force on the beam.

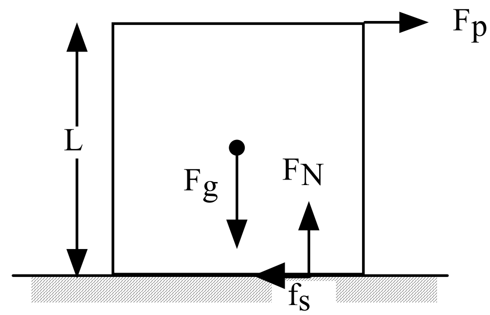


4. Old Exam Question

A 5 kg block of uniform density is balanced on two points as shown. What are the normal forces on the block from each of the points?



5. Static Equilibrium Problems



Where does the normal force act? The normal force exerted on an object by a surface is actually a sum of a large number of such forces distributed over the area of contact of the two surfaces. The effective point of application of the full normal force is such that the torque is the same as that produced by the distributed normal forces. A horizontal force of magnitude $F_p = (1/3)F_g$ is applied at the top of the uniform cubic block shown.

(a) Locate the effective point of application of the normal force, assuming that the block does not slide.
(b) What is the minimum value of the coefficient of static friction needed to keep the block from sliding?

Ladder

A 4.00 m, 15.0 kg ladder leans against a smooth wall (this means no friction on the wall). The ladder's lower end touches the floor 1.00 m from the wall. The ladder has a uniform mass distribution. A 52.0 kg engineer stands on the ladder at a point 1.50 m (along the ladder: not horizontal or vertical) from its top. Determine

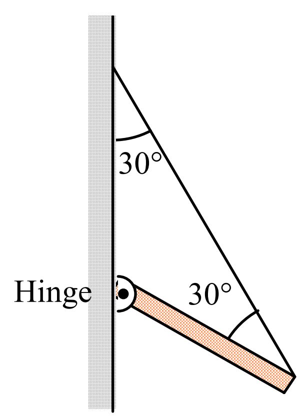
(a) the (horizontal) force exerted by the wall,
(b) the normal and frictional forces exerted by the floor, and
(c) the minimum coefficient of friction needed at the ladder-floor interface to keep the ladder from sliding.

(Hint: There will be a total of five forces acting on the ladder)

Beam

One end of a uniform beam that weighs 217 N is attached to a wall with a hinge. The other end is supported by a wire making the angles shown.

(a) Find the tension in the wire.
(b) What is the horizontal component of the force of the hinge on the beam?
(c) What is the vertical component of the force of the hinge on the beam?



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Equilibrium and Elasticity Problems

1. A helicopter is lifting a 2.10×10^3 kg Jeep. The steel suspension cable supporting the Jeep has a cylindrical cross-section with a radius of 5.00 mm and an unstrained length of 48.0 m. The Jeep is hoisted upward with a constant acceleration of 1.50 m/s^2. By what amount does the cable stretch while the Jeep is moving with the constant acceleration of 1.50 m/s^2, upward?

2. A uniform 120 N ladder rests against a smooth vertical wall. The bottom of the ladder rests on a smooth horizontal surface making an angle 53.0^o with respect to the ground. A horizontal steel wire of radius 0.361 mm is tied between the bottom of the ladder and the vertical wall to keep the ladder from sliding. A 10.0 kg monkey begins to climb the ladder.

   a) Derive an expression for the tension in the wire as a function of the distance *d* the monkey climbs up along the ladder.
   b) How far up the ladder (expressed as a fraction of the ladder's length) can the monkey climb if the stress in the wire must not exceed its tensile yield strength (250×10^6 N/m^2)?

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Linear Elasticity of Materials

You are already familiar with Hooke's Law for a spring, $F = - k \Delta x$, where Δx measures the stretch of the spring from its relaxed state. Let's first quickly review this experimentally, as we'll be using springs in the upcoming labs on simple harmonic motion, and it's important to understand the distinction between the position x and the extension Δx.

Hang the *small end* (can you think of why the small end?) of the spring from the support and measure the Δx extension of the spring in some reasonable manner. Consider the relaxed state to be when the mass holder alone is on the cord, and use this as the zero of your force. The mass holder may stretch the spring a little bit, but this can still be taken as $\Delta x = 0$ providing the plot in a linear relationship and we consider the force to be the weight added to the holder.

Data Table 1: Now add the following masses and measure the length of the spring, including uncertainties.

Mass added to holder (grams)	Length of spring (cm)	Elongation Δx (cm)
0		
5		
10		
15		
30		
60		
80		
100		
150		
200		
250		
300 (Don't exceed this for the spring!)		

(Substitute other masses here if you can't arrange the ones shown, just don't exceed 300 grams).

Compute and plot the added force in newtons (not including the mass hanger) versus the stretch Δx in cm. Estimate your uncertainties and draw error bars on the graph. Properly plan out and label the graph, and attach one copy to this report. Identify the part of the graph that most closely obeys Hooke's Law, if not all of it, and indicate this on the graph.

How well does it agree with Hooke's Law? If there is a linear part of the graph, determine its slope and find k. If only part of the graph is linear, use that part to find the slope.

Show your calculations below, and be sure to include units in your final answer. Also make sure you have one significant figure on your error (possibly two if it starts with a "1"), and make sure your digit placement on the error matches that of the value.

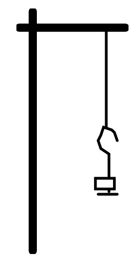
k value calculations:

Is the graph you obtained a proportionality? WHY? (Hint: Does it pass through the origin?)

Now you will measure the behavior of something else to show that other materials besides springs also display this sort of linear behavior.

Take a length of physics cord and stretch it by various forces. The force will again be the weight of the applied masses. Use a meter stick to measure the length, but take care to do this as accurately as possible as we want a nice graph to characterize this material.

Hang the cord from a support and come up with some way of measuring the length of the cord. Consider the relaxed state to be when the mass holder alone is on the cord, and use this as the zero of your force (It's probably safe to ignore the very small extension the holder causes and take this to be the zero point).



Use masses ranging up to roughly 500 or 600 grams, and fill in the table on the next page. The maximum weight per cross sectional area is not close to the yield strength, so the uppermost part of a typical stress-strain curve, where the material starts to permanently deform or break, will not be seen, but there are some features to be aware of if the measurements are done accurately. **Be careful that the rubber part of the cord does not slip within the nylon sheath, as this will cause a sudden discontinuity in your data.** This might happen if the cord is loosely held at one of the ends.

Compute and plot the added force (not including the mass hanger) in Newtons versus the elongation in cm. Estimate your uncertainties for *k* and draw error bars on the graph. Properly plan out and label the graph, and attach one copy to this report.

Identify the part(s) of the graph that most closely obeys Hooke's Law, if any. Indicate this on the graph. If there is a linear part of the graph, determine its slope and find *k*. If only part of the graph is linear, you will have to use just that part. How well does it agree with Hooke's Law?

k values(s):

Data Table 2: Add the masses and carefully measure the extension of the cord. (Pick some convenient mass intervals here, ranging up to 600g).

Mass added to holder (grams)	Length of string (cm)	Elongation Δx (cm)
0		
600 (max)		

Puzzler for this lab: If you've done this carefully, you'll notice that your plot is different than the simple plot obtained for the spring. Discuss this with your lab partners and try to come up with a plausible explanation of what might be going on here, and write it in the space below:

(For those of you who will grow up to be engineers, this is the sort of thing to be very careful of.)

What would be the danger here if you design a structure using these cords and decide to take just 2 or 3 data points, as some non-science students might do?

On the back of this page, write a quick abstract of this lab report. This is good practice for technical writing and all your lab partners should be involved in deciding how to state the basic numerical results and any insights in as concise a manner as possible. Pretend that it's like one of those classified ads where you get charged for every word you use.

ABSTRACT:

If you finish early:

Q1: Does it matter what length of material you choose? Compare your plot with one from a nearby group. Chances are they used a different length of cord. Do the graphs look the same? Should they?

Q2: Try starting from a large force and going back down to a small force. Are the plots still the same? (Any lack of retraceability in a plot due to direction is known as hysteresis).

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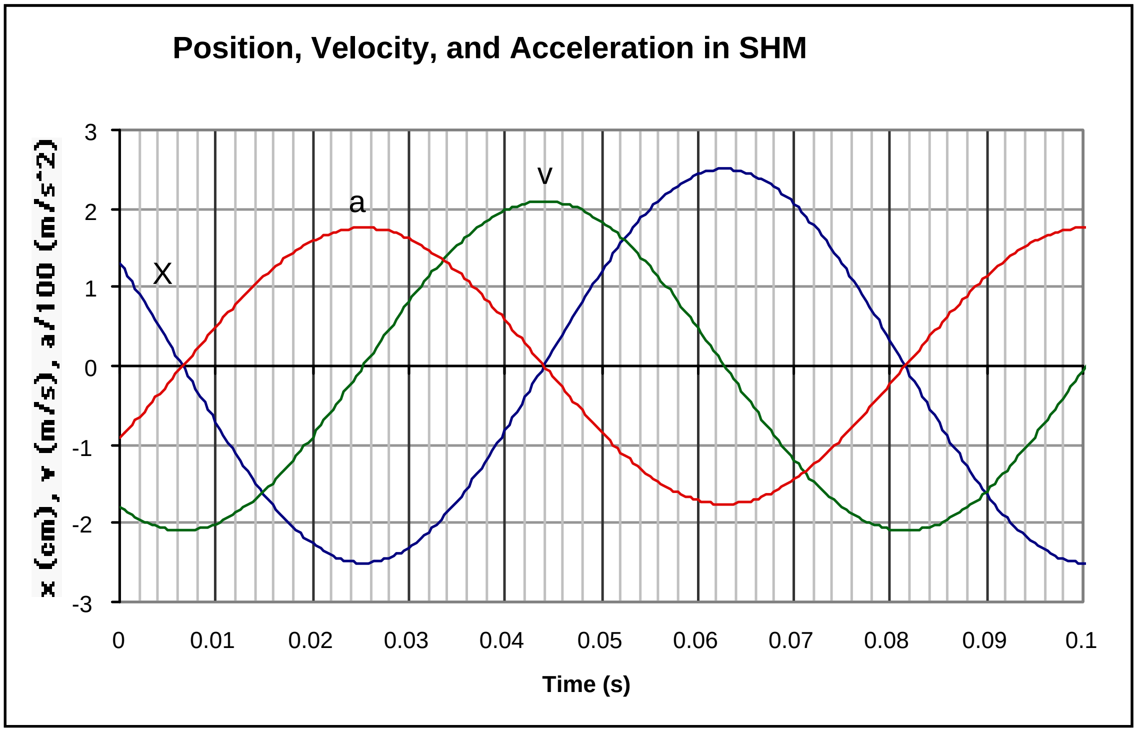
Aspects of Simple Harmonic Motion

A) Determining Parameters from Data Plots



Using the plots above, determine the following parameters for this simple harmonic motion:

Period, T ________________________________

Frequency, f ________________________________

Angular frequency, ω ________________________________

Amplitude, A ________________________________

Maximum speed, v_m ________________________________

Maximum acceleration a_m ________________________________

Phase angle, ϕ ________________________________

Suppose the plots describe a 50 g mass attached to a spring and set into simple harmonic motion. What are the

Spring Constant, k ________________________________

Total Mechanical Energy, E ________________________________

B) What factors determine the period of oscillation? (Intuitive predictions)

Consider two oscillating systems: a mass *m* on the end of a vertical spring (spring constant *k*) and a simple pendulum (mass *m* at the end of a string of length *L*). Complete the following table indicating which variables you expect to have the most influence on the period of motion, which will have smaller but easily measurable effects, and which will have little or no measurable effect. Try to come to a consensus as a group, and don't worry about whether the answer is actually correct.

Rate the following according to their influence on the period as: <u>Definite effect</u>, <u>small but possibly measurable effect</u>, or <u>no effect</u>.

Mass on a spring:

Mass hanging from the spring: ____

Amplitude of motion: ____

(We will only use the one spring so the spring constant will not be a variable in the experiment).

Pendulum:

Mass: ____

Length: ____

Amplitude of motion: ____

C) Measurement of Periodic Parameters

In this section, you will make measurements on a pendulum and a mass/spring system to determine which variables have the most influence on the period of the motion. You will investigate all five of the variables described above.

This portion is designed to be an exploration of the variables that determine the period of oscillation. It is exploratory; so-called "quick and dirty" measurements. You will design and perform a more careful set of measurements in a following workshop.

First, your team should decide on a strategy or procedure to use to determine how important these variables are. You need to make enough measurements to rate how important the variables are, but you should not make so many measurements that you'll be sitting there all day. Make each measurement by timing about five to ten cycles of the motion with a stopwatch. Since these are "quick and dirty" measurements, determine the period only once for any given set of variables. Do not average several measurements.

Summarize your team's strategy here, and explain why you thought the number of trials you did was sufficient:

Why is measuring over roughly ten cycles a better idea than trying to carefully time **one** cycle? Is there any danger in timing too many cycles within a given measurement? If so, why?

Write your measurements in an easy-to-understand table on the back of this page. Summarize your conclusions by indicating which variables had the ***most influence*** on the period of motion, which had ***small but still measurable*** effects, and which had ***no measurable*** effect.

Do not attach more than 300 grams to the spring.

Mass on a spring: Mass: ____________________

Amplitude of motion: ____________________

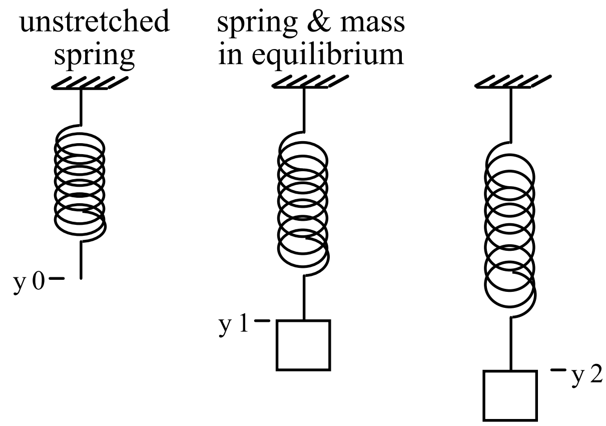
Pendulum: Mass: ____________________

Length: ____________________

Amplitude of motion: ____________________

If you finish early: Using the period *T* and the mass *m* of the spring on the mass, calculate the force constant *k* of the spring, and see if it agrees with the value you found in a recent activity. (It won't be the same spring, but the spring constants for the brass springs are essentially the same).

D) Technical Point: Is a mass on a <u>vertical</u> spring SHM?



If we hang a mass on a spring, there's the extra complication that gravity pulls the spring down in its equilibrium state. Does the math all work out that it's still simple harmonic motion? (SHM is when the restoring force is proportional to the displacement from equilibrium).

More specifically, consider a mass *m* attached to a spring with spring constant *k*. If we lower the mass slowly, it will come to equilibrium, stretching the spring by an amount $y_1 - y_0$. Note that the displacement down is negative, and the restoring force up is positive. If we pull the mass farther down, to a position y_2, and release it, will the mass and spring move with simple harmonic motion?

Steps to determine this:

1. Draw the force diagram for the mass when it is in <u>equilibrium</u>, and find an expression for y_1-y_0 in terms of *m*, *g*, and *k*. (Don't be a trouble-maker; pick up to be positive).

2. Draw the force diagram for the mass when it is pulled down farther (right hand diagram) and then released. Write Newton's Second Law for the mass just *after* it is released in terms of *m*, *g*, y_0, y_2, *k* and *a*. Be careful with the signs!

3. Simple harmonic motion occurs when there is a restoring force proportional to the displacement. Combine 1) and 2) to show that this is true for the mass hung from a vertical spring. From where must the displacement be measured?

Your Name (**Print**): ______________________ Date: ________

Group Members: ______________________

______________________ Group: ________

Simple Harmonic Motion Lab

Verifying the relation between period, *T*, spring constant, *k*, and mass, *m*, for a system of a mass on a spring.

Your text and workshop notes claim that for a mass *m* attached to a light spring executing simple harmonic motion, the period is:

$$T = 2\pi\sqrt{\frac{m}{k}}\ .$$

Your task is to investigate this using good experimental technique. Note that the book looks at the situation for a mass on a horizontal frictionless surface, while we use a mass hung vertically. You have seen in a previous activity that the two systems are mathematically equivalent.

1) **Static method:** In a previous activity, you found that the spring obeys Hooke's Law; $F = -k\,x$. Accurately measure *k* for your spring, the way you did in the previous activity with the spring not moving. (We have to redo this because all the springs are slightly different). Measure the extension from a reference point such as the bottom of the mass holder and not the spring itself as the spring might rotate as it extends. You want to be as accurate as possible here.

2) **Dynamical method**: Your second task is to check the validity of the above equation. Use a **graphical** approach. What variables will you use? How can you measure period accurately and precisely? Think about what you will plot on the axes. You have the use of the computers, and you should try out the motion detectors to see how accurately things can be measured with it. You might want to also use a stopwatch to make sure the computer is doing things right. Terrible accidents have occurred (not in this lab but in designing aircraft and cars, etc.) because engineers have blindly trusted computer readouts without checking them.

Equipment: a spring and place to hang it from, some masses and a mass hanger, a meter stick, a computer with a motion detector, anything else you want to use in the boxes, and anything you don't see - just ask.

Very accurate results are possible in this experiment. Students usually obtain values of the spring constant from the two methods that agree within experimental error, sometimes within errors less than 1%. You should be able to get this close also! Note that this handout is deliberately sparse to encourage you to think about and discuss how measurements are to be made. Sometimes students even come up with ideas the professors hadn't thought of.

General experimental notes:

- Do not attach more than 300 grams to the spring.

- Pay careful attention to experimental conditions which do not match the ideal: for example you need to be careful that the oscillation is purely vertical and does not swing from side to side. There are **many** possible imperfections that could compromise the experiment. Perform the experiment so that these imperfections are not present, or at least are minimized. Be cognizant of whether your amplitude is the same each time or different.

- Good experiments require attention to uncertainties. You need to consider ways to reduce the effect of uncertainties. If you curve fit a sine wave, fitting it to a larger number of cycles reduces the total uncertainty of the fit in general.

- Notice that the spring oscillates even with no mass hanger on it. This means that the mass of the spring (or some fraction of it) is part of the oscillating mass.

- A single data point says very little, and in this case won't even work because *m* in the formula is not the true total *m* of the system; you want to have many data points (many means at least 10) for different masses. Good experimental design also says that these points should be spaced widely, and roughly equally when plotted on linear graph paper. Masses of 400, 401, 402, 403, and 404 grams would not be a sober choice!

- If you have extra time, take more data points. Plotting them out during the lab so you can tell ahead of time if there are any screw-ups would be a really good idea, and <u>don't take the equipment apart until you are certain everything is done correctly</u>.

- A major hint here: You'll want to think carefully about what you're going to plot on the graphs. Recall there is the pesky problem of the mass actually including part, but not all, of the spring mass moving back and forth. Plot a graph such that you can actually determine what this extra mass term should be. We essentially have

$$m = m_{\text{weights}} + m_{\text{portion of spring}}$$

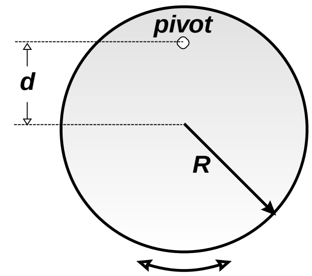
where $0 < m_{\text{portion of spring}} < m_{\text{spring}}$. You must determine a numerical value for $m_{\text{portion of spring}}$.

- **<u>The due date and report format will be announced by your instructor</u>**.

Your Name (**Print**): ________________________ Date: ________
Group Members: ________________________
________________________ Group: ________

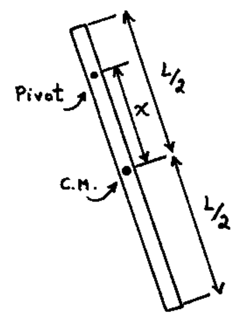
Simple Harmonic Motion Problems

1. A physical pendulum consists of a uniform solid disk of radius R supported in a vertical plane by a pivot located a distance d from the center of the disk. The disk is displaced a small angle and released.



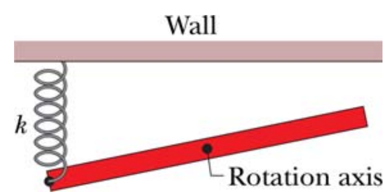
   a) Derive an expression for the period of the resulting simple harmonic motion in terms of the given quantities.
   b) Calculate a numerical value for the period if R = 2.35 cm and d = 1.75 cm.

2. A stick of length L oscillates as a physical pendulum.



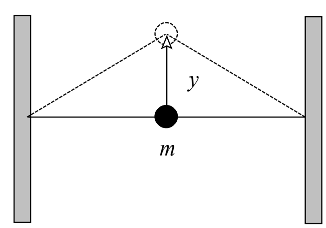
   a) Derive an expression for the period as a function of the distance x between the pivot point and the center of mass of the stick.
   b) Derive an expression for the value of x that gives the <u>shortest</u> period.
   c) Calculate a numerical value for the shortest period if L = 1.85 m.

3. In the overhead view shown in the diagram, a long uniform rod of length L and mass M is free to rotate in a horizontal plane about a vertical axis through its center. A spring with force constant k is connected horizontally between one end of the rod and a fixed wall. When the rod is in equilibrium, it is parallel to the wall.



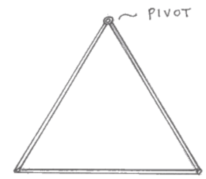
   a) When displaced slightly from equilibrium, is the motion simple harmonic motion? Why?
   b) If the motion is simple harmonic, derive an expression for the period in terms of the given quantities.

4. A mass m is connected to two identical rubber bands of length L, each under tension T as shown in the diagram. The mass moves on a smooth horizontal surface, and the diagram shows a top view. Assume the magnitude of the tension in the rubber bands does not change when the mass is displaced a small distance y from the equilibrium position.

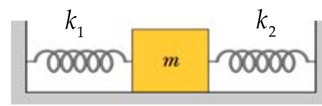


   a) Does the mass execute simple harmonic motion when it is displaced a small distance y and released? Why?
   b) If the motion is simple harmonic, derive an expression for the period in terms of the given quantities.

5. a) Calculate the period of a physical pendulum formed by joining three identical meter sticks into a triangle as shown and letting it pivot through a small angle about one of the corners.
   b) Measure the period using the assembled triangle in the room. Does your calculated value agree with the measured value within a reasonable uncertainty? If not, repeat part a)!

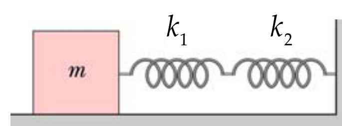


6. Two springs of spring constants k_1 and k_2 are attached to a mass m as shown in the diagram. The mass oscillates on a smooth horizontal surface.



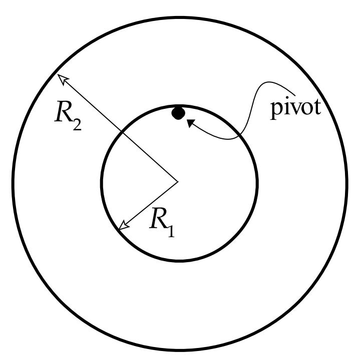
   a) Show that the mass executes simple harmonic motion when it is displaced a distance x from the equilibrium position and released.
   b) Do you expect the period with both springs attached to be the greater than, less than, or equal to the period if one of the springs is removed? Explain.
   c) Derive an expression for the period in terms of the given quantities.
   d) Is the period different if the springs are initially stretched when m is at the equilibrium position?

7. Two springs of spring constants k_1 and k_2 are attached to a mass m as shown in the diagram. The mass oscillates on a smooth horizontal surface.



   a) Show that the mass executes simple harmonic motion when it is displaced a distance x from the equilibrium position and released.
   b) Derive an expression for the period in terms of the given quantities.
   c) Suppose the spring constants are equal. What is the ratio of the period for <u>two</u> identical springs to the period using just <u>one</u> spring?
   d) Measure the period for two vertical springs and then for one vertical spring using $m \sim 200$ g. Does the ratio of the two measured periods agree with your calculated value within a reasonable uncertainty?

8. A uniform annular ring has a mass M and inner and outer radii R_1 and R_2, respectively. The ring is pivoted at a point along its inner radius.



   a) Derive an expression for the period for small oscillations in terms of the given quantities.
   b) What is the value of the period in the limiting case of a <u>thin</u> annular ring, that is, $R_2 = R_1 = R$? Is this the expected result? Why?
   c) Measure the period for two rings of different inner radius. Do the measured periods agree (within a reasonable uncertainty) with your calculated values?
   d) Using the result from b), how would you graph period/radius data in order to generate a straight line? What is the slope of your straight line?

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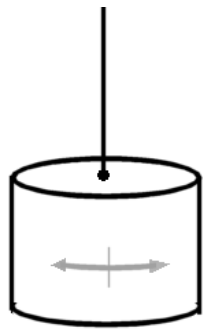
Torsion Pendulum

1) If you hang a mass on the end of a steel wire, then twist it, and release it, it will oscillate back and forth rotationally. This is called a torsional spring, and it's a nice example of how the same basic steps in the mathematics of SHM can be applied to a large range of different problems.



(a) State what the rotational version of Hooke's law would be for such a spring. Think about Hooke's law for a "regular" spring and replace the linear variables with rotational analogies.

(b) How would you define the spring constant for such an object? Let's call it κ ("kappa"). What units would this quantity have? On what properties of the wire do you think the torsional spring constant would depend?

(c) Apply Newton's Second Law (rotational form) and write down the resulting differential equation. (Assume the I is just some unknown moment of inertia, not necessarily a cylinder).

(d) What type of motion results from this? Write down an expression for the position angle of the object as a function of time.

(e) Use the answer in (d) to determine the period of motion of this system? Clearly define any quantities that you have used in your expression.

2) Now a specific torsional problem…..

A solid wooden cylinder has a mass of 12 kg and a radius of 25 cm. It is hung from its cylindrical axis using a steel piano wire. It is observed that a tangential force of 1.5 N is required to "twist" the mass through a 20^{o} angle. From this position, the mass is released and allowed to rotate.

(a) What is the period of motion?

(b) What is the total mechanical energy of the system?

(c) What is the maximum angular speed of the object? At what point in the motion does this occur? (There are two different things using the symbol ω here, it happens sometimes, so this is an example of not getting them mixed up).

(d) What is the maximum angular acceleration of the object? At what point in the motion does this occur?

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Traveling Wave Functions

A linear (non-dispersive) traveling wave keeps a constant shape as it travels. The text makes the claim that all such waves must have the form *y(x,t)* = f(*x* - *vt*) or *y(x,t)* = f(*x* + *vt*).

1) In the space to the right of each function, write it in a traditional form (as shown for the first three functions). <u>Predict</u> which of the following will be linear traveling waves and write "yes" on the line; write "no" for the ones that are not linear traveling waves. Note: In case you haven't seen this computer notation, exp(…) mean e to the power of the argument, and sqr(…) means the argument squared, NOT the square root.

(a) _____ *y*(*x*,*t*) = 3*cos(pi*x/2-pi*t) $$y(x,t) = 3\cos\left(\pi\left(\frac{x}{2} - t\right)\right)$$

(b) _____ *y*(*x*,*t*) = 3*exp(-sqr(pi*x/2-pi*t+3*pi)/2) $$y(x,t) = 3e^{-\frac{1}{2}\left[\pi\left(\frac{x}{2} - t + 3\right)\right]^2}$$

(c) _____ *y*(*x*,*t*) = 3*cos(pi*x/2 -pi*t)/(2+sin(pi*x -2*pi*t)) $$y(x,t) = \frac{3\cos\left(\pi\left(\frac{x}{2} - t\right)\right)}{2 + \sin\left(\pi\left(x - 2t\right)\right)}$$

(d) _____ *y*(*x*,*t*) = 3*cos(((x/2) - t)*((x/2) + t))

(e) _____ *y*(*x*,*t*) = 3*cos(((x/2) - t)*(x - (2*t)))

(f) _____ *y*(*x*,*t*) = 3*cos(x/2 -t)*sin(2*x-t)

(g) _____ *y*(*x*,*t*) = 3*cos(x/2 -t)*sin(2*x+t)

(h) _____ *y*(*x*,*t*) = 1*exp(-sqr(pi*x/2 - pi*t + 3*pi)/2)* (pi*x/2 + pi*t)

If the function in (h) is NOT a travelling wave, what ***<u>single</u>*** change can you make to the function to convert it to a travelling wave? TRY IT!

(i) _____ $y(x,t)$ = 3*cos(((x/2) - (5*t))*((x/2) - (5*t)))

After you have made your predictions, use the physlet at

http://people.rit.edu/vwlsps/312_s03/Physlets/pulse.html

to check whether each function is a traveling wave or not. (Hit 'enter' not 'return').

Wave Simulations

2) Let's start with an exercise in determining parameters. Load the webpage:

http://people.rit.edu/vwlsps/312_s03/Physlets/Wave1A.html

For the wave shown in the top box, find the:

(a) amplitude

(b) wavelength

(c) direction of motion

(d) wave speed

(e) period

(f) Phase angle

(g) Write the equation for your successful function here, and test it by entering it into the bottom box. If it's correct, the top and bottom box should now be exactly the same.

$g(x, t) =$

(h) What must you do to the equation to make the wave travel in the opposite direction? Write the new equation below.

(i) What must you do to the equation to make the wavelength half its original value, but keep the same wave speed? Write the new equation below.

(j) Show that you can switch the use of the sine and cosine function by adding or subtracting a phase constant. Write an equation that's the same result as (f) but using the other trig function.

3) Now let's look at a wave pulse and a sinusoidal wave having same amplitude and velocity. Go back to the original webpage:

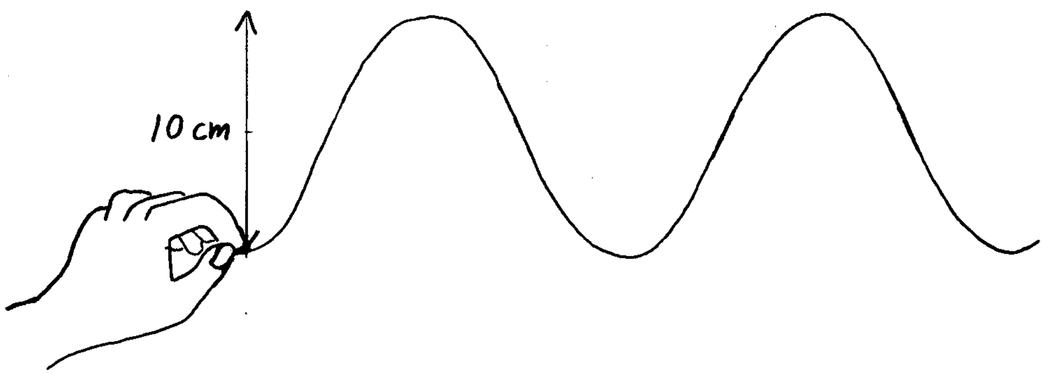
http://people.rit.edu/vwlsps/312_s03/Physlets/pulse.html

(a) Write the original equation for the pulse. Again note that sqr(..) means the quantity squared, not square-rooted.

(b) What must you do to the equation to make the pulse travel in the opposite direction? Write the new equation below. Check your answer by entering the function in the bottom box and running the applet.

(c) What must you do to the equation to make the pulse start at a different initial position? Write the new equation below (pick any new position). Check your answer by entering the function in the bottom box and running the applet.

4) Jasmine moves a string up and down through a distance of 10.0 cm with a frequency of 6.00 Hz. The resulting waves travel at a speed of 0.500 m/s in the positive x-direction as shown. Assume that we start at $t = 0$ when the end of the string is at the bottom of the motion as shown. Write down an equation that describes the wave, putting the values and units of all constants into the equation. (There's a few different ways of writing any given wave equation, as long as it's correct).



5) Now let's look at the superposition of two pulses going in opposite directions. Load the page:

http://people.rit.edu/vwlsps/312_s03/Physlets/pulseSuperposn.html

Write the bottom function $g(x,t)$ so that it travels in the $-x$ direction and change its phase angle from "+1.5" to "-1.5". Hit "Enter" and run the simulation.

(a) When the pulses overlap would you describe the signal as **constructive** or **destructive**?

(b) Try changing the second function by making the coefficient "-3".
Constructive or **destructive**?

(c) Make the amplitudes both positive, but with different values, say 3 and 4.
Constructive or **destructive**?

(d) Make the amplitudes of opposite sign and of different values, say +3 and -4.
Constructive or **destructive**?

(e) Describe how the combined signal [technically called the "linear superposition" of the two waves] is mathematically related to the two individual wave pulses.

6) Try the superposition of two sinusoidal waves going in the same direction:

http://people.rit.edu/vwlsps/312_s03/Physlets/SinSuperposn.html

(a) As shown, two waves travel in the same direction.
Constructive or **destructive**?

(b) Change the amplitude of the first function to -3.
Constructive or **destructive**?

(c) Make the amplitude of the first function +3 again, but add a phase of pi.
Constructive or **destructive**?

(d) You can also experiment with making the waves have different amplitudes, different phase angles, different speeds, or traveling in different directions. To begin with, **change only one thing for one wave at a time**. Sketch what happens to the sum of the two waves when they have different:

(i) amplitudes

(ii) phase angles

(iii) speeds

(iv) directions of travel

7) Things to try if you finish early:

(a) Make your wave exactly out of phase with the top wave.
(b) Make your wave move to the left instead of to the right. [No fair using the reverse button!]
(c) Make a wave with half the wavelength but with the same speed as the top wave.
(d) Make two Gaussian wave pulses that travel together in the same direction on the same string.

Your Name (**Print**): ______________________________ Date: __________

Group Members: ______________________________

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Superposition, Phasors and Standing Waves

1. Equal amplitudes, same direction, same frequency and wavelength

Use the waves $y_1(x,t) = A\cos(kx-\omega t)$ and $y_2(x,t) = A\cos(kx-\omega t+\phi)$

If they are equal amplitude, then the sum can be obtained from the trig identity:

$$\cos(a) + \cos(b) = 2\cos[\tfrac{1}{2}(a+b)]\cos[\tfrac{1}{2}(a-b)]$$

Q1: Add y_1 and y_2 using this trig relation.

Q2: Could we still use this identity somehow if they are not of equal amplitude?

Note that in your solution to Q1, depending on the phase angle, the interference can be

Completely (or totally) constructive, amplitude = $2A$ when $\phi = 0, 2\pi, 4\pi, \ldots$

Completely (or totally) destructive, amplitude = 0 when $\phi = \pi, 3\pi, 5\pi, \ldots$

Q3: Go to the same Physlet we used recently

http://people.rit.edu/vwlsps/312_s03/Physlets/SinSuperposn.html

and verify that changing the phase by any amount still gives a traveling wave that has the same velocity (magnitude and direction), the same frequency, and the same wavelength.

Problems:

Q4. Write an expression for the superposition of

$$y_1 = (3.00\text{ mm})\cos(kx-\omega t) \quad \text{and} \quad y_2 = (3.00\text{ mm})\cos(kx-\omega t+\pi/3)$$

Q5. The superposition of

$$y_1 = (3.00\ mm)\ cos(kx-\omega t) \text{ and } y_3 = (3.00\ mm)\ cos(kx-\omega t+\phi)$$

has amplitude of 4.00 mm. What is the phase angle ϕ? Find all possible values in the range $0 \leftrightarrow 2\pi$.

Verify that your answers to Q4 and Q5 match the correct waves using the applet linked above.

2. Standing Waves (Equal amplitudes, opposite directions, same frequency and wavelength)

These waves can be represented by adding

$$y_1(x,t) = A\ cos(kx-\omega t) \text{ and } y_2(x,t) = -A\ cos(kx+\omega t)$$

Q6. Add these using the trig relation $cos\ a - cos\ b\ = -2sin\left[\frac{1}{2}(a-b)\right] sin\left[\frac{1}{2}(a+b)\right]$ to show that:

$$y_1+y_2 = 2A\ sin(\omega t)sin(kx)$$

Q7. How do we know that this is no longer a traveling wave?

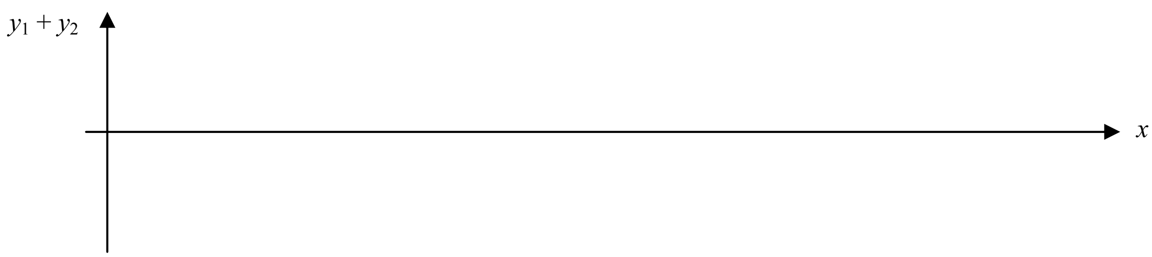
Q8. Explain how we can find the locations of nodes [where the amplitude is always zero] along the *x*-axis.

Q9. To find antinodes [where the amplitude is a maximum], we must set *kx* equal to ____________

Q10. i) On the axes below, sketch graphs of the sum of

$$y_1 = A\cos(kx-\omega t) \text{ and } y_2 = -A\cos(kx+\omega t)$$

at times t = 0, t = T/4, t = T/2 and t =3T/4. Label the three graphs to identify which one is which.



ii) If the wave is fixed at $x = 0$ and $x = L$, what are the possible values of the wavenumber k?

Q11. Find the result of adding these three waves in linear superposition:

$$y_1(x,t) = (1.0\ cm)\ cos(kx+\omega t)$$

$$y_2(x,t) = (2.0cm)\ cos(kx+\omega t+ 2\pi/3)$$

$$y_3(x,t) = (3.0\ cm)\ cos(kx+\omega t+4\pi/3)$$

Write the result in the form $y(x,t) = A\ cos(kx+\omega t+\phi)$ where the amplitude and phase angle are replaced by numerical values.

Note: applet showing the addition of two phasors;
http://vnatsci.ltu.edu/s_schneider/physlets/main/phasor1.shtml

Your Name (**Print**): ____________________________ Date: __________
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____________________________ Group: __________

Waves on a String

The wave speed for any wave is related to the wavelength and frequency by

$$v = \lambda f \qquad (1)$$

For transverse waves on a stretched string, the wave speed is given by the relation

$$v = \sqrt{F/\mu} \qquad (2)$$

where F is the tension in string, and μ is the mass per unit length of the string. The goal is to see if these two expressions for the wave speed give the same result.

There is a vibrating string apparatus on your table as shown in Figure 1. An audio generator and an amplifier (neither are shown in the figure) drive the mechanical vibrator with a frequency set on the generator.

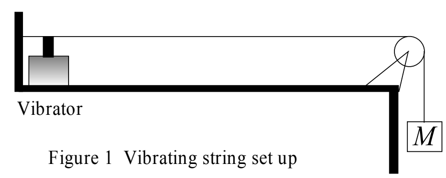

Figure 1 Vibrating string set up

Rules: • **Do not allow amplitude of oscillation of slotted rod in the vibrator to exceed 0.5 cm.**
• **The sound of the vibrator should be barely audible.**
• **Do not put your ears, or anybody else's ears, up against the apparatus.**

a) Hang a mass $M \approx 750$ g on the string. Find any frequency at which the string resonates: i.e., where the amplitude is a **maximum** and an integral number of half-wavelengths fit on the string. Record the frequency from the dial (with an estimated uncertainty) and sketch the pattern of the string. Note that the node is not exactly at the end due to the action of the mechanical vibrator.

b) Estimate the other frequencies at which this system should resonate. For example, once you've found one resonance pattern, it's just a matter of a simple fraction ratio to estimate where the next one should be. List your estimated frequencies below.

Be sure to include uncertainties in ALL measured quantities.

c) Verify your estimated frequencies experimentally. You will make detailed measurements on at least two of the resonance patterns. Construct a data table on a separate sheet of paper that lists the harmonic number (n), your <u>estimated</u> resonance frequency ($f_{n,est}$), and the <u>actual</u> measured frequency (f_n) for at least 9 patterns. The table must also include the measured distance between adjacent nodes for any two of the patterns corresponding to $n \geq 3$ and a sketch of the shape of the string. Do not attempt to locate the node at the end near the mechanical vibrator. (It is claimed that the all-time record is 32 nodes, not counting the ends, but you needn't go anywhere near that high).

d) For the patterns that you recorded the nodal spacing(s), use Equation (1) to calculate the wave speed. Calculate the uncertainty in each wave speed. Are the wave speeds for the different resonance patterns equal within experimental uncertainty? Should they be equal? Why? SHOW ALL WORK BELOW.

e) In order to calculate the wave speed using Equation 2, you'll need the linear mass density μ of the **<u>stretched</u>** string. Make appropriate measurements of a piece of **<u>unstretched</u>** string and record the results in the space below. The catch: **<u>Do not cut the string</u>**. Non-destructive testing only. A sample piece of string is at the center table if you need to weigh one.

f) Describe how you will account for the fact that the string stretches when the mass M is attached to the end of the string.

g) Calculate the **stretched** value of μ and its uncertainty. Show all your work in the space below.

h) Use Equation 2) and the stretched value of the linear mass density to calculate the wave speed and its uncertainty. Show all work below. Compare the speed to the values calculated using Equation 1).

i) Briefly summarize your conclusions in the space below.

j) If the distance between the vertical rod / vibrator (the left end of the string in the Figure 1) and the pulley is shortened,

i) Will the wave speed increase, decrease, or remain the same? Explain.

ii) Will the fundamental frequency of the vibrating string increase, decrease, or remain the same? Explain.

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Beat Frequencies

Beats: equal amplitudes, same direction, slightly different frequencies and wavelengths

The two waves can be represented by

$$y_1 = y_m \sin\left(k_1 x - \omega_1 t\right) \quad \text{and} \quad y_2 = y_m \sin\left(k_2 x - \omega_2 t\right)$$

Since they are equal amplitude, the sum can be obtained from the trig identity:

$$\sin\alpha \pm \sin\beta = 2\sin\left[\frac{\alpha \pm \beta}{2}\right]\cos\left[\frac{\alpha \mp \beta}{2}\right]$$

Use this trig identity to combine the two waves:

Notice that the argument of the sine function contains just the average wave number and the average angular frequency, while the argument of the cosine term contains a small wave number and a small frequency (the difference between the two frequencies is small).

Go to the following website: **http://vnatsci.ltu.edu/s_schneider/physlets/main/beats.shtml**

The applet generates the superposition of two sound waves. The time dependence of the sum of the waves is displayed mathematically. The amplitude of each wave defaults to a value of 1 on a relative scale. The relative amplitude of either wave can be changed by simply multiplying the sine function by a numerical value. For example, to completely remove the second wave and hear only a pure tone associated with the 400 Hz sound, write the function

f(*t*) = sin(2*pi*400*t)+0*sin(2*pi*402*t)

and hit 'New'. Notice the "0*" in front of the second sine term.

(1) Listen to the 400 Hz and 402 Hz tones separately (the default values when the applet first loads). Can you tell these two sounds apart? If your lab partner claims that they can hear the difference separately, make sure to test them on this by not letting them see the screen.

(2) Now listen to the two sounds together (adjust the relative amplitudes to be equal to 1). What do you hear?

(3) Now listen to 400 Hz and 403 Hz sounds -- separately, together -- What do you hear?

(4) How is this different from part (2)?

This technique of using beat frequencies provides a nice way of telling frequencies with small differences apart (401 Hz, 402 Hz, 403 Hz, …) by comparing them to a standard fixed one (400 Hz in this case). This is used a lot in physics measurements involving waves, including light as well.

(5) Calculate the beat frequency you should be hearing in the two cases above, and listen again to see if it seems correct. Is there a difference in the structure of the waveform made from the pair (400 Hz, 402 Hz) and (404 Hz, 406 Hz)? Is there a difference in the beat frequencies?

(6) Using the (400 Hz, 402 Hz) pair, determine the effect of adding a phase constant to one wave. Do you still detect a beat pattern? Does the waveform change? Does a constant phase difference affect the beat frequency?

(7) Make the relative amplitudes of the two waves unequal. If the amplitudes differ by a factor of 2, can you still detect the beats? What if the amplitudes differ by a factor of 10?

(8) Try the combination 350 Hz and 440 Hz. (Don't delete the 2*Pi part). Recognize this distinctive sound?

Also try 480 Hz mixed with 620 Hz and pulse it on and off in half-second intervals. Recognize this sound?

The two examples above are from standard Touch-Tone frequency pairs. They are distinctive pairs of frequencies that electronic equipment can be easily configured to recognize, and they deliberately avoid standard musical frequencies so that they will not be falsely triggered by possible background music. If you're interested in more tone-pairs, or what phones sound like in different parts of the world, check out

www.hypertextbook.com/physics/waves/beats/

(9) Experiment with adding three or more frequencies by editing the equation in the box (for example, 400 Hz, 402 Hz and 404 Hz). Try other combinations yourself. Limit the entries to integer frequencies. Hit 'New' to reload a new equation, **not** the 'add' button (this adds the new equation to the previous). Notice that adding three or four tones together can make some fairly annoying sounds for certain integer combinations of frequencies. As a very general rule, integers that share no common divisors will sound a little more annoying than sets that do. This is because your brain has a harder time sorting out the components it hears. Try to find a set that is so annoying it would make a really good fire alarm or alarm clock. Record the most annoying one your group can come up with and we'll take a vote on the worst one.

(10) Load the following website to get a better visual sense of how waves combine to form beats:

http://www.mta.ca/faculty/science/physics/suren/Beats/Beats.html

By following the bouncing ball, you can see clearly when there's constructive and destructive interference.

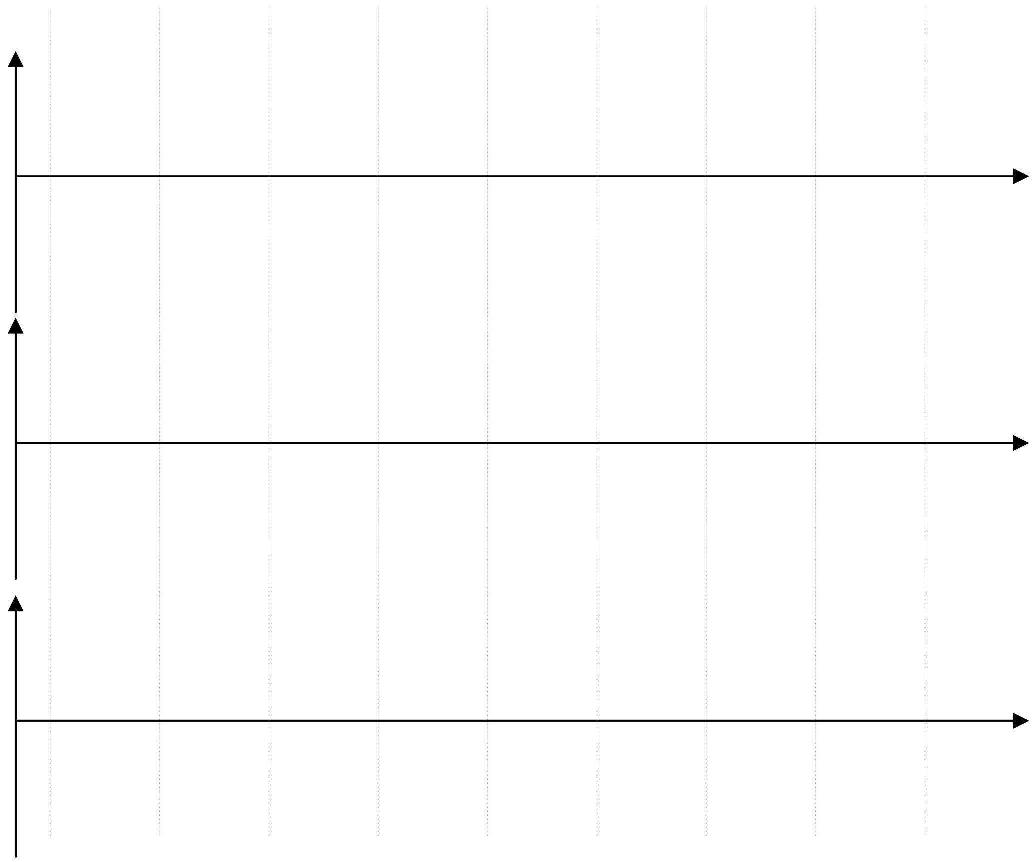
(11) Load the following page to see how the relative amplitude and phase affect the beat pattern.

http://www-math.mit.edu/daimp/Beats.html

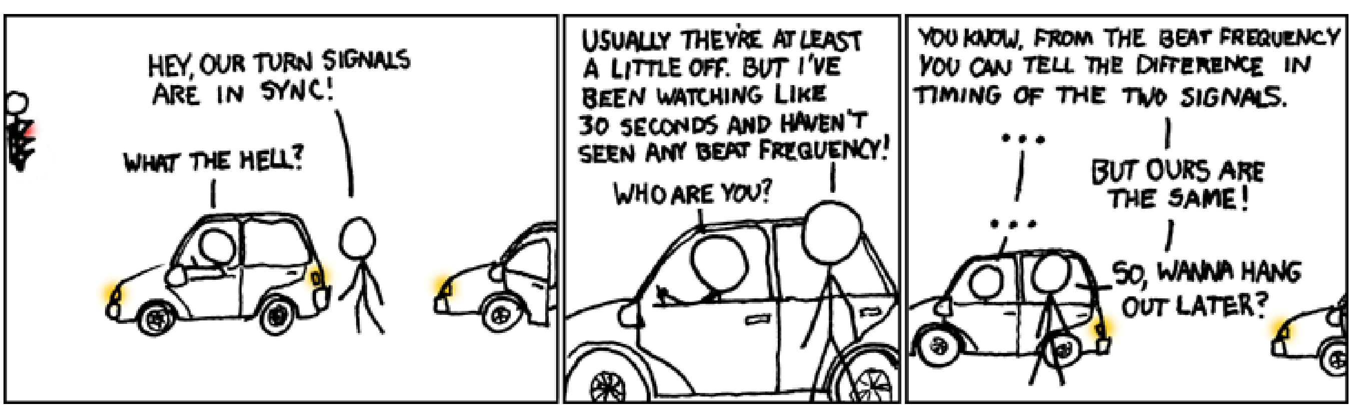
The green function displayed in the top graph is the superposition of the red and yellow functions shown in the bottom graph. The red function is fixed. The amplitude and phase constant of the yellow function are adjusted by moving the sliders at the bottom of the display. Verify what you concluded earlier regarding the effect of unequal amplitudes and different phase constant.

(12) Just to make sure everyone gets the overall idea here, sketch two sine waves that have **slightly** different frequencies and sketch the resulting beat frequency. Be careful to line up the sketches vertically, so each point on the combined wave is directly below the points on the two sine waves that create it.

How does the beat frequency change as the original two frequencies become further apart?



From: http://xkcd.com/



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Measuring Sound: Beats and Fourier Transforms

Never strike the tuning fork against a hard surface like the edge of the table, or your lab partner's head, as this will damage the tuning fork. Instead use the striking block provided, or the heel of your hand. Do not put the tuning forks in your mouth, as the vibrations can shatter fillings.

1. Hold a tuning fork and strike it against the heel of your hand. Hold it near your ear and rotate the tuning fork. Describe what you hear, and explain why you hear what you hear.

2. Hook up the LabPro to your computer and connect the microphone to Ch 1. **Plug the power into the LoggerPro box first**. Open the Student Shares to *Team Physics 312: LoggerPro.* Copy the file *Measuring Sound minilab* to your MyDocuments folder, then drag the icon onto the LoggerPro icon to open it.

3. Check the settings on the collection (*Experiment → Data Collection*): you should be set to collect for 0.5 sec at 10000 samples per second. Zero the microphone when there is little or no sound (control zero). Test the setup by recording a nice sine wave for a single fork. (If it's not a nice sine wave, it's possible that particular fork was dropped or struck too hard). Check that the frequency on the computer matches the frequency labeled on the tuning fork.

4. Get two tuning forks that have different frequencies, but not too different. About 5 to 10 percent difference is good.

   Strike one of the tuning forks firmly against the heel of your hand or the striking block (not a hard surface!), hold the fork in front of the microphone and collect data for the 0.5 sec. Click and drag on the horizontal axis to scale it out.

   Make sure you can see the sinusoidal wave (if you can't, try changing the time range to 0.2 seconds). Measure the period of the sound by counting cycles on the graph and calculate the frequency from the period. Compare the frequency calculated this way to the number stamped on the tuning fork. Try to fit a sine function to the graph to check the frequency you got from the period.

5. Strike both forks and hold in front of the microphone as you collect. You will need to adjust distances, and repeat this a few times until the microphone hears both forks. What does the wave look like now? Can you tell that there are two different frequencies, and what they are?

Can you see a beat frequency? If not, you may need to adjust the time scale of the graph. What is the period *T* of the beats? How does the beat frequency (1/*T*) compare to the difference in frequency of the two forks stamped on the stems? Once you get a really nice one, print out a copy for your notes.

6. Now consider the Fourier transform. A Fourier transform takes the "sinusoidal" signal that is a mixture of sounds of different frequencies, and figures out the frequencies of the tuning forks that produced the sound. If you click the box that reduces the size of the graph, the Fourier Transform Graph (FFT) should appear (or go to ‘page’→‘next page’). First use one tuning fork and collect data. You should have one very large peak in the Fourier transform. What is its frequency, and how does it compare to the tuning fork frequency? Are there any other peaks in you data? If so record their frequencies (*Analyze → Examine*) and find the ratio of the new frequencies to the first frequency.

   Repeat for the second tuning fork.

7. Now strike both tuning forks and collect data as you did in part 5. You may have to adjust the distance from the forks to the microphone until you see two major peaks on the Fourier Transform. How do the two frequencies compare to the frequencies of the two tuning forks?

8. Repeat for two tuning forks when the frequencies are quite different. Can you pick out the frequencies on the sound graph? Can you pick out the frequencies on the Fourier Transform graph?

9. Now you will measure your voice. Use one of the tuning forks to get a pitch. Sing the vowel sound "oo" (as in food) at the same frequency and collect data. Do you sing a single frequency? Record the frequencies that you see and their relative amplitude and make a sketch of the FFT graph.

10. Repeat 9, but now sing the vowel sound "ee" (as in beet). How do the "oo" and "ee" sounds differ? Record the frequencies that you see and their amplitude and make a sketch of the FFT graph.

11. Finally hiss into the microphone and collect data. How does an "sss" differ from the vowel sounds? Make a sketch of the FFT graph. This pattern is technically known as ‘white noise’ and in similar to what causes static on you television when the signal is completely randomized.

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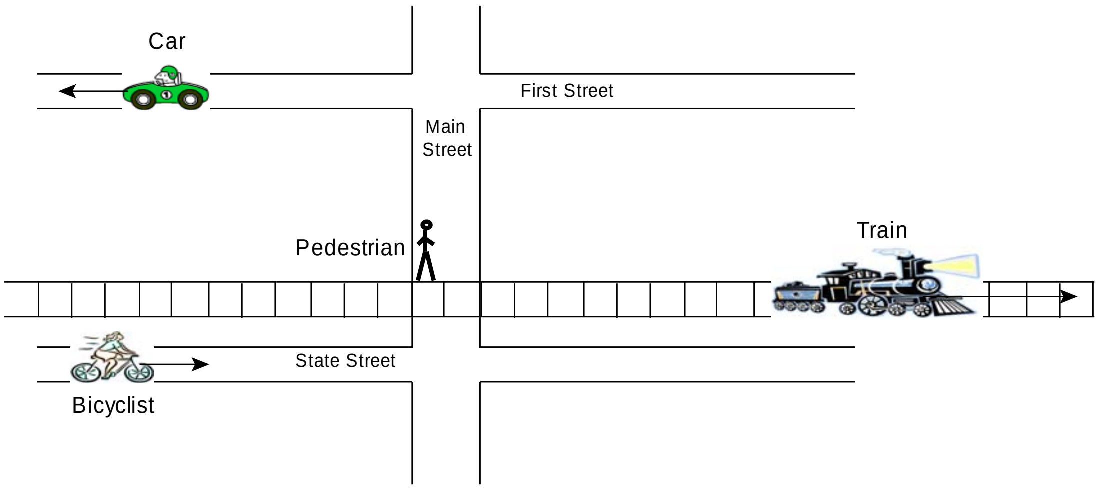
Doppler Effect

The train traveling east on the tracks at 50.00 mph (22.35 m/s) is blowing its whistle which emits a frequency of 400.0 Hz. Take speed of sound in air as 340.0 m/s. Assume all motion is along a straight line.



For each part, first write the equation you used to determine an answer, substitute numerical values, and then write the numerical answer to **four significant figures.**

1. a) A pedestrian is waiting at Main Street to cross the tracks. What is the whistle frequency heard by the pedestrian?

   b) A bicyclist is riding east on State Street at 3.000 m/s. What is the whistle frequency heard by the bicyclist?

   c) A car is speeding west through town at 28.00 m/s. What is the whistle frequency heard by the driver of the car?

2. Suppose the pedestrian hears the echo of the sound reflected off the moving car, what frequency is observed? Write the equation you used to determine the answer, substitute numerical values, then write the numerical answer to **four significant figures**.

HINT: The wave fronts are reflected from the moving car at the same rate that they are incident on the moving car.

3. A wind is blowing ***from*** the east at 15.00 m/s. Repeat the calculations of Part 1 to include the effect of the wind. **SHOW ALL WORK**.

   (a) *A pedestrian is waiting at Main Street*:

   (b) *A bicyclist is riding east at 3.000 m/s*:

   (c) *The car driving west at 28.00 m/s*:

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Comparing Waves on Strings and Sound Waves

So far we have discussed mostly waves in a string. The wave is a function of time and a single position variable, x. Next we will look at sound waves that are a function of time and the three position variables, (x, y, z). You can predict several features of sound based on the features of waves in a string. Use the text where needed.

For waves on a string moving along x-axis $y(x,t)$	For sound waves moving along x-axis $s(x, y, z, t)$
String is characterized by linear density μ with units of …	Air is characterized by … with units of …
Wave is (Transverse Longitudinal)	Wave is (Transverse Longitudinal)
Amplitude measures the change in the variable …	Amplitude measures the change in the variable …
For sinusoidal waves, in terms of the angular frequency ω and wavenumber k, the speed is	For sinusoidal waves, in terms of angular frequency ω and wavenumber k, the speed is
The speed of a string wave is determined by (source, medium, both, neither)	The speed of sound is determined by (source, medium, both, neither)
The frequency of a string wave is determined by (source, medium, both, neither)	The frequency of a sound wave is determined by (source, medium, both, neither)
The wavelength of a string wave is determined by (source, medium, both, neither)	The wavelength of a sound wave is determined by (source, medium, both, neither)
An expression for a wave moving to the left is	An expression for a wave moving to the left is
For a string characterized by μ and tension F, the speed is …	For air the speed is …
Waves in string carry power having units of …	Sound waves have intensity in units of …
The relation between power, amplitude, and angular frequency is…	The relation between intensity amplitude, and angular frequency is…
We can have pulses, interference, and standing waves for waves in a string.	For sound, we can have …

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Sound Wave Problems

1. Sound from a speaker has an amplitude equal to the amplitude of the SHM of the speaker. You have two speakers, one high frequency and the other low frequency. You hear equal intensity sound waves, with the frequency of the second wave is 9 times the frequency of the first.

   a) Which speaker will have larger amplitude?
   b) What is the ratio of amplitudes?

2. A 3.00 kHz sound wave travels through air at 343 m/s. The intensity of the sound wave is equal to the threshold of hearing (10^{-12} W/m^2).

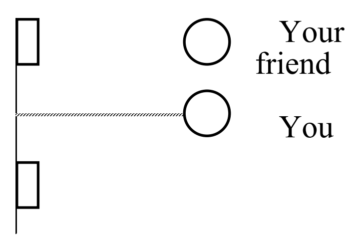
   a) What is its sound level?
   b) What is the pressure amplitude of the sound wave? Compare this to atmospheric pressure.
   c) What is displacement amplitude of the sound wave? Compare this distance to the size of an atom.

3. I produce sound with the same frequency, first in a pure oxygen atmosphere, then in a pure helium atmosphere. The bulk modulus is the same for the two gases.

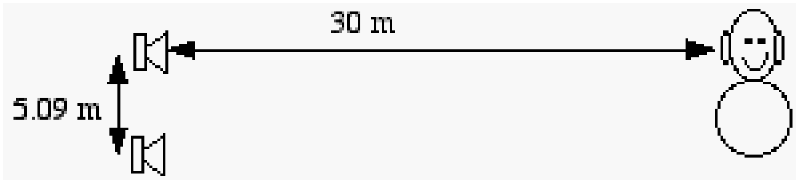
   a) In which gas is the speed larger?
   b) What is the ratio of speeds?

4. In your room you listen to two identical speakers mounted on a wall and separated by 4.16 m. They emit identical sounds of frequency 686 Hz. You are sitting in a chair at a distance of 5.0 m from the wall, and your friend is standing 2.08 m to your right. Use 343 m/s for the speed of sound.

   a) Will you hear constructive or destructive interference?
   b) Will your friend hear constructive or destructive interference?
   c) Find a different frequency of sound for which you and your friend **both** hear completely constructive interference.



5. Suppose I have two speakers, separated by 5.09 m, each emitting an 800 Hz sound, and I listen at a point 30.0 m away as shown (not to scale). Use v = 343 m/s.

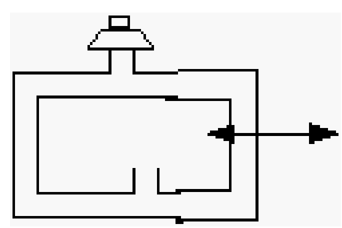


a) Assume that the speakers are in phase–that is they both emit a peak at the same time. What will I hear, constructive or destructive interference?
b) Now I move the bottom speaker vertically. At what speaker separation will I hear the other type of interference from part (a)?
c) Suppose I double the frequency of the speakers, what will I hear for the two separations in a) and b) above?

6. Two identical sources emit sound of frequency 2.00 kHz and speed v = 343 m/s. They are separated by 25.7 cm.

a) What type of interference is heard at a distance of 5 m away along the axis joining the speakers?
b) You now increase the frequency of both speakers. Find the next frequency for which you will hear destructive interference.
c) Now return to 2 kHz. What, if anything, can you do to get the opposite type of interference as in part (a)?

7. The apparatus demonstrates sound wave interference. Sound enters the tube at the top, goes left and right around the tube, and is detected at the bottom. The right hand C-shaped section can be slid left or right. Assume that we have a 3.00 kHz sound in air with v = 343 m/s.



a) As I slide the tube out, what will I hear?
b) Suppose I start with a maximum sound and pull the right hand tube out 5.71 cm. Describe what I hear, a loud or quiet sound?

8. Consider a string of length L with nodes at each end.

   a) Draw the four lowest frequency standing wave patterns and complete the other items. Use v to represent the speed of the wave when oscillating in the fundamental mode.

Pattern (equilibrium position of string is shown)	**# of Wavelengths in *L***	**Wavelength**	**Frequency**
N *N*			

   b) Now label each picture with successive Nodes [*N*] and Antinodes [*A*], thus the lowest pattern would be *N*-*A*-*N*.

   c) Now suppose I touch my finger lightly at the center of the string, at $L/2$. Which of the patterns in part a) could appear on your touched string? WHY?

   d) On the following page, repeat step (a) and (b) for the first three N-A and A-A cases. These would be the cases for a tube open on one end and open on two ends, respectively.

N-A Patterns

Pattern (equilibrium position of string is shown)	# of Wavelengths in *L*	Wavelength	Frequency
N ______ *N*			
______			
______			

A-A Patterns

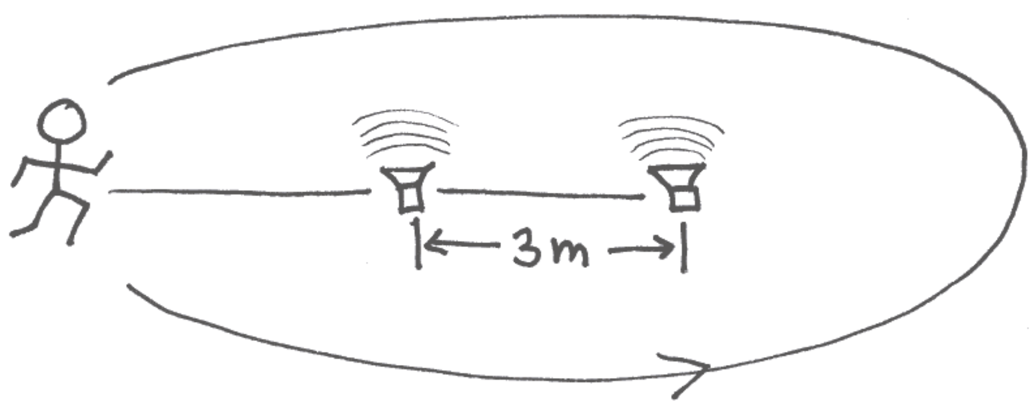
Pattern (equilibrium position of string is shown)	# of Wavelengths in *L*	Wavelength	Frequency
N ______ *N*			
______			
______			

10. You have a sound of some unspecified intensity. You then reduce the intensity by a factor of three ($I \rightarrow I/3$). By what amount does the sound level (dB) change?

11. A rock concert is roughly 110 dB. What is I / I_0 in this case?

12. How many dB is $I_0 / 100$ (cat hearing)?

13. Brain-Buster: You go out onto the RIT fields on a balmy 20.0 ^{o}C afternoon and set up two speakers 3.00 meters apart, both emitting the same 340 Hz tone, and run around them in a big loop (everyone needs a hobby). How many times in one circuit around the speakers do you hear destructive interference (where the two sounds are completely out of phase)? Use 340 m/s for the speed of sound.



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Reflection and Refraction Applets

1. Reflection: Go to: micro.magnet.fsu.edu/primer/java/scienceopticsu/reflection/

What do you notice about what happens when you change the angle of incidence?

What do you notice about what happens when you change the wavelength?

2. Refraction: (a) Go to: micro.magnet.fsu.edu/primer/java/scienceopticsu/refraction/

When the applet begins, it shows white light passing through a prism. **Begin by switching to monochromatic light** = light of one color (one wavelength).

What do you notice about what happens when you change the angle of incidence?

What do you notice about what happens when you change the wavelength? Pay attention to the numbers too!

What do you notice about what happens when you change materials? How does refraction (the amount the light is bent) vary with n?

Now change back to **white light**, and explore **dispersion**. Note the dispersion = different angles for different colors (or wavelengths or frequencies).

How does the dispersion change if you switch from ice → crown glass → diamond → lead sulfide? [Each material has a number next to it which is its value of n = the index of refraction.] How does dispersion seem to vary with n?

What do you notice about what happens to the dispersion when you change the angle of incidence? Be certain to look at 0°.

(b) Go to: www.phy.ntnu.edu.tw/ntnujava/index.php?topic=16 (scroll down a little)

How does the speed of the wave front vary from one material to the other? How does it depend on the index of refraction? Note: $n_{air} = 1$.

How must the frequency of the light in one medium compare to its frequency in another?

Slow the animation down enough so you can see the individual wavefronts being emitted as the light hits the boundary. This is Huygen's model of how light propagates. How does the wavelength in air compare to the wavelength in diamond (or glass or water)?

Why does the wavelength change in this way?

Index of refraction, *n*, is defined as the ratio of the speed of light in vacuum, *c*, to the speed of light in the material, *v*: $n = c / v$

Does this help explain why light does not change angle on reflection, but does change angle on refraction (transmission)?

TIR = total internal reflection

Choose one of the situations where $n_2/n_1 < 1$. What happens to the angle of refraction?

Change the angle of incidence and look for the "critical angle" of incidence, the first angle at which there is no transmitted light. How does this change as you change the ratio of n_2/n_1? What is the refracted angle when the incident angle is the critical angle?

You have encountered Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

What should the value of the critical angle be for one case you looked at above?

If you finish early, check these out while the others catch up;

World's coolest ripple tank applet: falstad.com/ripple
Everybody likes rainbows: www.atoptics.co.uk/bows.htm

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Thin Lenses and Image Formation

Materials/Equipment:

Optical bench with lens assortment
1 short focal length convex lens (f_{short} of approx. 48 mm)
1 long focal length convex lens (f_{long} of approx. 127 mm)
1 negative focal length concave lens (f_{neg} of approx. -150m)

Light source (goose-neck lamp with 40W clear bulb). Use a rubber band to hold it lower.
Meter stick and/or ruler
3x5 cardboard cards

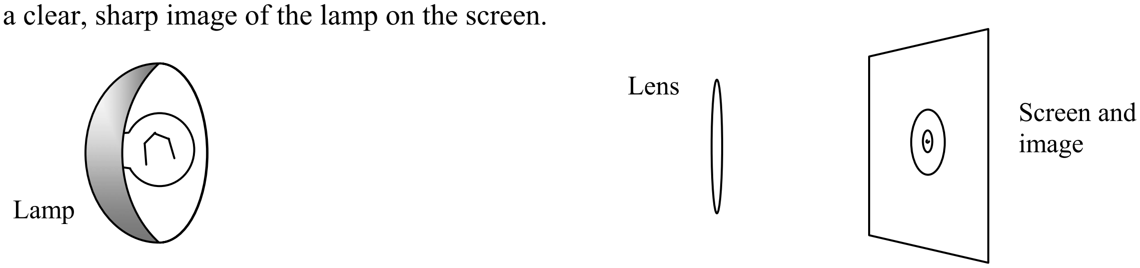
Objectives:

Observe the behavior of lenses and lens combinations, including pinholes. Understand the thin-lens equation, and especially the **sign** choices that arise.

Procedure:

1. The focal length of a convex lens is the distance from the lens to the image of a light source that is infinitely far away. Determine the approximate focal lengths of your convex lenses by measuring this distance with a ruler when forming an image of a distant object (out a hallway window), just to make sure you've got the right ones in the right bags, they may be mislabeled. Write the results here:

2. Arrange the image screen and lamp about one meter apart on your lab table. Hold the long focal length lens between them, nearer to the screen than the lamp, and adjust the **screen's** position until you see a clear, sharp image of the lamp on the screen.



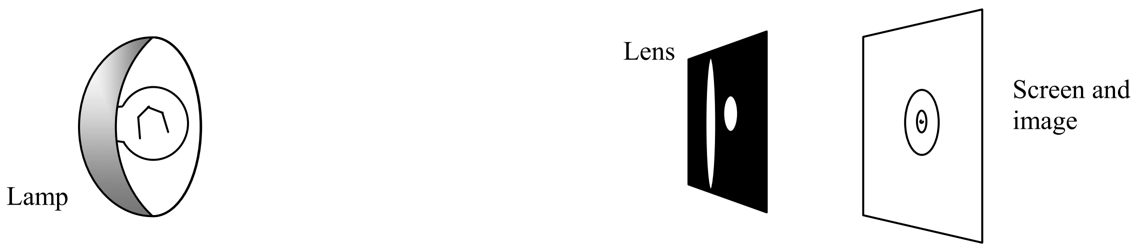
Is the image upright or inverted; that is, is it flipped vertically? Is it flipped horizontally? How can you tell? [You may want to hold a pencil right in front of the lamp and move it around while watching the image of the pencil.]

3. What do you predict will happen to the image when the top half of the lens is covered so no light gets through that part of the lens? (Answer below before attempting this).

Now try it. Use a 3x5 card to gradually cover the top half of the lens, making sure the card is close enough to touch the lens. What do you observe? Does it agree with your prediction? [Don't change the prediction! It is OK if you predicted wrong, as long as we're learning here.]

What happens if you cover the bottom half or a side half of the lens?

4. What do you predict will happen to the image if you cover the entire lens except for a small hole?



Try it. **Safely** punch a small hole (about one or two mm in diameter) in a card and hold it against the lens. Try moving the pinhole around so you see how the image looks when the light goes through different parts of the lens. What do you observe?

5. Draw a conclusion: Does light from one point on the lamp go through only one particular part of the lens, or does light from all points on the lamp go through all parts of the lens? Justify your answer in terms of your observations.

6. Predict what will happen to the image if you remove the lens and leave the card with the small hole in it.

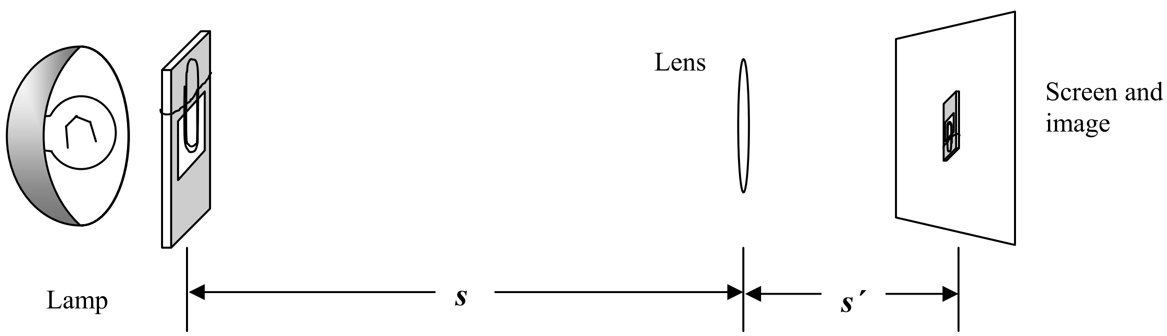
7. Try it. Instead of removing the lens from its holder, just replace the lens and holder with a different holder that has a card with a small hole in it. What do you observe?

You may have heard of a "pinhole camera." This is just a camera that has a tiny hole, like the one you are using, instead of a lens. For better results, you may want to try a smaller hole, but there is always a trade-off as this reduces the brightness of the image.

Brain-buster: Does a focal length for your pinhole make sense? What do you find when you try to measure it?

Thin-lens equation and magnification

Attach a paperclip in a lens holder (it's magnetic) as shown below and use this as your object for the remainder of the lab. You can bend them if needed; they're not special physics paperclips.



The thin-lens equation relates the focal length f to the object distance s and the image distance s'

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

When a lens forms an image of an object, the size of the image is generally different from the size of the object. We define the lateral magnification m to be the ratio of image height h' to object height h (where "height" means the distance from the bottom of the image to the top of the image, for example, not the distance from the table top). If an image or object is inverted, we take its height to be negative.

$$m = \frac{y'}{y}$$

Measurements

8. Set up the long focal length lens so it forms an image of a **distant** object on the screen. (This step was done earlier, so only repeat it if you think your measurements might be more accurate now that you've fiddled with things a little). Now **accurately** measure the distance from the screen to the **center** of the lens. Since the object is far away ($s \to \infty$), record this result as the focal length of the lens:

$$f \approx s' =$$

9. Set up the paperclip and lens as in the previous diagram. Can you see the chromatic aberration around the edges of the image? **Accurately** measure the object and image distances:

$s =$ $s' =$

10. Use the thin lens equation to compute the focal length from your measurements in step 9, and record the result here. How closely does this agree with your measurement from step 8?

11. Measure the size of the paperclip (some convenient reference distance on it) and measure the size of its corresponding image. Remember: $y' < 0$ for an inverted image. Compute the magnification:

$y =$ $y' =$ $m =$

12. Compare the magnification from step 11 to the negative of the ratio of the image and object distances. (Remember that an inverted image has a negative height.)

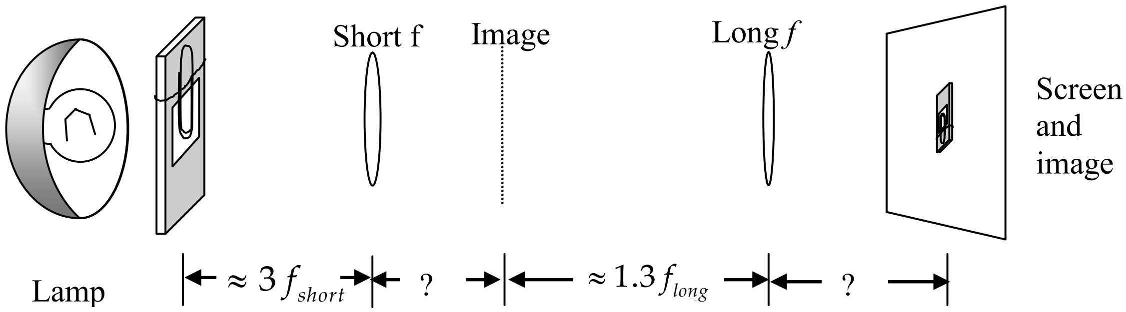
$-s'/s =$

13. Repeat steps 8 through 12 using the short focal length lens, and record the results here:

14. Notice that the thin lens equation is symmetric in the variables s and s'. Try setting up the paperclip and long focal length lens with the distances interchanged ($s \leftrightarrow s'$). That is, set it up so that the lens is now as far from the *paperclip* as it was from the screen in step 9. It will now be as far from the *screen* as it was from the paperclip in step 9. What do you see on the screen? Draw a ray diagram for the two cases.

15. Measure the magnification and record the result here. Does it agree with $m = - s'/s$?
The image may be very large now and difficult to measure, but give it your best try. If it's really screwed up, try starting with different numbers in step 9.

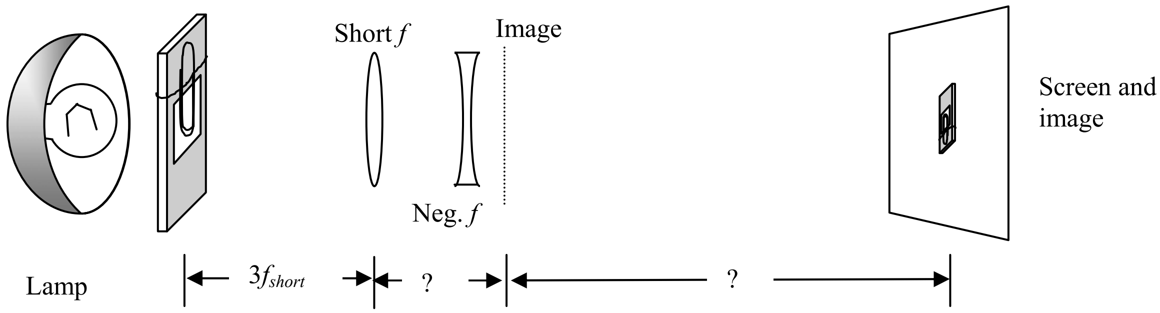
16. Put the short focal length lens in a holder and place it approximately a distance of $3f_{short}$ from the paperclip. Using the screen, locate the image of the paperclip. Measure the image distance and compare it to what you calculate with the thin-lens equation.



Put the long focal length lens in a holder and place it approximately a distance of 1.3 f_{long} from the image. (These are ***<u>very rough estimates</u>*** of the distances, and they'll vary for the different lenses, so you may have to alter this configuration significantly to get it to work). Using the screen, locate the image created by the long focal length lens (see diagram above). You might have to fiddle with the distances a little to get this to work, as the lenses are of various focal lengths. Measure this image distance and compare it to what you calculate with the thin-lens equation. Is the resulting image due to the two-lens system inverted or erect? Why? What is the overall magnification of the two-lens system? Is it what you expect ($m = m_1 \cdot m_2$)?

<u>Show work here</u>:

17. Leave the short focal length lens where it is and remove the long focal length lens from the setup. Place the negative lens in a holder and arrange it *in between* the short focal length lens and the image position you found earlier for the short focal length lens. Using the screen, locate the image formed by this two-lens system.



Is the resulting image due to this two-lens system inverted or erect? Why? Measure the image and object distances for the negative lens (Note that its object and image are both on the *same side of the lens* in the drawing above, so the Image goes in with a minus sign and the screen is positive in the last stage). Be careful with the signs! Use the thin-lens equation to calculate the focal length of the negative lens. (Note: You couldn't otherwise do this trying to project a real image with just the concave lens by itself. This is one way to determine a focal length if $f < 0$).

<u>Show work here</u>:

18. If you have extra time: Experiment with one short and one long focal length convex lens used together to see if you can make a simple telescope. Draw the ray diagram to explain what is going on here. (hint: you'll want to look through the short focal length lens with your eye fairly close to it.)

Circle one: (comments from previous quarters)

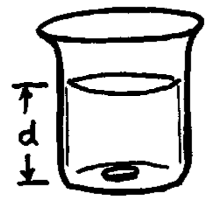
(a) This lab was both exciting and challenging. My head is spinning with the possible industrial applications of the thin lens equation. I'll have problems sleeping tonight.
(b) This lab sucks; I could have been sleeping now. (Please attach revised lab script).
(c) I know all this already. (Please attach full derivation of the thin-lens equation).
(d) What happened to burning ants?

Other: ____

Your Name (**Print**): ______________________ Date: __________
Group Members: ______________________
______________________ Group: __________

Refraction Problems

1. A coin sits on the bottom of a beaker at a depth d in a transparent fluid of refractive index n. What is the apparent depth d' of the coin when viewed from above (assume near normal incidence)?

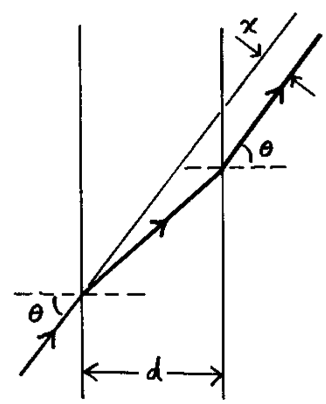


2. a) Prove that a ray of light incident on the surface of a sheet of plate glass of thickness d emerges from the opposite face parallel to its initial direction but displaced sideways, as shown in the diagram.

   b) Show that, for small angles of incidence θ, this transverse displacement is given by

$$x = d\theta\left(\frac{n-1}{n}\right)$$

   where n is the index of refraction of the glass and θ is measured in radians.



3. A letter on a page is 0.60 mm tall. A magnifying glass (a single thin lens) held 4.5 cm above the page forms an image of the letter that is 2.4 cm tall.

   a) Is the image real or virtual?
   b) Where is the image?
   c) What is the focal length of the lens? Is it converging or diverging?
   d) Sketch a ray diagram of this system to verify the results make sense.

Your Name (**Print**): ____________________ Date: __________

Group Members: ____________________

____________________ Group: __________

Double Slit Interference - Exploration

Caution: Class IV lasers (mostly harmless) are used in this experiment. Under no circumstances should you look directly into the laser, even if it seems like a really good idea at the time and your lab partners bet you a quarter it won't hurt. Laser light reflected from non-metallic surfaces is safe, but be careful with reflective surfaces. <u>Please help save batteries</u> and turn them off if you're doing calculations for any length of time.

Place the laser at the far end of an optical bench. Make sure that the laser is pointing **toward** the wall. Use the slit set marked "Double Slits". The slit set should be located near the laser and a piece of paper should be taped to the wall.

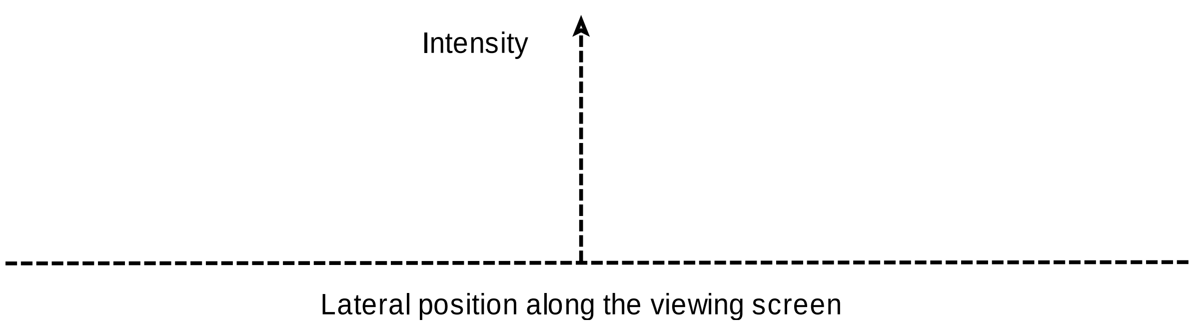
Record the distance between the plane of the slits and the viewing screen (the paper taped to the wall):

$D =$

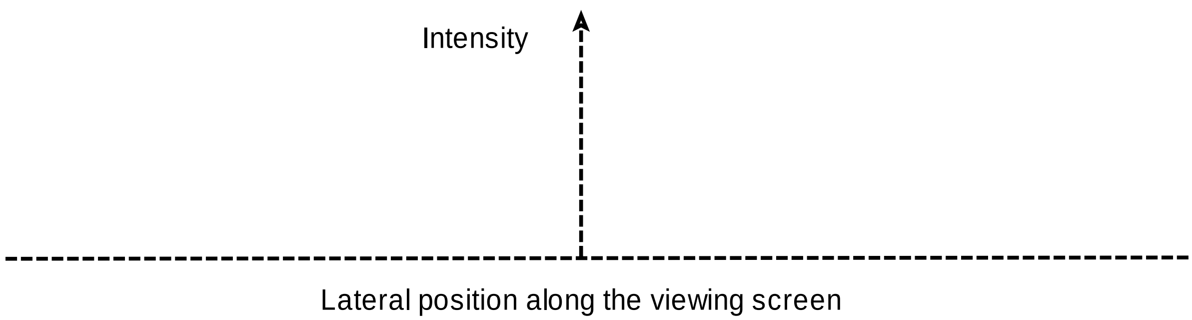
You will be using the combinations shown in the table. Be sure the laser shines through the correct slit set.

	Double-Slits			
Label	**A**	**B**	**C**	**D**
Width *a* in mm	0.04	0.04	0.08	0.08
Separation *d* in mm	0.250	0.500	0.250	0.500

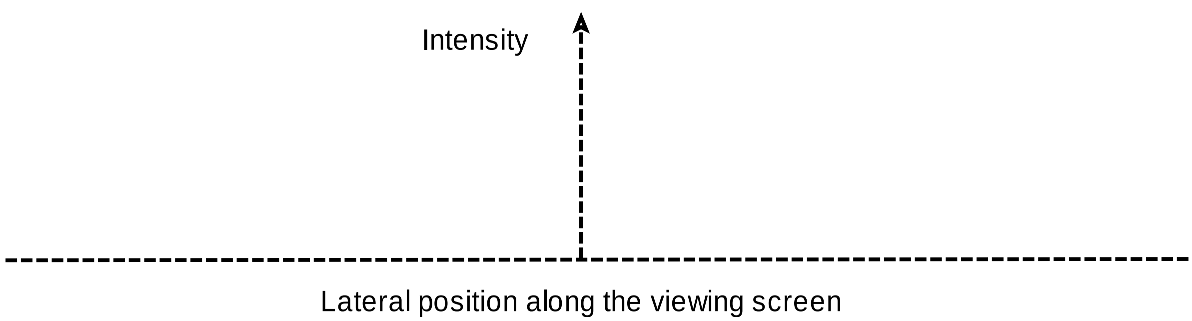
Starting with the combination labeled **A**, mark the ***centers of all the closely-spaced bright fringes*** on the paper taped to the wall. Mark as many as you can see. Label the pattern with the values of *a* and *d*. Also, sketch a graph of intensity as a function of position for the pattern you have drawn above. Capture the overall larger pattern as well as the fine fringes and label the features due to the slit separation *d* and the slit width *a*; that is, include the closely spaced interference fringes within the broad diffraction envelope.

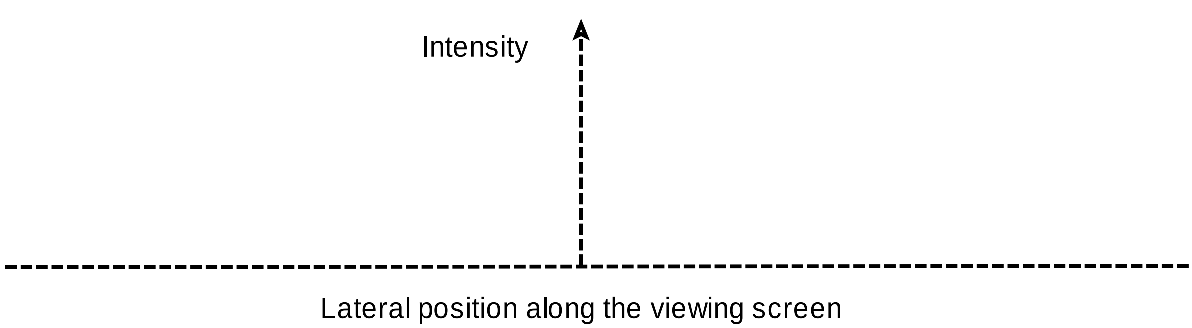


Move the paper up slightly and adjust the slit set so the laser passes through the double slits labeled **B** (*a* = 0.04 mm, *d* = 0.5 mm). Mark the ***centers of all the closely-spaced bright fringes***. Label the pattern with the values of *a* and *d*. Sketch a graph of intensity as a function of position for this pattern. Capture the overall larger pattern as well as the fine fringes and label the features due to the slit separation *d* and the slit width *a*; that is, include the closely spaced interference fringes within the broad diffraction envelope.



Repeat for the remaining two combinations.





Use your results to answer the following questions:

1. What happens if the slit separation increases, but the slit width is constant?

2. Explain the case above in terms of the formula we derived in class.

3. What happens if the slit separation is constant, but the slit width decreases? What would happen to the pattern if the slit width became infinitesimally small?

Your Name (**Print**): ______________________________ Date: __________
Group Members: ______________________________
______________________________ Group: __________

The Double Slit Experiment – Wavelength Calculation

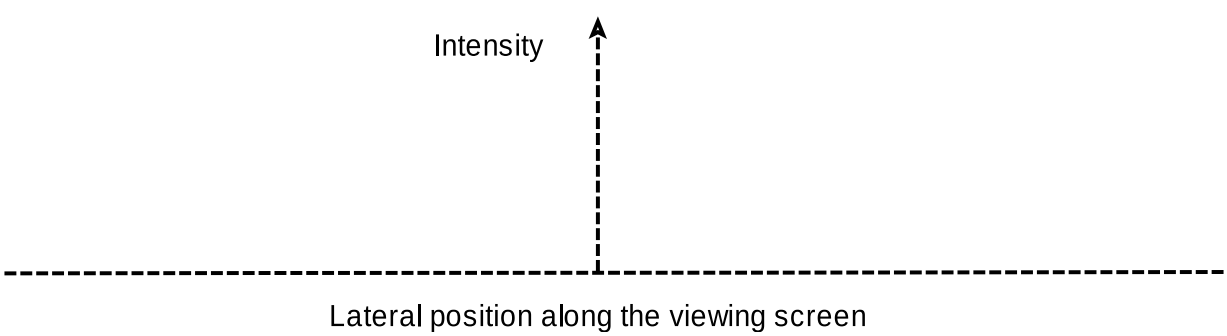
Caution: Class IV lasers (mostly harmless) are used in this experiment. Under no circumstances should you look directly into the laser, even if it seems like a really good idea at the time and your lab partners bet you a quarter it won't hurt. Laser light reflected from non-metallic surfaces is safe, but be careful with reflective surfaces. Please help save batteries and turn them off if you're doing calculations for any length of time.

Review the experimental set-up for Young's double slit experiment. Set up the laser and slit set on an optical track and prepare to **carefully** measure the parameters, along with all accompanying uncertainties.

If you have a complete set of good data from the previous activity ("Double Slit Interference – Exploration") then proceed to the top of the next page.

(a) Sketch the bright and dark spots you see on the screen for a clear double slit pattern that you've made with the laser. Pick one where the pattern is large and easy to measure. You should have a fairly large distance to the screen or wall to minimize the uncertainties. Use your pen/pencil to make it darker on the paper where it is brighter on the screen. (Pencil mark = bright spot on screen and white paper = dark spot on screen). The sketching-challenged can try tracing it right over the pattern projected onto this sheet.

(b) Sketch a graph of intensity as a function of position for the pattern you have drawn above. Match the horizontal scales so the two sketches line up. Try to capture the overall larger pattern as well as the fine fringes and label the features due to the slit separation *d* and the slit width *a*.



Let $\Delta y = y_{m+1} - y_m$ be the spacing between adjacent bright fringes. *D* is the distance between the plane of the slits and the viewing screen.

(c) **Carefully** measure the separation of the bright spots on the screen and the distance from the slits to the screen. Calculate the wavelength of the red light, and **include an uncertainty in the result**. This uncertainty should be based on the uncertainties you measured for the three measurements needed for the calculation. (Use the product/quotient rule for relative errors). Triple check your numbers and spend some time making sure you're doing everything as carefully as possible. You'll have to estimate what you think the error in *d* would be, but it's probably safe to assume it's manufactured to within a percent or so at least.

Measurements (include uncertainty):

$\Delta y =$

$D =$

$d =$

(d) Which error in the things you measured had the greatest relative error? Which had the smallest?

If your total error below is a little large, try increasing *D* and measuring the setup again. This should help decrease the relative error in two of your quantities.

Calculations:

Wavelength Result:

$\lambda =$ (________________ ± ________________) nm

(Remember, always: 1 sig. fig. on error, placement of cutoff on digits matching)

(e) Does your result agree with the range printed on the laser, within uncertainties?

(If not, better go back and check things because this is one of those beautifully elegant experiments that yields very accurate results from just a few measurements)

(f) What general conclusions could you make about the fundamental nature of light even if you had not made any measurements and just examined the pattern? (If you had set this up 200 years ago with monochromatic light (no lasers back then), you'd have won shiny awards, and been invited to all the right parties.) Why don't we see diffraction of light more in everyday life?

(g) Use the numbers you recorded in part c) to find the wavelength without using the small angle approximations for $\sin\theta$ that we used in the derivation, and be careful not to round off in the middle of calculations. At what number of significant digits does using the approximations start to matter? Is this well within the uncertainty range, so we didn't really have to worry about using the small angle approximation?

Calculations:

(h) Notice that although the laser beams all look like basically the same color of red, different groups are getting slightly different wavelengths. This is to be expected because the lasers 'drift' slightly with age, and variations in manufacturing, etc., and your eyes can't discriminate between pure wavelengths very well. Now, two lasers at slightly different wavelengths should make a beat frequency, so try aiming your laser and another group's at the same spot. Can you see a beat frequency? Explain why or why not.

Calculations:

Your Name (**Print**): ______________________ Date: __________
Group Members: ______________________
______________________ Group: __________

Multiple Slits and Diffraction Gratings

Caution: Class IV lasers (mostly harmless laser pointers) are used in this experiment. Under no circumstances should you look directly into the laser, even if it seems like a really good idea at the time and your lab partners raise the bet to a dollar it won't hurt. Laser light reflected from non-metallic surfaces is safe, but be careful with reflective surfaces. Please help save batteries and turn them off if you're doing calculations for any length of time.

In a previous workshop, you've fiddled with the interference pattern arising from two slits, so you should be familiar with the setup at this point. Switch to the slit set labeled "Multiple Slits" with 2, 3, 4, and 5 slits and sketch each pattern. As before, mark the ***centers of all the closely-spaced bright fringes*** on the paper taped to the wall. Mark as many as you can see. Label the pattern with the values of a, d and number of slits. Here both a and d are fixed, and only the number of slits changes. As the number of slits changes, what changes about the pattern?

Notice the extra structures now present between the brightest interference fringes? Carefully sketch the intensity pattern that includes three of the brightest interference fringes. You may want to make D, the distance to the screen, as large as possible in order to see the patterns better.

2 slits:

3 slits:

4 slits:

5 slits:

Theory of Interference with 3, 4, ... *N* slits

Consider an aperture with *N* identical slits, each having the same small width, *a*, with the center-to-center spacing from one slit to the next slit being the same value, *d*, for all slits. We can use phasors to add up the light at a given point on the screen (essentially the phase of an electric field, and you'll learn more about that in University Physics III). The paths from the different slits will be slightly different, and this can be represented by some phase angle for each of the phasors that gets added together. This is a little abstract, so we'll use an applet to help picture what is going on here. It's useful to try playing around with it first so you get the idea.

Go to the applet: http://vnatsci.ltu.edu/s_schneider/physlets/main/phasorslits.shtml

The phasors represent the amplitudes of the waves, with each slit contributing an amplitude of 1 and (except at $\theta = 0$) the same phase difference. Remember that the relative intensity is the square of the amplitude. Use the applet to help you fill in the table below for the cases listed.

Start off with $N = 2$ slits and advance the phase angle of the second slit a few degrees at a time. This is what would happen if the path length of the second slit to the screen became slightly longer as a result of looking away from the center $y = 0$ of the screen.

The red vector is the resultant, and the square of its length is proportional to the intensity we should see on the screen as we move along. Notice that only the length of the resultant matters, so we don't have to rotate the first phasor, we can just leave it on the '*x*-axis' since it's only the relative phase difference between the phasors that matters.

Notice that the whole thing cycles around and comes back together in phase again, and this explains the repetitive pattern of fringes we see on the screen. Note that this particular use of phasors does not include effects from the width of the slits, so the broader dark minima structure will not be predicted here.

Now try $N = 3$ slits, and again tour through a cycle to see what's going on with the phasors. Make sure you understand why there's a relative phase shift between each phasor. Does the extra structure in the intensity diagram match what you recorded in the previous sketches?

In going from $N = 2$ to $N = 3$ slits, does the intensity drop off faster or slower as you move away from the center? Does this agree with what you recorded in the previous sketches?

Identify some general trends when comparing the different numbers of slits by recording some of the results from the applet. Note that you can read the intensity off of the phasor plot at a given point along the intensity plot corresponding to the blue dot. The units here are just relative to some standard, we just want to see the qualitative pattern here.

N	Intensity of Principle Maximum (θ=0 center)	Intensity of 1st Secondary Maximum (next one over)	Ratio of Principle Intensity to 1st Secondary Intensity	Angle at which Intensity First Equals 0	Angle for 1st Secondary Maximum	Number of Secondary Maxima Between Principle Maxima
2	4	N/A	N/A	180º	N/A	0
3						
4						
5						
6						
7						
10						
100						

Look at the sketches you made in the previous pages. Describe how those results compare with what you wrote in the table above. Check the specific details of the structure you saw between the principal maxima, don't just say, "They were sweet!" It's difficult to judge intensities, but the counting of the number of secondary maxima should agree.

Q: What can you say about the general trend if we increase the number of slits to 500, or 1000? Note that you can insert a larger number like this into the program by typing it in and clicking on the ‘update manually’ button. See what happens when you do this. The calculations are more intense now, so give the applet a second or two if it’s a large number.

Diffraction Gratings

(a) Try putting the diffraction grating in place of the slits. This is the slide which has sort of a rainbow color to it. This is caused by thousands of small rulings on the slide which is like having thousands of small slits. They are very close together, so you may have to look at a much larger angle to find the first principal maximum. Does what you see make sense in terms of carrying the previous examples to very large N?

(b) **Carefully** measure the separation of the bright spots (principal maxima) on the screen and the distance from the grating to the screen (D). Calculate the wavelength of the red light, and **include an uncertainty in the result**. This uncertainty should be based on the uncertainties you measured for the three measurements needed for the calculation. The spacing between the rulings (slits) can be determined from the number written on the grating. Note that it’s much smaller than previous slit spacing d you’ve used, so the spacing between the bright principal maxima ($\Delta y = y_{m+1} - y_m$) will be much larger now. Estimate what you think the error in d would be, assuming the number written on the slide is accurate to the number of digits shown.

Measurements:

$\Delta y =$ ____________________ ± ________________________

$D =$ ____________________ ± ________________________

$d =$ ____________________ ± ________________________

(c) Which error in the things you measured had the greatest relative error? Which had the smallest?

Calculate the wavelength λ of the laser. Note that this is exactly the same setup as we had for the two slit experiment, so the math is the same, it's just that having multiple slits allows us to determine the Δy value far more accurately now.

The only catch is that because Δy is much larger now, we can't use the small angle approximation anymore, so we'll have to leave the $\tan(\theta)$ and $\sin(\theta)$ functions as they are when calculating λ. (Recall that we tried this in a previous workshop with the 2 slits to see if it made a difference then).

However, when calculating the uncertainty in the final result, the uncertainty is just an estimate to one significant figure, so we can still just add the relative errors from the three terms as in the previous workshop. The very slight change from using the trig functions doesn't matter in the error analysis to one sig. fig. anyway.

Calculations of λ and $\Delta\lambda$:

Wavelength Result:

λ = (__________________ ± __________________) nm

(Remember, always: 1 sig. fig. on error, placement of cutoff on digits matching)
Note that this result is far more accurate than the wavelength result from the previous workshop.

(d) Does your result agree with the range printed on the laser, within uncertainties?

(e) Calculate where the second order (m=2) principle maxima for the grating should be, and see if it is there. Repeat this for the higher orders, and see how many of them should be visible. What limits whether they will appear or not?

(f) Look at a white light source through the diffraction grating, such as a desk lamp or outside a window. Explain why there is a rainbow spectrum at roughly the same angle as where the first red maxima appeared. Which has the smaller angular position on the viewing screen (θ), the red light or blue light, and why?

Your Name (**Print**): ______________________ Date: __________

Group Members: ______________________

______________________ Group: __________

Diffraction

Caution: Class IV lasers (mostly harmless) are used in this experiment. Under no circumstances should you look directly into the laser, even if it seems like a really good idea at the time and your lab partners bet you a quarter it won't hurt. Laser light reflected from non-metallic surfaces is safe, but be careful with reflective surfaces. Please help save batteries and turn them off if you're doing calculations for any length of time.

The single-slit diffraction geometry is shown in Figure 1. As discussed in class, the first position of fully destructive interference (the center of a dark fringe) occurs when the light from the top half of a slit of width a interferes destructively with the light from the bottom half of the slit. In general, the angular positions of the dark fringes θ_m are determined by

$$a \sin\theta_m = m\lambda \text{, where } m = \pm 1, \pm 2, \pm 3, \cdots \textbf{ but not } m = 0 \qquad (1)$$

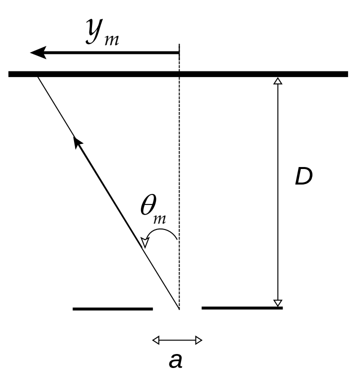
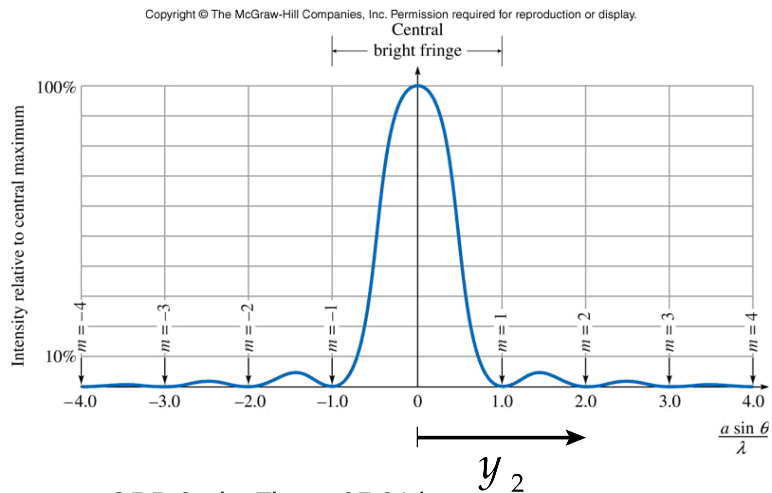


Figure 1 Single-slit Diffraction Geometry



GRR 2nd Figure 25.31 b

Instead of measuring the angle θ_m directly, you will measure the linear distance y_m along the screen from the center of the pattern ($\theta = 0$) to the center of the m^{th} dark fringe. The angle θ_m is related to the distance y_m and the distance D from the slit to the screen; specifically,

$$\tan\theta_m = \frac{y_m}{D}. \qquad (2)$$

If θ_m is small, then

$$\sin\theta_m \approx \tan\theta_m = \frac{y_m}{D}. \qquad (3)$$

PREDICTION:

1. The width of the central bright fringe is defined as the linear distance between the centers of the first dark fringes on each side of the central bright fringe (corresponding to m = -1 and m = +1). Use Equations 1 and 3 to **derive an equation** for the <u>width of the central bright fringe</u> in terms of λ, D and a. **SHOW ALL WORK.**

2. **Derive an expression** for the <u>width of any lateral fringe</u> (defined as the linear spacing between adjacent dark spots on either side of the lateral bright fringe). Are the lateral fringes equally spaced? **WHY**? **SHOW ALL WORK.**

3. What is the ratio of the width of the central bright fringe to the width of any lateral fringe?

$$\frac{width\ of\ central\ fringe}{width\ of\ lateral\ fringe} =$$

   a) Does your result depend on the width of the slit?

   b) Does it depend on the wavelength of the light?

MEASUREMENT

Safety Precaution: Avoid looking directly into the laser. Reflections from the screen or wall are safe.

The single-slits that you will use are mounted on a slide holder ("Single Slit Set") and are identified as follows:

	Single-Slits	
Label	**B**	**C**
Width a in mm	0.04	0.08

Place the laser at the end of an optical bench farthest from the wall. Make sure that the laser is pointing **toward** the wall. The slit set should be located near the laser and a piece of paper should be taped to the wall. Adjust the set so that the laser shines through slit **B** (a = 0.04 mm). Be sure the laser shines through the correct slit. Record the distance from the slit to the screen (D) in Table 1.

Mark the **center of the dark spots** on both sides of the central bright fringe. Mark as many dark spots as possible on each side (but at least three). Label the pattern "a = 0.04 mm". Move the paper and adjust the slide set so that the laser shines through slit **C** (a = 0.08 mm), mark the centers of the dark spots, and label the pattern "a = 0.08 mm". Be sure the laser shines through the correct slit.

Remove the paper and measure the width of the central bright fringe for both patterns and record the result in Table 1. Include an uncertainty (how well were you able to locate the center of the dark fringe?).

Measure and record the widths of the lateral fringes for each pattern. Record the measurements in Table 1. Calculate the average width of the lateral fringes.

Are all the lateral fringes on both sides of the central bright fringe equally spaced? Is this consistent with your prediction (Prediction 2)? Explain.

For both slits, calculate the numerical value of the ratio of the width of the central bright fringe to the average width of the lateral fringes? Is this consistent with your prediction (Prediction 3)? Explain.

Compare the diffraction patterns produced by the two slits. Describe how the single-slit diffraction pattern changed when the slit width doubled.

Use the measured width of the central bright fringe and your result from Prediction 1 to calculate the wavelength of the light in each case. Calculate the uncertainty in the wavelength (assume the uncertainty in the slit width is 1%). Make sure you get reasonable values - the laser light is visible! Show one wavelength calculation (and its uncertainty) in the space below.

Do the two wavelengths agree within experimental uncertainty?

DATA AND RESULTS

Table 1. Determination of Wavelength

Single-Slit	Width of **central** bright fringe (cm)	Width of **lateral** bright fringes (cm)	Average width of lateral fringes (cm)	D (m)	Wavelength (nm)
B (a = 0.04 mm)					
C (a = 0.08 mm)					
Average λ (nm)					
Uncertainty $\Delta\lambda$ (nm)					

Single slit with phasors

Go to: surendranath.org/Applets.html

and select Applet Menu → Optics → Single Slit.

Try changing the slit width. Describe the effect. Does it do what you expect? Explain.

Try changing wavelength. Describe the effect. Does it do what you expect? Explain.

Now switch from "Experiment" to "Results" in the lower left corner. Watch the phasor (sum of all the individual "point" sources in the slit width) change as you move the mouse along the line representing the x-axis of the plot. Work with this until you understand why the phasors and their sum look the way they do.

Your Name (**Print**): ______________________________ Date: __________
Group Members: ______________________________
______________________________ Group: __________

Interference and Diffraction Problems

1. Single slit: A single slit has a width of 0.040 mm and is illuminated by 600 nm light. The diffraction pattern is observed on a screen 90 cm away from the slit. Determine the locations of the first three dark fringes on one side of the central maximum.

2. Double Slit: You have a double slit with slit width 0.040 mm and slit separation 0.150 mm.
(a) How many interference fringes would appear in the central diffraction envelope? (This is mixing both the effects of the double slit spacing and the width of the slits, as you observed in the experiment. Also note that the λ doesn't matter in this problem).

(b) Suppose you double the separation. What happens to (i) the location of the first diffraction minimum and (ii) the number of interference fringes in the central maximum?

(c) Suppose you keep the same separation as in part (a) but now make the slit width half of its previous value. What happens to (i) the location of the first diffraction minimum and (ii) the number of interference fringes in the central maximum?

3. Diffraction Grating: We shine light onto a diffraction grating with 550 lines/mm. (a) Work out the angles for all orders of diffraction maxima for blue light of wavelength 450 nm (b) What are the angles for all orders of diffraction maxima for red light of wavelength 650 nm?

4. Soap Bubble: Consider a thin layer of soap bubble with air on both sides. Find the thicknesses of the soap bubble for which there is (a) constructive interference (b) destructive interference of reflected light of wavelength 550 nm (assume the water/soap solution has an index $n = 1.33$) (c) Find the type of interference you expect when the film becomes very thin. In all cases assume the light is incident perpendicular to the film.

5. Anti-Reflection Coating: Suppose we put a thin layer, thickness L, of MgF_2, $n = 1.38$, onto glass, $n = 1.5$. Light of wavelength 500 nm is shone onto the system perpendicular to the film. Find the conditions that lead to (a) constructive and (b) destructive interference for the reflected light. (c) What is the minimum thickness of the material that will cause destructive interference of the reflected light?

6. Michelson Interferometer: Suppose the vertical path, d_2, has a pipe around it that can be evacuated. Also, make the starting condition $d_2 = 1.600$ m. Initially both paths are filled with air, and we have a bright fringe. As we slowly evacuate the pipe to produce a vacuum ($n = 1$), we count 1548 bright fringes of 600 nm light appearing and disappearing in the telescope. Find the index of refraction of the air.

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Measuring with Interference and Diffraction

Purpose: In this activity you will accurately measure the width of a human hair using the interference and diffraction properties of light.

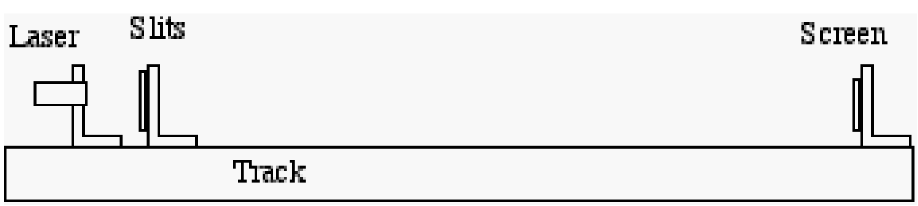
Method: By shining laser light onto a hair, a diffraction pattern can be observed beyond the hair on a screen. According to **Babinet's Principle**, the diffraction pattern of an object is **identical** to the diffraction pattern of the negative of the object. Thus a single thin barrier such as a hair should have the same pattern as a single slit. By knowing the wavelength of the laser and making measurements on the setup, you can find the width of the hair using the single-slit formula. Recall, we do not know the wavelength of individual laser diodes in the lab. The wavelength depends on minute variations in the manufacturing process, etc. Typical values range from about 630 to 680 nm. To check the wavelength of the laser, you will use three different techniques to measure the wavelength: Double Slit, Single Slit and Diffraction Grating. These have been done previously, and serves as a nice review at this point, but be careful to measure accurately.

Safety Precautions:

Again, avoid looking directly into the laser or reflections off glass or mirror-like surfaces. Detach hair from head **before** mounting onto holder.

Setup Procedure: Mount the laser at one end, the slit holder near the laser, and the screen at the far end of the optical track.

Note: You can make the patterns easier to measure if you put the screen on the wall instead of the optical track. You can tape a sheet of paper to the wall on which to record positions.



Calibration Procedure I -- Double Slit: Choose a slit pair in the section of the set containing double slits to measure the wavelength of the laser. Be sure to measure the *closely-spaced* fringes due to double-slit interference and not the wide pattern due to single-slit diffraction. As always, reduce the relative uncertainty by measuring the total width of a number of fringes and divide the result by that number of intervals. Attach the sheet you used to record this pattern.

Also estimate the uncertainties in your measurements and calculate the uncertainty in your value of the wavelength. Show your calculations and results for ($\lambda \pm \Delta\lambda$) here:

Calibration Procedure II -- Single Slit: Mount the set with single slits on the optical bench and choose a slit to measure the wavelength of the laser. Try to choose a larger *m* value to reduce uncertainties. Estimate the uncertainties in your measurements and calculate the uncertainty in your value of the wavelength. Attach the sheet you used to record this pattern. Show your calculations and results for ($\lambda \pm \Delta\lambda$) here:

Calibration Procedure III – Diffraction Grating: This should be the most accurate of the three methods. Record the number of slits/mm for the diffraction grating (most of them are 600 lines/mm, but check regardless). Note that we can't use the small angle approximation for this case, but when it comes to the uncertainties we can still just sum up the three relative errors because the uncertainty is only needed to one or two significant figures and the trigonometry used doesn't change the uncertainty very much at all. Calculate the wavelength of the laser and its uncertainty. Show your calculations and results for ($\lambda \pm \Delta\lambda$) here:

Calibration Analysis: Compare the values of the wavelengths you obtained. Do they agree within their uncertainties? If they agree or approximately agree within the uncertainties, then average the values to find the wavelength of your laser and the uncertainty of the wavelength. If they do not agree, check your calculations and uncertainty estimates. If one happens to be a much better measurement than the others, you may wish to use only that one and discard the others. Make your own judgment call on this.

Final Measurement: Obtain a single clean human hair. Please ask nicely if it is still attached to someone else. Long straight hairs have a more uniform cross-section than curly hair and are easier to mount on the lens holders. Tape it vertically across the aperture in the slide holder and carefully adjust the laser to hit the hair. The pattern should resemble a single slit pattern by Babinet's Principle (and NOT the negative of it as you might first think). On a new piece of paper mark the location of the first dark spot on each side of center. Use this data and the wavelength you have calculated to accurately determine the diameter of the hair (in μm) and the uncertainty in this value. Attach the sheet you used to record this pattern.

Super-fun extra stuff:

1) This is much trickier to rig up, but if you have extra time, see if you can get two identical hairs parallel and very close together. Is the pattern the same as that of the double-slit experiment?

2) Try one of those 0.5 mm pencil leads. A larger object will have a smaller pattern, which is one reason why you don't see diffraction too much in everyday life (the other is that you need monochromatic light). You might have to project it something like 6 meters to see the pattern clearly. Does the calculated value agree?

3) What would the negative of a diffraction grating be? What would the resulting pattern on the screen? Does this make sense in terms of Babinet's principle?

4) Young's original 'double-slit' experiment used a strip of paper edgewise to split the beam into two. See if you can rig this up to show a diffraction pattern.

Circle one: (comments from previous labs)

1) I came in today with a full set of hair.
2) I'm going to make a million dollars using lasers to do fake hair analysis in rich salons because nobody's thought of it yet.
3) Now I have two reasons why this class hurts my head.
4) Something keeps moving on my hair.
5) Other: ____

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<u>Thin Films</u>

Key Point: In thin films there are <u>two</u> possible sources of phase difference:

(1) The optical path length difference between rays: $L_{optical} = nL$

(optical path length) = (index of refraction)*(physical path length)

If the light is perpendicular to the film we need to use $2nL$ because it traverses the thickness of the film twice, once going down and once back up.

(2) The phase introduced upon reflection: If light is in a material of lower index, and reflects from a boundary with a material of higher index, the phase change is π. **"Low off high, change of pi"**, and conversely, "**High off low, change of zero**". There is no 'in-between' in this case, it's either one or the other.

Let's quickly look at (2) first with the applet at:

www.surendranath.org/Applets.html

Click on the "Applet Menu" box in the upper left corner, then "waves" → "Transverse waves" → "reflection and transmission". Try the different cases available. It's using string, but the analogy to light waves is exactly the same. This is a lot safer using applets for this part than rigging up springs all over the room.

Now let's look at part (1) as well, including the difference in the optical path length between two routes. Suppose a piece of glass is coated with a thin layer of another material, and we shine light on it. The sources of the two rays of light are the reflection from the top layer, and the reflection from the boundary. Load the animation:

webphysics.davidson.edu/physlet_resources/bu_semester2/c26_thinfilm.html

This one is drawn with the two light rays off to the side so you can see what's going on a little more clearly. Click the "1 wavelength" box on the first ($n_1 < n_2$ and $n_2 > n_3$) choice.

For $n_1 < n_2$, what is the phase change off the top surface? ______________
For $n_2 > n_3$, what is the phase change off the bottom surface? ______________

Check that what's going on in the applet agrees with the reflection song (**"Low off high, change of pi"**)

For a thickness of 1λ, what is 2 $L_{optical}$ in terms of phase? _______________

(Remember here that $k = 2\pi / \lambda$, so 1λ in distance is 2π in phase)

So the total difference in phase, taking into account both effects, will be _______________

and will this be **Destructive** or **Constructive** interference? _______________

(Check that this agrees with what's going on in the applet)

Insert this result into the first row of the following table and predict whether the remaining thicknesses will be constructive or destructive interference.

Film Thickness t in wavelengths	Top reflection phase change	Bottom reflection phase change	Optical path difference in terms of phase	Total phase change	Constructive of destructive interference
λ					
¾ λ					
½ λ					
¼ λ					
2 λ					

Check that your prediction match what's going on in the applet in terms of interference between the two reflected waves.

Now repeat the same steps for the second choice of $n_1 < n_2 < n_3$.

Film Thickness t in wavelengths	Top reflection phase change	Bottom reflection phase change	Optical path difference in terms of phase	Total phase change	Constructive of destructive interference
λ					
¾ λ					
½ λ					
¼ λ					
2 λ					

Now let's look at a site where you can change the thickness continuously:

mysite.verizon.net/vzeoacw1/thinfilm.html (That's a "one" after the vzeoacw)

Note that rays are drawn at an angle in the following simply for clarity, and we will always use waves that are incident perpendicular to the surface (called "normal incidence") in this course, so the angle of incidence will be zero since it is measured from the normal. If anyone does ever run up to you with a project where the angles are different, you could figure it out, there's just a bunch of extra geometry involved that we don't need at this point.

1. Set your simulation so the incoming waves make an angle of almost 90˚ with the surface -- not exactly 90˚ because we want to be able to still see what's going on with both waves. You can grab it with the mouse to change the angle.

(a) Try dragging the thick bar separating the blue film from the gray glass and see the interference change from constructive to destructive as the thickness of the thin film changes. The red line provides a reference line to see whether the reflected rays are in phase or not.

(b) What are the phase changes upon reflection from both of the surfaces?

Top surface phase change: **0** **π** (Circle one.)

Bottom surface phase change: **0** **π**

(c) What is the smallest thickness (in number of wavelengths) of the "blue" film that gives fully constructive interference?

(d) What is the total path difference in wavelengths (not including the reflection contributions yet) in this case? (Remember that you have to go twice the distance through the film).

(e) What is the next thickness (in number of wavelengths) of the "blue" film that gives fully constructive interference?

(f) What is the path difference in this case? How many wavelengths in the film is this?

(g) How does the wavelength in the "blue film" compare to the wavelength in air?
(Recall that the index of refraction is defined as $n = c / v$).

- Is the speed of light in the film **faster**, **slower** or the **same** as in air?
- Is the frequency of light in the film **larger**, **smaller** or the **same** as in air?
- So, must the wavelength of light in the film be **longer**, **shorter** or the **same** as in air?

(h) Write the relationship between the wavelength in air λ and the wavelength in the film λ_f.

(i) In terms of actual distances in nm, assuming we had green light at around 550nm, what would be the answers to (c) and (e)?

(j) What is the smallest thickness of the film (in nm) that gives fully **destructive** interference?

2. Now change the index of refraction of your film to $n = 1.65$, and see how this changes things;

(a) What is the smallest thickness (in number of wavelengths) of the "blue" film that gives fully constructive interference?

(b) What is the total path difference in wavelengths (not including the reflection contributions) in this case?

(c) What is the next thickness (in number of wavelengths) of the "blue" film that gives fully constructive interference?

(d) What is the path difference in this case? How many wavelengths in the film is this?

(e) When you changed to the higher index film:

• Was the light reflecting off the air-film interface (the top reflection) reflecting off the **more dense (higher index)** or the **less dense (lower index)** material?

What was its phase change? **0** **π** ?

• Was the light reflecting off the film-glass interface (the lower reflection --where the line is thick) reflecting off the **more dense** or the **less dense** material?

What was its phase change? **0** **π** ?

• What is the total phase difference due to reflection? **0** **π** ?

(f) What is now the smallest thickness of the film (in nm) will give fully **destructive** interference?

3. Wait for everyone to catch up before we move on. Everyone likes soap bubbles;

http://public.lanl.gov/wdaniel/science/soapfilms/soapfilms.html

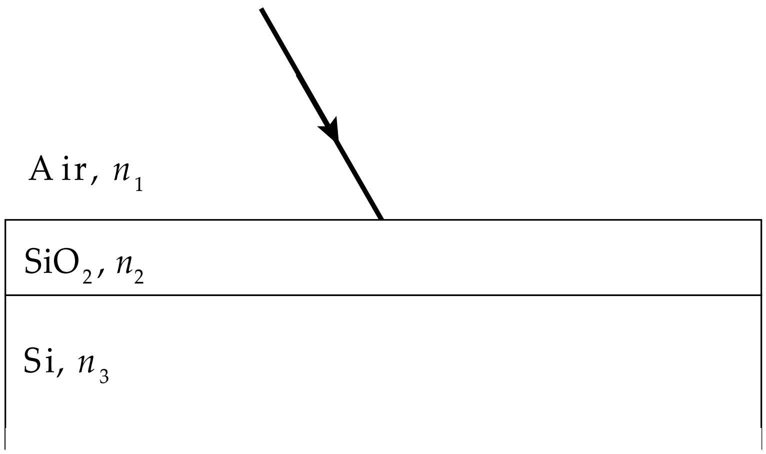
http://www.weather-photography.com/techniques.php?cat=miscellaneous&page=soap_films

Why do you see different colors in the interference pattern?

Question

A flat sheet of silicon (Si, $n_3 = 3.50$) is coated with a transparent thin film of silicon dioxide (SiO_2, $n_2 = 1.544$) to minimize reflective losses from the surface.

a) Draw the rays that will undergo interference in the air.
b) Determine the minimum film thickness that produces the least reflection at a wavelength of 550 nm for incident light rays in air that are perpendicular to the surface.



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______________________________ Group: __________

Polarization

1. Look through one piece of the polarizer material at the overhead room lights. Describe what happens as you rotate the polarizer through 360^o. What can you conclude about the polarization of the light emitted from the ceiling lights?

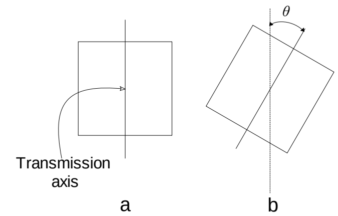
2. Look through one piece of the polarizer material at the reflections of the overhead room lights on the floor (the bright glare spots on the floor). Include both a glare spot very near to you (viewed at near-normal incidence), as well as a glare spot very far away (viewed at a glancing angle of incidence). Describe what happens when you rotate the polarizer through 360^o. What can you conclude about the polarization of the reflected light?
   HINT: The observations are not related to the Law of Reflection!

   www.colorado.edu/physics/PhysicsInitiative/Physics2000/applets/polarized.html

   is an applet that shows the effect of viewing both direct sunlight and sunlight reflected from a surface of water. Notice the orientation of the "sun glasses" in the upper left hand corner.

3. Put two pieces of the polarizer material together so that no light gets through (this is called "crossed polarizers"). Now put a third one between the other two. Describe and explain what happens as you rotate the middle polarizer. Clearly explain the physical basis for each situation. A drawing that shows the polarization of the light through each filter should be included in your explanation.

4. Consider a single polarizer square. Assume that it is oriented as in Figure (a) to the right and that light is being transmitted at maximum intensity. This corresponds to the electric field in the incident electromagnetic wave oscillating in the vertical direction (along the transmission axis). The intensity of the transmitted light is proportional to the square of the amplitude of the transmitted electric field:



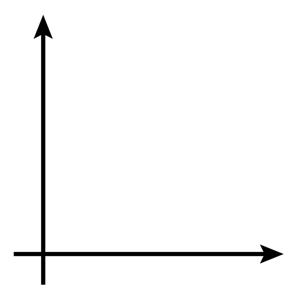
$$I_o \propto E_o^2 \text{ or } I_o = CE_o^2,$$

where C is a constant of proportionality.

When the polarizer is rotated through an angle θ as in Figure (b), the transmitted intensity is proportional to the square of the **component** of the electric field vector that is parallel to the transmission axis. Write an expression for the intensity (I) of the transmitted light in terms of I_o and θ. This result is known as Malus' Law.

5. Measurements of the fractional intensity (I/I_o) transmitted through a polarizer as a function of the angle θ between the transmission axis of the polarizer and the direction of the oscillating electric field in the incident electromagnetic wave are shown below. How would you **easily** prove that your **formula** in Part 4 for the intensity is correct ***using a graphical approach***? In other words, what quantities would you graph in order to generate a straight line with a slope of 1? Clearly identify the quantity you propose to plot on the vertical axis AND the quantity you propose to plot on the horizontal axis. Make sure to specify the units of any quantities that you select to graph. Explain your rationale clearly. You do not have to actually draw the graph, just describe HOW you would graph the data.

θ (rad) (± 0.03 rad)	I / I_0 (± 0.1)
0.00 (0°)	1.0
0.52 (30°)	0.8
0.79 (45°)	0.5
1.05 (60°)	0.3
1.31 (75°)	0.1
1.57 (90°)	0.0
2.09 (120°)	0.3
2.36 (135°)	0.5
2.62 (150°)	0.8
2.88 (165°)	0.9
3.14 (180°)	1.0



6. a) Is light from a laser polarized? How can you show this? Does the source of the laser light make a difference?

b) Look at the laser beam on the wall (or a piece of paper). Now put a polaroid piece in that beam and rotate it. Is the original beam polarized or unpolarized? How do you know?

7. Polarization by scattering.

a) Shine an unpolarized laser beam through milky water onto the wall (or a piece of paper). Is the beam transmitted through the milky water polarized? How do you know?

b) Suppose you **look down** at the milky water (perpendicular to the laser light) and rotate a piece of polaroid sheet, what do you see? What can you conclude about light scattered off of molecules?

8. Birefringence

Birefringent materials are materials with different indices of refraction in different crystallographic directions.

a) Look at a printed letter on a page through a piece of calcite. Rotate the calcite crystal. Describe what you see. Notice that there are 2 images of the printed letter. The one that matches up with the original letter of text on the paper is the ***ordinary*** image; the one that rotates around that is the ***extraordinary*** image

b) Now, put a piece of polaroid material over the calcite crystal and rotate the polaroid. Describe what you see. What can you conclude about the light passing through the calcite?