Appendix A: Important Equations in University Physics 1

1 Uncertainties

(Equations using average deviation are shown. Standard deviations could be used instead.)

$$x_{avg} = \frac{\sum x_i}{N} \tag{1}$$

$$\text{Average Absolute } \Delta x = \frac{\sum |x - x_i|}{N} \tag{2}$$

$$\text{Add or subtract } \Delta z = \Delta x + \Delta y \tag{3}$$

$$\text{Multiply or divide } \frac{\Delta z}{z} = \frac{\Delta x}{x_{avg}} + \frac{\Delta y}{y_{avg}} \tag{4}$$

2 Vectors & Other Math

$$\text{If } Ax^2 + Bx + C = 0 \text{ then } x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{5}$$

$$\vec{A} = \mathbf{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \tag{6}$$

$$\left|\vec{A}\right| = A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{7}$$

In two dimensions with angle θ measured from the x-axis,

$$A_x = A\cos\theta \tag{8}$$

$$A_y = A\sin\theta \tag{9}$$

$$A = \sqrt{A_x^2 + A_y^2} \tag{10}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} \text{ or this angle -180°} \tag{11}$$

With ϕ as the angle between two vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{B} = AB\cos\phi = A_x B_x + A_y B_y + C_x C_y \tag{12}$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\phi \tag{13}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\,\hat{i} + (A_z B_x - A_x B_z)\,\hat{j} + (A_x B_y - A_y B_x)\,\hat{k} \tag{14}$$

3 Kinematics

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{15}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{v}}{dt^2} \tag{16}$$

If acceleration $a_x =$ constant, and if at $t = 0$, $x = x_0$ and $v_x = v_{x0}$ then

$$v_x = v_{x0} + a_x t \tag{17}$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \tag{18}$$

$$v_{ave} = \frac{v_{x0} + v_x}{2} \tag{19}$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0) \tag{20}$$

Consider two dimensional motion with only a constant force of gravity acting and up being positive y. If at $t = t_0$ $x = x_0$, $y = y_0$, and velocity is v_0 at an angle θ_0, then

$$y = y_0 + \tan\theta_0(x - x_0) - \frac{g}{2v_0^2\cos^2\theta_0}(x - x_0)^2 \tag{21}$$

$$\text{Centripetal acceleration } a_c = \frac{v^2}{r} \tag{22}$$

4 Dynamics

Forces are interactions between objects. If the force acting on object 2 because of its interaction with object 1 is denoted $\vec{F}_{12}$ then Newton's Third Law says

$$\vec{F}_{12} = -\vec{F}_{21} \tag{23}$$

$$\text{Second Law } \sum \vec{F} = \vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt} \tag{24}$$

$$\text{Weight near earth } \vec{W} = \vec{F}_g = m\vec{g} \tag{25}$$

$$\text{General force of gravity between two point objects } |\vec{F}_G| = \frac{Gm_1m_2}{r_{12}^2} \tag{26}$$

$$\text{If unstretched spring has } x = 0 \;\; \vec{F}_{x,spring} = -kx\hat{i} \tag{27}$$

$$|\vec{f}_{static}| \leq \mu_s N \tag{28}$$

$$|\vec{f}_{kinetic}| = \mu_k N \tag{29}$$

5 Work and Energy

$$W = \int_{init}^{final} \vec{F} \cdot d\vec{r} \tag{30}$$

$$\text{If the force is constant, } W = \vec{F} \cdot \Delta\vec{r} \tag{31}$$

$$K = \frac{1}{2}mv^2 \tag{32}$$

$$\sum W = W_{net} = (K_f - K_i) = \Delta K \tag{33}$$

$$\text{For a conservative force, } \Delta U = U_f - U_i = -\int_i^f \vec{F} \cdot d\vec{r} \tag{34}$$

$$\text{Mechanical Energy } E = \sum K + \sum U \tag{35}$$

$$F_x = -\frac{dU}{dx} \tag{36}$$

$$\text{General work-energy theorem } E_f = E_i + W_{non-cons} \tag{37}$$

$$\text{Mechanical Energy Conservation, If } W_{non-cons} = 0, E_f = E_i \tag{38}$$

$$\text{Power } P = \frac{dW}{dt} \tag{39}$$

$$P = \vec{F} \cdot \vec{v} \tag{40}$$

6 Momentum, Impulse, Center of Mass

For a single particle the next 3 equations,

$$\text{Linear Momentum } \vec{p} = m\vec{v} \tag{41}$$

$$\text{Impulse } \vec{J} = \int \vec{F} dt = \vec{F}_{avg} \Delta t \tag{42}$$

$$\text{Impulse-momentum theorem for particle } \vec{J}_{net} = \Delta \vec{p} \tag{43}$$

For a system of particles, the rest of the equations,

$$\text{Center of mass coordinates } \vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \tag{44}$$

$$\text{Center of mass coordinates, continuous object } \vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm} \tag{45}$$

$$\text{System momentum (NOT power) } \vec{P} = \vec{P}_{net} = \sum \vec{p}_i \tag{46}$$

$$\text{Newton's second law for a system } \sum \vec{F}_{ext} = \frac{d\vec{P}_{net}}{dt} \tag{47}$$

$$\text{Impulse-momentum theorem for system } \sum \vec{J}_{ext} = \vec{P}_f - \vec{P}_i \tag{48}$$

$$\text{Linear Momentum conservation. If } J_{ext} = 0, \vec{P}_f = \vec{P}_i \tag{49}$$