Ballistic Pendulum analysis

mass of ball $m = 0.066 \pm 0.0005$ kg

$$\frac{\Delta m}{m} = \frac{0.0005}{0.066} = 0.0076$$

mass of arm $M = 0.239 \pm 0.0005$ kg

$$\frac{\Delta M}{M} = \frac{0.0005}{0.239} = 0.0021$$

length from pivot to center of mass $L = 0.285 \pm 0.002$ m

$$\frac{\Delta L}{L} = \frac{0.002}{0.285} = 0.0070$$

angle of swing

trial	θ (degrees)
1	27.5
2	26.5
3	26.0
4	26.0
5	27.3
6	26.5

$\theta = 26.6^\circ \pm 0.6^\circ$

$\cos\theta = 0.894 \pm 0.005$

$(1 - \cos\theta) = 0.106 \pm 0.005$

$$\frac{\Delta(1-\cos\theta)}{(1-\cos\theta)} = \frac{0.005}{0.106} = 0.0472$$

height of catapult above floor $H = 0.780 \pm 0.005$ m

$$\frac{\Delta H}{H} = \frac{0.005}{0.780} = 0.0064$$

The largest fractional uncertainty by far is in $(1-\cos\theta)$, based on θ. It is nearly $\pm 5\%$. All others less than $\pm 1\%$.

Speed of arm + ball $V_2 = \sqrt{2gL(1-\cos\theta)} = \left[2gL(1-\cos\theta)\right]^{1/2}$

$$\frac{\Delta V_2}{V_2} = \frac{1}{2}\left\{\frac{\Delta L}{L} + \frac{\Delta(\cos\theta)}{\cos\theta}\right\} = \frac{1}{2}\left\{0.0070 + 0.0472\right\}$$

$$= 0.0271$$

$$\rightarrow V_2 = 0.7689 \pm 0.0271\,(0.7689)$$
$$= 0.7689 \pm 0.0208 \quad m/s$$

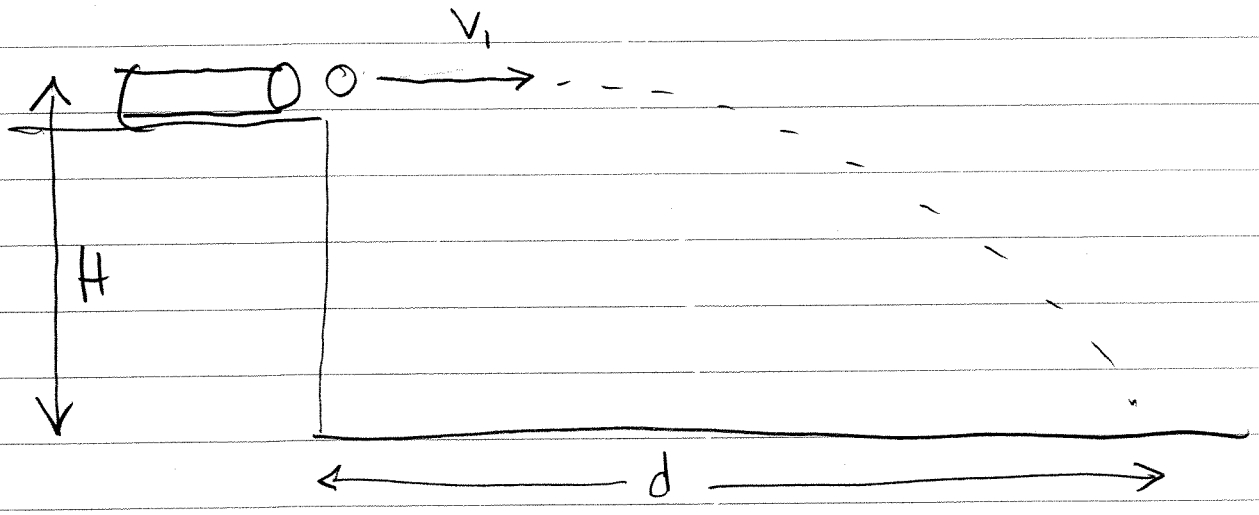
Speed of ball $V_1 = V_2\left(\frac{m_1 + m_2}{m_1}\right)$

$$\frac{\Delta V_1}{V_1} = \frac{\Delta V_2}{V_2} + \frac{\Delta(m_1+m_2)}{m_1+m_2} + \frac{\Delta m_1}{m_1}$$

$$= 0.0271 + \frac{0.001\ kg}{0.305\ kg} + 0.0076$$

$$= 0.0380$$

$$\rightarrow V_1 = 3.553\ \tfrac{m}{s} \pm 0.0380\,(3.553\ m/s)$$
$$= 3.553 \pm 0.135 \quad m/s$$



Time to hit floor $t = \sqrt{\frac{2H}{g}} = 0.399\ s$

$$\frac{\Delta t}{t} = \frac{1}{2}\left\{\frac{\Delta H}{H}\right\} = \frac{1}{2}\{0.0064\}$$
$$= 0.0032$$

$$\rightarrow \quad t = 0.399\ s \pm 0.0032\,(0.399)$$
$$= 0.399 \pm 0.001\ s$$

Distance travelled horizontally $d = V_1 t = 1.418\ m$

$$\frac{\Delta d}{d} = \frac{\Delta V_1}{V_1} + \frac{\Delta t}{t} = 0.0380 + 0.0032$$
$$= 0.0412$$

$$\rightarrow \quad d = 1.418\ m \pm 0.0412\,(1.418\ m)$$
$$= 1.418\ m \pm 0.058\ m$$