EASTER VACATION WORK AT PORT ERIN.

THE Easter vacation party at the Port Erin Bio-
logical Station has this year been larger than
ever before, and has carried out a longer programme
of work—both in the laboratory and on the seashore.
During the last few weeks (March and April) the
number of researchers and senior students enrolled in
the books of the station has reached the record total
of eighty-five, including half a dozen professors and
a dozen university lecturers and demonstrators, while
nearly half of the total number were post-graduate
researchers. Altogether twelve universities or univer-
sity colleges have been represented. Practically all
the senior students and post-graduate workers of the
botanical and zoological departments of the Univer-
sity of Liverpool, under Profs. Harvey Gibson and
Herdman, migrated to the Port Erin laboratory for
the vacation. Prof. Cole brought a considerable con-
tingent from the University College of Reading, and
Dr. Stuart Thomson a number from Manchester; Mr.
Holden came with some students from University
College, Nottingham, and smaller groups came from
Birmingham, Cambridge, Oxford, Bristol, Bangor,
Cardiff, London, and Melbourne. In addition to the
laboratory work of the students and their collecting
expeditions on the seashore, the activities of the bio-
logical station at this time of year are threefold:
first, the flat-fish hatching (seen at its best during
March and April); secondly, the plankton investiga-
tion going on at sea from the s.y. *Runa*; and thirdly,
the special investigations of the post-graduate re-
searchers.

The spawning of the mature plaice in the open-air
fish-ponds started at the beginning of February this
year, at least a fortnight earlier than usual, and it is
by no means finished yet. Already more than eight
millions of eggs have been skimmed from the ponds,
and about seven millions of young fish have been set
free in the sea round the south end of the Isle of
Man.

Work at sea was much hampered by bad weather
during the earlier part of the time, and it was some-
times difficult to get the periodic plankton hauls taken.
This is now the eighth year of Prof. Herdman's
scheme of intensive study of the nature and distribu-
tion of the plankton, of which it is hoped to com-
plete ten years' statistics before winding up the inves-
tigation. Up to the present the phytoplankton this
spring has been characterised by the prevalence of
Coscinodiscus.

In addition to the collecting and recording of rare
species, both of animals and sea-weeds, which has
gone on very much as in former years, there has
been a large amount of special investigation both at
sea and in the laboratory on the part of those who
are engaged in the preparation of L.M.B.C. Memoirs,
and also of others who are at various researches. For
example, Mr. R. D. Laurie has been making observa-
tions on the movements of Amphidinium in the sand,
Mr. S. T. Burfield has been working at Sagitta, Miss
Gleave at Archidoris, Mr. H. G. Jackson on Decapod
larvæ in the plankton, and Prof. B. Moore and Mr.
E. Whitley on the nutrition of marine animals and
the variations in the alkalinity of the sea-water. The
memoir on Echinoderm larvæ which Mr. Chadwick
has been engaged on for some years is now in the
printer's hands, and will be published at an early
date. The pressure on the laboratory accom-
modation has been very great during this vacation,
and the need of further extension of the building is
urgent.

W. A. H.

RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS. *

I.

Historical.

INVESTIGATIONS into the nature of the stars
must necessarily be very largely based upon the
average characteristics of groups of stars selected in
various ways—as by brightness, proper motion, and
the like. The publication within the last few years
of a great wealth of accumulated observational mate-
rial makes the compilation of such data an easy pro-
cess; but some methods of grouping appear to bring
out much more definite and interesting relations than
others, and, of all the principles of division, that
which separates the stars according to their spectral
types has revealed the most remarkable differences,
and those which most stimulate attempts at a
theoretical explanation.

In the present discussion, I shall attempt to review
very rapidly the principal results reached by other
investigators, and shall then ask your indulgence for
an account of certain researches in which I have been
engaged during the past few years.

Thanks to the possibility of obtaining with the
objective prism photographs of the spectra of hundreds
of stars on a single plate, the number of stars the
spectra of which have been observed and classified
now exceeds one hundred thousand, and probably as
many more are within the reach of existing instru-
ments. The vast majority of these spectra show only
dark lines, indicating that absorption in the outer and
least dense layers of the stellar atmospheres is the
main cause of their production. Even if we could not
identify a single line as arising from some known
constituent of these atmospheres, we could nevertheless
draw from a study of the spectra, considered merely
as line-patterns, a conclusion of fundamental import-
ance.

The spectra of the stars show remarkably few
radical differences in type. More than 99 per cent. of
them fall into one or other of the six great groups
which, during the classic work of the Harvard Col-
lege Observatory, were recognised as of fundamental
importance, and received as designations, by the pro-
cess of "survival of the fittest," the rather arbitrary
series of letters B, A, F, G, K, and M. That there
should be so few types is noteworthy; but much more
remarkable is the fact that they form a continuous
series. Every degree of gradation, for example, be-
tween the typical spectra denoted by B and A may
be found in different stars, and the same is true to
the end of the series, a fact recognised in the familiar
decimal classification, in which B5, for example,
denotes a spectrum half-way between the typical
examples of B and A. This series is not merely con-
tinuous; it is *linear*. There exist indeed slight differ-
ences between the spectra of different stars of the
same spectral class, such as A0; but these relate to
minor details, which usually require a trained eye for
their detection, while the difference between successive
classes, such as A and F, are conspicuous to the
novice. Almost all the stars of the small outstanding
minority fall into three other classes, denoted by the
letters O, N, and R. Of these O undoubtedly pre-
cedes B at the head of the series, while R and N,
which grade into one another, come probably at its
other end, though in this case the transition stages,
if they exist, are not yet clearly worked out.
From these facts it may be concluded that the prin-

cipal differences in stellar spectra, however they may originate, arise in the main from variations in a single physical condition in the stellar atmospheres. This follows at once from the linearity of the series. If the spectra depended, to a comparable degree, on two independently variable conditions, we should expect that we would be obliged to represent their relations, not by points on a line, but by points scattered over an area. The minor differences which are usually described as "peculiarities" may well represent the effects of other physical conditions than the controlling one.

The first great problem of stellar spectroscopy is the identification of this predominant cause of the spectral differences. The hypothesis which suggested itself immediately upon the first studies of stellar spectra was that the differences arose from variations in the chemical composition of the stars. Our knowledge of this composition is now very extensive. Almost every line in the spectra of all the principal classes can be produced in the laboratory, and the evidence so secured regarding the uniformity of nature is probably the most impressive in existence. The lines of certain elements are indeed characteristic of particular spectral classes; those of helium, for instance, appear only in Class B, and form its most distinctive characteristic. But negative conclusions are proverbially unsafe. The integrated spectrum of the sun shows no evidence whatever of helium, but in that of the chromosphere it is exceedingly conspicuous. Were it not for the fact that we are near this one star of Class G, and can study it in detail, we might have erroneously concluded that helium was confined to the "helium stars." There are other cogent arguments against this hypothesis. For example, the members of a star-cluster, which are all moving together, and presumably have a common origin, and even the physically connected components of many double stars, may have spectra of very different types, and it is very hard to see how, in such a case, all the helium and most of the hydrogen could have collected in one star, and practically all the metals in the other. A further argument—and to the speaker a very convincing one—is that it is almost unbelievable that differences of chemical composition should reduce to a function of a single variable, and give rise to the observed linear series of spectral types.

I need not detain you with the recital of the steps by which astrophysicists have become generally convinced that the main cause of the differences of the spectral classes is difference of temperature of the stellar atmospheres. There is time only to review some of the most important evidence which, converging from several quarters, affords apparently a secure basis for this belief.

The first argument is based upon the behaviour of the spectral lines themselves. To appreciate its full force, one must familiarise himself with a multitude of details. A typical instance is that of the heavy bands in the region of longer wave-length, which are the most characteristic feature of spectra of Class M, appear faintly in Class K5, and are absent in Class K and all those higher in the series. Fowler has shown[1] that these bands are perfectly reproduced in the spectrum of the outer flame of an electric arc charged with some compound of titanium, while the spectrum of the core of the arc, though showing conspicuously the bright lines of titanium, does not contain the bands. Here we are evidently dealing with some compound—perhaps titanium oxide—the vapour of which is present in the relatively cool flame of the arc, and emits a spectrum of the banded type, char-

[1] Proc. Roy. Soc., vol. lxxii., pp. 219-225, 1904.

acteristic of compounds, while in the hotter core it is dissociated, and only the lines of the metal are seen. There seems then to be no escape from the conclusion that the atmospheres of stars of Class M are cool enough to permit the existence of this compound, and hence cooler than the core of the arc, and that the temperature of its dissociation is approached in Class K5, and surpassed in Class K. In general, those metallic lines which are relatively strong in the spectra produced in the oxyhydrogen flame or the electric furnace are also strong in spectra of Classes M and K; the lines most prominent in Class G are the typical arc lines; and the relatively few metallic lines which persist into Classes A and B are those which appear exclusively, or with greatly enhanced intensity, in the spark spectra of the laboratory.

The second line of evidence is afforded by the distribution of intensity in the continuous background of the spectra, the differences of which from type to type are obvious to the eye as differences in the colour of the stars. This characteristic is fortunately capable of accurate measurement. For the brighter stars, spectro-photometric comparisons may be made with a terrestrial light-source the energy curve of which is known, as has been done visually by Wilsing and Scheiner,[2] and photographically by Rosenberg.[3] Much fainter stars may be reached by the comparison of their brightness as measured visually (or on iso-chromatic plates with a suitable colour-screen), and photographically on ordinary plates. The "colour-index" so obtained, which expresses, in stellar magnitudes, the relative photographic brightness of stars of equal visual brightness, is found to be very intimately related to the spectral type, the differences within each spectral class being scarcely greater than the errors of observation. The results of King,[4] Parkhurst,[5] and Schwarzschild,[6] working with different instruments and on stars of very different brightness, are in excellent agreement, as is shown in Table I. The near approach to equality among the differences in colour-index from class to class is very remarkable, when it is considered that these types were picked out somewhat arbitrarily according to the general appearance of the photographic spectra. The judgment of the Harvard observers in selecting the really important points of difference was evidently very good.

TABLE I.

Spectrum	Colour-index King	Colour-index Parkhurst	Colour-index Schwarzschild	Temperature
B0	−0.32			20,000
B5	−0.17	−0.21	−0.20	14,000
A0	0.00	0.00	0.00	11,000
A5	0.19	0.23	0.20	9,000
F0	0.30	0.43	0.40	7,500
F5	0.42	0.65	0.60	6,000
G0	0.72	0.86	0.84	5,000
G5	0.98	1.07	1.10	4,500
K0	1.10	1.30	1.35	4,200
K5	1.62	1.51	1.80	3,200
M	1.62	1.68		3,100
N		2.5		2,300

If the spectral sensitiveness of the plates used in such investigations has been determined (as Parkhurst has done) it is possible to calculate the temperature at which a black-body would emit light of the same colour as that observed; and similar calculations can be made, with greater accuracy, from the spectro-photometric data. The last column of Table I. gives the effective temperatures thus derived (based mainly on the work of Wilsing and Scheiner). The absolute

[2] Potsdam Publications, vol. xix., part 1.　　[3] A.N., 4628, 1913.
[4] Harvard Annals, vol. lix., p. 179.
[5] Astrophys. Jour., vol. xxxvi., p. 218, 1912.
[6] Göttingen Aktinometrie, Teil B, p. 19.

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values of the temperatures here given may be considerably in error, especially at the top of the scale (in fact, Rosenberg's work indicates a much greater range), but there can be no doubt about the relative order.

Of a third independent confirmation of the temperature hypothesis, based on the determination of the surface brightness of the stars, I shall have occasion to speak later.

It should be expressly stated that the "temperatures" here spoken of are the effective "black-body" temperatures corresponding to the spectral distribution of the radiation. Unless the surfaces of the stars possess decided selective emissivity for certain wavelengths, these effective temperatures should also indicate with tolerable accuracy the energy-density of the flux of radiation which escapes from them. This tells us little about the temperature of the deeper regions; but it must be the main, if not the only, factor in determining the temperature of those outer and nearly transparent layers of the atmospheres in which the characteristic line absorption takes place. If we further assume, in accordance with Abbot's studies of the solar atmosphere,[7] that the absorption is nearly complete in so small a thickness of the atmosphere that wide variations in its depth and density would modify its total absorption but little, it becomes easy to see how the influence of its temperature (which presumably determines the relative strength of absorption in different lines) may predominate so greatly over all that of all other factors in determining the spectral type.

We may now review rapidly some of the relations which have been brought to light between other characteristics of the stars and their spectral types. First, as regards the relative numbers of stars of the different classes, we have in Table II some results of counts made at Harvard.[8]

TABLE II.

Spectrum	O	B	A	F	G	K	M	N
No. above 3.25m.	3	52	32	16	20	35	21	0
„ 6.25m.	20	696	1885	720	609	1719	457	8
Percentage in Galactic region	100	82	66	57	58	56	54	87

Classes A and K make up more than half of all the stars brighter than 6·25 m—that is, of the stars visible to the naked eye. The remaining stars are divided fairly evenly among the other four principal classes, while only one star in 300 is of Class O, and only one in 800 of Class N. The relative proportions of the different classes are, however, different in different parts of the heavens, as is indicated by the last line of the table, which give the percentage of stars of each class which lie in a belt covering one-half of the celestial sphere, and extending for 30° each side of the Milky Way. All the stars of Class O are close to the central line of the Galaxy (except for a few in the Magellanic Clouds). The stars of Class B are very strongly concentrated in the galactic region; those of Class A are considerably so; those of the following classes very little, except in the case of Class N (for which the tabular percentage is derived, not from the eight brightest stars of this class alone, but from a much larger number of fainter ones).[9]

The relative proportions of the different classes vary also with the apparent brightness of the stars. Among the stars brighter than 3·25m., as the table shows, Class B has more representatives than any other; but the percentage of this type steadily diminishes as we pass to fainter stars. The percentage of stars of Class A at first increases with diminishing visual

7 Abbot, "The Sun," p. 252, 1911.
8 Harvard Annals, vol. lxiv., p. 134.
9 Harvard Annals, vol. lxvi., p. 213.

brightness; but there is good reason to believe that, at least in regions remote from the Galaxy, the relative proportion of these too falls off rapidly in the neighbourhood of the ninth magnitude[10]; and Fath's work on the integrated spectrum of the Milky Way[11] shows that, even there, the bulk of the very faint stars which form the galactic clouds must be of Secchi's second type (F, G, or K).

Counts of the stars down to any given magnitude may, however, be very misleading unless we bear in mind the enormous preference which this method of observation gives to the stars of great actual luminosity, which can be seen afar off, and hence are being sought in a much greater volume of space than those of small luminosity. A difference of but five magnitudes in the real brightness of two groups of stars gives the brighter kind (if both are uniformly distributed in space) a thousand-fold better chance of getting into our catalogues; and this example understates the actual conditions in some cases. Mere counts of stars need therefore to be supplemented by such knowledge as we can obtain concerning their distances.

Much information can be obtained from the average proper-motions of the stars of the various classes, and still more by deriving their average parallaxes from the mean parallactic drift due to the motion of the solar system in space. Studies of this character have been made by several investigators of the first rank. Their results, which are summarised in Table III., show certain apparent discrepancies, which, however, arise principally from differences in the methods according to which the various workers have selected the groups of stars for investigation.

TABLE III.

Spectrum	Mean centennial proper-motion Kapteyn	Boss	% rejected	Mean parallax Kapteyn	Boss	Campbell	% rej.
	"	"		"	"	"	
O		1·6	0		0·004		
B	2·6	2·4	0	0·007	0·007	0·006	0
A	5·8	4·6	3	0·010	0·010	0·016	3
F	14·5	7·7	28	}	0·012	0·035	3
G	27·0	5·2	20	} 0·022	0·008	0·022	8
K	13·0	5·7	6	}	0·010	0·015	9
M	5·9	5·0	6	0·011	0·008	0·011	3
N		3·2		0·0007			

Kapteyn's data[12] represent the mean proper motions and parallaxes of all the stars of the fifth magnitude of each class, except for Class N, in which, to get enough stars, it was necessary to include faint objects, so that the average magnitude is here 8·3. His results show a conspicuous maximum of average proper-motion and parallax for Class G, with a rapid fall on both sides of it. The stars of Class N would have to be brought about five times nearer to appear as bright as the others, but even then they would have the smallest mean parallax of all.

Boss,[13] in his investigation of the solar motion, had at his disposal very accurate proper-motions of all the stars down to 5·7m., and about half as many more between this and the seventh magnitude. The average magnitude of his stars is therefore nearly the same as that of Kapteyn's. But, for very good reasons, he excluded from his main solution all stars with proper-motions exceeding 20" per century. The percentage of stars thus excluded (which differs greatly from class to class) is given in the fourth column of Table III. It is natural that this often drastic rejection of the large proper-motions, and hence in general

10 *Astronomical Journal*, vol. xxvi., p. 153, 1910.
11 *Astrophys. Journal*, vol. xxxvi., pp. 362–367, 1912.
12 *Astrophys. Journal*, vol. xxx., p. 295: vol. xxxii., p. 91, 1909–10.
13 *Astronomical Journal*, vol. xxvi., pp. 187–201, 1911. The mean proper-motions of the few stars of Classes O and N which appear in Boss's Catalogue have been added by the writer.

of the nearer stars, should greatly diminish his mean values. Among the classes in which the mean proper-motion is small, the percentage of exclusion is also small, and the results are but little modified. But it is noteworthy that the exclusion of 6 per cent. of the stars of Class K has reduced the mean proper-motion in a greater ratio than that of 28 per cent. of those of Class F, and also that the removal of one-fifth of the stars of Class G decreases the mean for the remainder to less than one-fifth of its initial value. It appears from these results that a large majority of the stars of Classes F, G, and K have nearly, if not quite, as small parallaxes and proper-motions as those of Classes A and M, though they are not quite so remote as the stars of Class B. The large mean values obtained for all the stars of these classes are due to the presence of a relatively small proportion of near and apparently rapidly moving stars, of which the percentage decreases, but the mean proper-motion and parallax increase, from F to K.

Campbell's results [14] are derived from a comparison of the radial velocities and proper-motions of nearly 1200 stars, mostly brighter than the fifth magnitude, and averaging about a magnitude brighter than Boss's stars, which would lead us to expect that their mean parallaxes should be 40 or 50 per cent. greater. In his work, "a few stars having proper-motions abnormally large for their classes were omitted in accordance with definitely set limits" (which unfortunately are not described more specifically). The approximate percentage of exclusion is given in the last column of the table. It appears on inspection that the differences between Campbell's and Boss's results for stars of Classes A, K, and M arise mainly from the greater brightness of Campbell's stars; those for Classes F and G are due mainly to the different percentages of exclusion, and that the only significant difference is that Campbell's B stars, though averaging much brighter to the eye than Boss's, have a slightly smaller mean parallax, and therefore must be, on the average, of greater real brightness.

Closely allied with these investigations is the determination of the mean peculiar velocity of the stars of each spectral class. The results of Boss and Campbell, reached almost simultaneously, and from quite independent data—proper-motions in one case and radial velocities in the other—are in extraordinary agreement. The values found for the average component of motion in any arbitrary direction are (in kilometres per second) :—

Spectrum	B	A	F	G	K	M
Campbell	6·5	10·5	14·4	15·9	16·8	17·1
Boss	6·3	10·2	16·2	18·6	15·1	17·1

The rapid increase of the mean velocity from B to F is very remarkable. The slow further gain from F to M would attract little attention if it were not in the same direction.

It should here be added that the phenomenon known as preferential motion, or "star-streaming"—the excess of the average peculiar velocity of the stars in a certain direction above those in the perpendicular directions—is almost absent in Class B, very conspicuous in Class A, and somewhat less so in the following classes, being partially concealed by the greater average magnitude of the velocities.

Another notable difference between the various spectral classes may be found in the number of binary stars, both visual and spectroscopic, among them. We may distinguish two classes of visual double stars: binary stars for which orbits have been computed (with periods rarely exceeding two centuries), ·and physical pairs, the real connection of which is proved

[14] Lick Observatory Bulletin, vol. vi., p. 134, 1911.

by common proper-motion, but the relative motions of which are slow, and periods long—probably often thousands of years. The counts of the two classes here given are from a list prepared in the course of my work, and include all stars for which the necessary data could be obtained, including many stars for which unpublished observations of spectra have been generously furnished me from Harvard. For the spectroscopic binaries, Campbell's counts have been taken from his catalogue of 1910.[15] They include all the systems the periods of which were then known, and are divided into two groups, one including all the periods of which are less than ten days, and also all those the periods of which, though not exactly known, are described as short; the other all the known periods exceeding ten days, and those which, though not precisely determined, are known to be long.

TABLE IV.

Spectrum	Visual binaries		Physical pairs		Spectroscopic binaries	
					Short period	Long period
B	...	0	...	52	... 33	... 15
A	...	14	...	152	... 15	... 14
F	...	33	...	115	... 11	... 9
G	...	24	...	74	... 8	... 14
K	...	12	...	62	... 0	... 13
M	...	0	...	11	... 0	... 2

It appears that, in Campbell's picturesque phrase, visual double stars of relatively short period "abhor" Classes B and M, the greatest number being of Class F, with G a good second. Among the physical pairs, of long period, the most favoured class is A. Class B is abundantly represented, and Class M very sparingly.

The percentage of stars which are found to be spectroscopic binaries is very probably greater among Classes B and A than lower down the list. As time goes on, indeed, more and more of the stars of these "later" types are found to be spectroscopically double, but of long period; but among these classes the detection of such systems, where the range of velocity is small, is much easier than among the stars of the first type, the lines of which are diffuse. In any case it is certain that short periods are almost confined to Classes B, A, and F, and are especially abundant in the first of these. The few short-period stars of Class G which appear in the table are all Cepheid variables, most of which were selected for observation on this account, and would not otherwise have got into the list.

Finally, we may note that, among variable stars, those of the eclipsing type, such as Algol or Beta Lyræ, are for the most part of Classes A and B, though there are a number of Classes F and G, and one at least of Class K; that the Cepheid variables are almost all of classes F and G, with a few A's and K's; and that almost all the irregular variables, and all the variables of long period, are of Classes M or N. Stars of Class M the spectra of which show bright hydrogen lines are without exception variable, and almost all the stars of Class N are also subject to changes in brightness.

(To be continued.)

UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

Cambridge.—Dr. James Ward, professor of mental philosophy and logic, has been nominated to represent the University on the occasion of the celebration at Oxford on June 10 of the seventh centenary of the birth of Roger Bacon, and Dr. Sorley, Knightbridge professor of moral philosophy, to represent the University.

[15] Lick Observatory Bulletin, vol. vi., p. 38, 1910.

RELATIONS BETWEEN THE SPECTRA AND OTHER CHARACTERISTICS OF THE STARS.*

II.

Brightness and Spectral Class.

HAVING thus made a rapid survey of the general field, I shall now ask your attention in greater detail to certain relations which have been the more special objects of my study.

Let us begin with the relations between the spectra and the real brightness of the stars. These have been discussed by many investigators —notably by Kapteyn and Hertzsprung—and many of the facts which will be brought before you are not new; but the observational material here presented is, I believe, much more extensive than has hitherto been assembled. We can only determine the real brightness of a star when we know its distance; but the recent accumulation of direct measures of parallax, and the discovery of several moving clusters of stars the distances of which can be determined, put at our disposal far more extensive data than were available a few years ago.

Fig. 1 shows graphically the results derived from all the direct measures of parallax available in the spring of 1913 (when the diagram was constructed). The spectral class appears as the horizontal coordinate, while the vertical one is the absolute magnitude, according to Kapteyn's definition— that is, the visual magnitude which each star would appear to have if it should be brought up to a standard distance, corresponding to a parallax of 0·1″ (no account being taken of any possible absorption of light in space). The absolute magnitude, −5, at the top of the diagram, corresponds to a luminosity 7500 times that of the sun, the absolute magnitude of which is 4·7. The absolute magnitude 14, at the bottom, corresponds to 1/5000 of the sun's luminosity. The larger dots denote the stars for which the computed probable error of the parallax is less than 42 per cent. of the parallax itself, so that the probable error of the resulting absolute magnitude is less than ±1·0m. This is a fairly tolerant criterion for a "good parallax," and the small

dots, representing the results derived from the poorer parallaxes, should scarcely be used as a basis for argument. The solid black dots represent stars the parallaxes of which depend on the mean of two or more determinations; the open circles, those observed but once. In the latter case, only the results of those observers whose work appears to be nearly free from systematic error have been included, and in all cases the observed parallaxes have been corrected for the probable mean parallax of the comparison stars
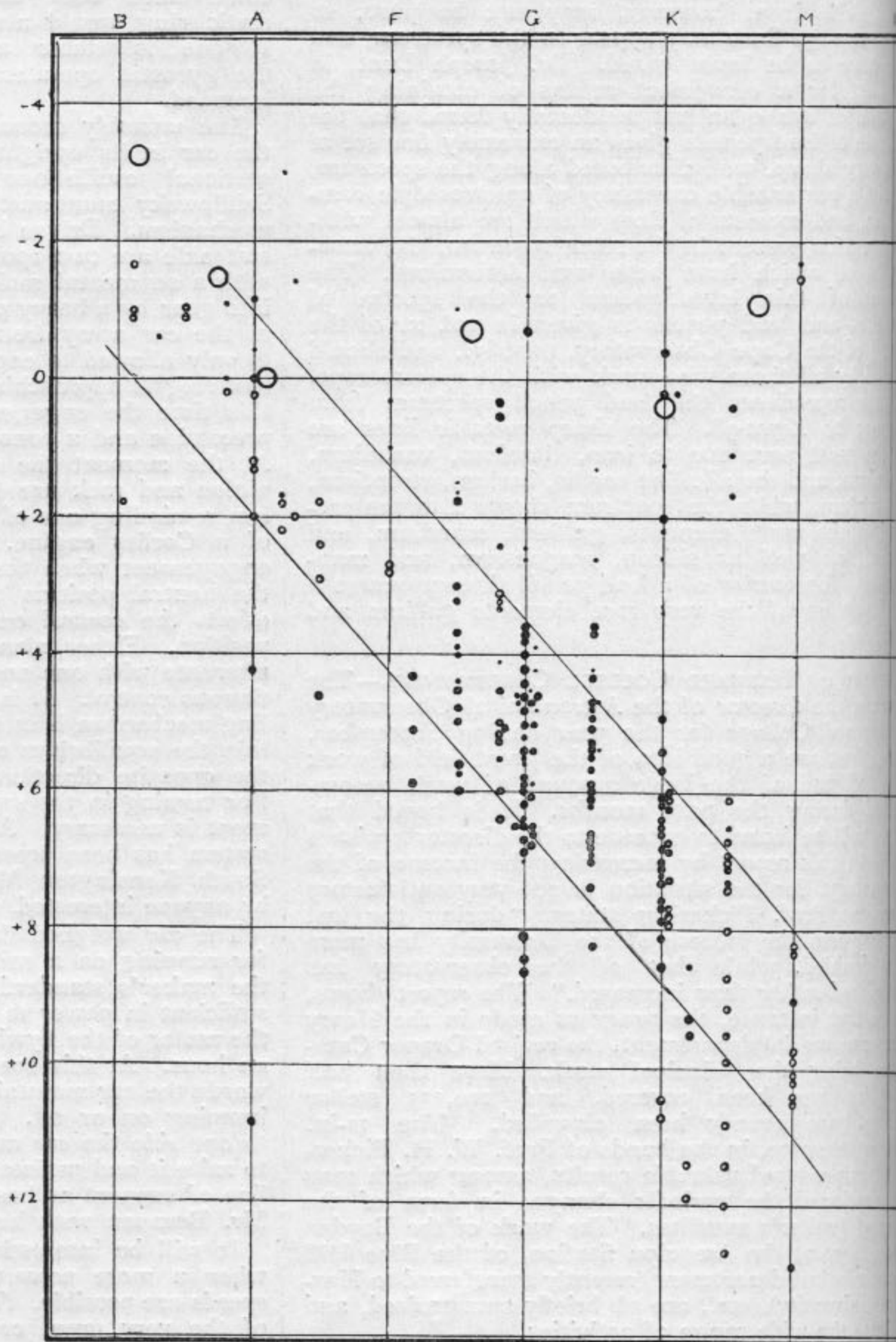

FIG. 1.

which they were referred. The large open circles in the upper part of the diagram represent mean results for numerous bright stars of small proper-motion (about 120 altogether) the observed parallaxes of which scarcely exceed their probable errors. In this case the best thing to do is to take means of the observed parallaxes and magnitudes for suitable groups of stars, and then calculate the absolute magnitudes of the typical stars thus defined. These will not exactly

correspond to the mean of the individual absolute magnitudes which we could obtain if we knew all the parallaxes exactly, but they are pretty certainly good enough for our purpose.

Upon studying Fig. 1 several things can be observed. (1) All the white stars, of Classes B and A, are bright, far exceeding the sun; and all the very faint stars—for example, those less than 1/50 as bright as the sun—are red, and of Classes K and M. We may make this statement more specific by saying, as Hertzsprung does,[16] that there is a certain limit of brightness for each spectral class, below which stars of this class are very rare, if they occur at all. Our diagram shows that this limit varies by rather more than two magnitudes from class to class. The single apparent exception is the faint double companion to O. Eridani, concerning the parallax and brightness of which there can be no doubt, but the spectrum of which, though apparently of Class A, is rendered very difficult of observation by the proximity of its far brighter primary.

(2) On the other hand, there are many red stars of great brightness, such as Arcturus, Aldebaran, and Antares, and these are as bright, on the average, as the stars of Class A, though probably fainter than those of Class B. Direct measures of parallax are unsuited to furnish even an estimate of the upper limit of brightness to which these stars attain, but it is clear that some stars of all the principal classes must be very bright. The range of actual brightness among the stars of each spectral class therefore increases steadily with increasing redness.

(3) But it is further noteworthy that all the stars of Classes K5 and M which appear on our diagram are either very bright or very faint; there are none comparable with the sun in brightness. We must be very careful here not to be misled by the results of the methods of selection employed by observers of stellar parallax. They have for the most part observed either the stars which appear brightest to the naked eye, or stars of large proper-motion. In the first case, the method of selection gives an enormous preference to stars of great luminosity, and, in the second, to the nearest and most rapidly moving stars, without much regard to their actual brightness. It is not surprising, therefore, that the stars picked out in the first way (and represented by the large circles in Fig. 1) should be much brighter than those picked out by the second method (and represented by the smaller dots). But if we consider the lower half of the diagram alone, in which all the stars have been picked out for proper-motion, we find that there are no very faint stars of Class G, and no relatively bright ones of Class M. As these stars were selected for observation entirely without consideration of their spectra (most of which were then unknown) it seems clear that this difference at least is real, and that there is a real lack of red stars comparable in brightness with the sun, relatively to the number of those 100 times fainter.

The appearance of Fig. 1 therefore suggests the hypothesis that, if we could put on it some thousands of stars instead of the 300 now available, and plot their absolute magnitudes without uncertainty arising from observational error, we would find the points representing them clustered principally close to two lines, one descending sharply along the diagonal, from B to M, the other starting also at B, but running almost horizontally. The individual points, though thickest near the diagonal lines, would scatter above and below it to a vertical distance corresponding to at least two magnitudes, and similarly would be

thickest near the horizontal line, but scatter above and below it to a distance which cannot so far be definitely specified, so that there would be two fairly broad bands in which most of the points lay. For Classes A and F these two zones would overlap, while their outliers would still intermingle in Class G, and probably even in Class K. There would, however, be left a triangular space between the two zones, at the right-hand edge of the diagram, where very few (if any) points appeared, and the lower left-hand corner would be still more nearly vacant.

We may express this hypothesis in another form by saying that there are two great classes of stars, one of great brightness (averaging, perhaps, a hundred times as bright as the sun), and varying very little in brightness from one class of spectrum to another; the other of smaller brightness, which falls off very rapidly with increasing redness. These two classes of stars were first noticed by Hertzsprung,[17] who has applied to them the excellent names of *giant* and *dwarf* stars. The two groups, on account of the considerable internal differences in each, are only distinctly separated among the stars of Class K or redder. In Class F they are partially, and in Class A thoroughly, intermingled, while the stars of Class B may be regarded equally well as belonging to either series.

In addition to the stars of directly measured parallax, represented in Fig. 1, we know with high accuracy the distances and real brightness of about 150 stars which are members of the four moving clusters the convergent points of which are known, namely, the Hyades, the Ursa Major group, the 61 Cygni group, and the large group in Scorpius, discovered independently by Kapteyn, Eddington, and Benjamin Boss, the motion of which appears to be almost entirely parallactic. The data for the stars of these four groups are plotted in Fig. 2, on the same system as in Fig. 1. The solid black dots denote the members of the Hyades; the open circles, those of the group in Scorpius; the crosses, the Ursa Major group; and the triangles, the 61 Cygni group. Our lists of the members of each group are probably very nearly complete down to a certain limiting (visual) magnitude, but fail at this point, owing to lack of knowledge regarding the proper motions of the fainter stars. The apparently abrupt termination of the Hyades near the absolute magnitude 7.0, and of the Scorpius group at 1.5, arises from this observational limitation.

The large circles and crosses in the upper part of Fig. 2 represent the absolute magnitudes calculated from the mean parallaxes and magnitudes of the groups of stars investigated by Kapteyn, Campbell, and Boss, concerning which data were given in Table III. The larger circles represent Boss's results, the smaller circles Kapteyn's, and the large crosses Campbell's.

It is evident that the conclusions previously drawn from Fig. 1 are completely corroborated by these new and independent data. Most of the members of these clusters are dwarf stars, and it deserves particular notice that the stars of different clusters, which are presumably of different origin, are similar in absolute magnitude. But there are also a few giant stars, especially of Class K (among which are the well-known bright stars of this type in the Hyades); and most remarkable of all is Antares, which, though of Class M, shares the proper motion and radial velocity of the adjacent stars of Class B, and is the brightest star in the group, giving out about two thousand times the light of the sun.

[16] *A. N.*, 4422, 1910.

[17] *Zeitschrift für Wissenschaftliche Photographie*, vol. iii., p. 442, 1905.

It is also clear that the naked-eye stars, studied by Boss, Campbell, and Kapteyn, are, for the most part, giants. With this in mind, we are now in a position to explain more fully the differences between the results of these investigators.

All the stars of Class B are giants, and, so far as we may judge from the Scorpius cluster, they do not differ from one another very greatly in absolute brightness. It is therefore natural that the results of all three investigators are in this case fairly similar, though Campbell, in employing stars that averaged brighter to the eye than did the others, has evidently been working with stars that are really brighter. In Class A the giants and dwarfs differ so little, and are so thoroughly intermingled, that the situation is about the same. In Class M, even the nearest and brightest of the dwarf stars are invisible to the naked eye: hence the stars of this class studied by the three investigators are all giants, and once more their results agree.

A number of the dwarf stars of Class K are visible to the naked eye; but these all lie very near us, and have such large proper motions that they are excluded as "abnormal" by both Campbell and Boss. The results of the two agree in indicating that the stars studied by them are typical giants. The few dwarfs, however, have such large parallaxes and proper-motions that their inclusion more than doubles the mean proper-motion, and presumably, also, the mean parallax of the whole, as shown by Kapteyn's figures in Table III. For Class G, the dwarf stars average much brighter, and a much greater number of them is visible to the naked eye. These have large parallaxes and proper-motions, and raise the average for all the stars of this class to greater values than for any other. But Boss's rigorous limitation to small proper-motions weeds them practically all out, leaving giant stars once more. Campbell's less drastic procedure omits only the nearer of the dwarfs (to be precise, those with the larger proper-motions), and his result lies about half-way between the others. In the case of Class F, the dwarf stars are still brighter—intermingling, in fact, with the giants. We can therefore see them farther off, and we get more of them in our catalogues, in proportion to the giants, than in any other class. Their mean parallax is, however, smaller than for the dwarfs of Classes G and K, and hence the mean proper-motion and parallax of all the stars of this class is less than for Class G. Campbell's criterion here excludes very few stars, and even Boss's admits a good many of the remoter and slower moving dwarfs, causing his mean parallax and proper-motion to be considerably greater for this class than for any other.

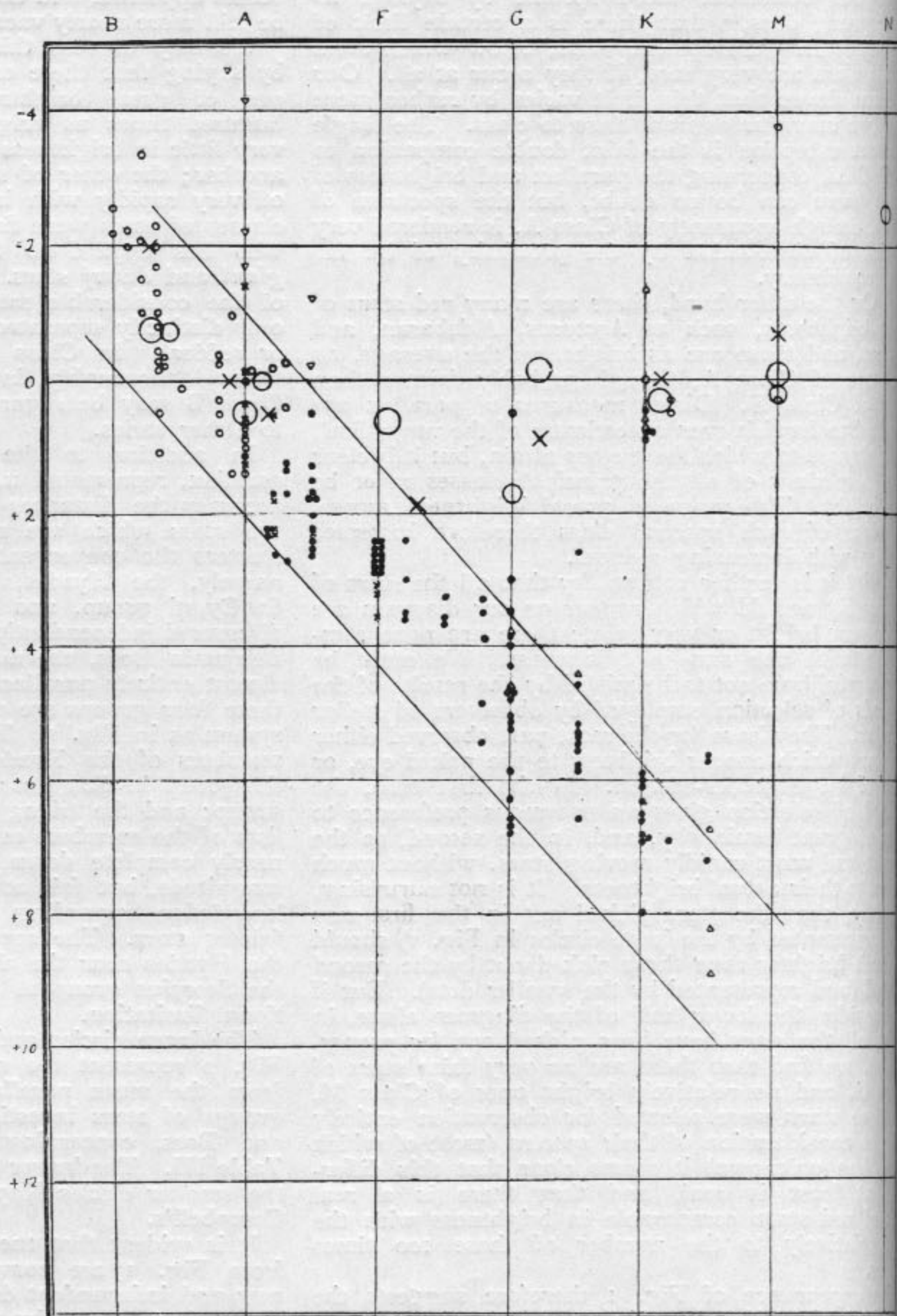It should finally be added that Kapteyn's discussion

FIG. 2.

shows that the stars of Class N are exceedingly bright, possibly surpassing any of the other giant stars.

We are now in a position to define more precisely the brightness of a typical giant or dwarf star of a given class of spectrum, and also to obtain a measure of the degree of divergence of the individual stars from this typical brightness. Taking first the stars of Class B and the dwarf stars of the other classes

we find, for the mean absolute magnitudes of all the stars of each class, the following values:—

TABLE V.
Mean Absolute Magnitudes.

Spectrum	Stars of measured parallax				Stars in clusters			
	No.	Abs.mag.	Formula	O-C m.	No.	Abs.mag.	Formula	O-C m.
B2	—	—	—	—	21	−1·2	−1·1	−0·1
B8	—	—	—	—	8	+0·3	+0·2	+0·1
A0	6	+1·4	+1·4	0·0	13	0·5	0·6	−0·1
A4	7	2·5	2·3	+0·2	26	1·7	1·5	+0·2
F0	—	—	—	—	15	2·4	2·7	−0·3
F1	5	4·2	3·7	+0·5	—	—	—	—
F3	—	—	—	—	7	3·3	3·3	0·0
F5	9	4·3	4·5	−0·2	—	—	—	—
F8	8	5·1	5·2	−0·1	5	4·2	4·4	−0·2
G0	29	5·7	5·6	+0·1	18	5·0	4·8	+0·2
G5	19	5·7	6·6	−0·9	9	5·1	5·8	−0·7
K0	28	7·1	7·7	−0·6	9	6·4	6·9	−0·5
K4	19	9·2	8·6	+0·6	7	+7·0	+7·7	(−0·7)
Ma	10	+9·9	+9·8	+0·1	—	—	—	—

The rate of decrease of brightness with increasing redness is very nearly the same for the stars with directly measured parallaxes and the stars in clusters, but the latter appear, with remarkable consistency, to be about 0·8m. brighter than the former. This seems at first sight very puzzling, but it is undoubtedly due to the way in which the stars observed for parallax were selected. Most observers, in preparing their working lists, have included mainly those stars which were brighter than a given magnitude and had proper-motions exceeding some definite limit. Of the stars above this limiting magnitude, those of greater actual luminosity will be, on the average, farther away, and have smaller proper-motions, than those of small luminosity, and selection by proper-motion favours the latter. The limitation of our present lists to stars the parallaxes of which have been determined with a probable error not exceeding 12 per cent. of their own amounts, though necessary to diminish the effects of casual errors of observation, works in the same direction, for, among the stars of any given visual magnitude, those of greatest luminosity have the smallest parallaxes, and are least likely to pass the test. The difference shown in our table need not therefore alarm us, but it is clear that the stars in clusters, rather than the others, should be taken as typical of the dwarf stars as a whole. For both sets of stars the absolute magnitude appears to be very nearly a linear function of the spectral class (if B is regarded as 1, A as 2, etc.) The columns headed "formula" in Table V. give the values calculated from the expressions M = 1·4m. + 2·1m. (Sp. − A) for the stars of directly measured parallax, and M = 0·6m. + 2·1m. (Sp. − A) for the stars in clusters. The residuals from these empirical formulæ, for the mean absolute magnitudes of the observed stars of different classes, average ±0·33m. in the first case and ±0·29m. in the second. They appear to be accidental in character, though in some cases (notably in Class G5) the residuals for the stars of the two sets are similar in sign and magnitude. The large negative residuals for Classes K and K5 in the clusters arise from the fact that in the Hyades, which contribute most of these stars, only the brighter ones have had their proper-motions determined, and get into our lists, as is clear from examination of Fig. 2.

Among the dwarf stars, therefore, a typical star of any spectral class is about seven times fainter than one of the preceding class, and seven times brighter than one of the following class.

The giant stars of all the spectral classes appear to be of about the same mean brightness, averaging a little above absolute magnitude zero, that is, about a hundred times as bright as the sun. Since the stars of this series which appear in Fig. 2 have been selected by apparent brightness, which gives a strong preference to those of the greatest luminosity, the average brightness of all the giant stars in a given region of space must be less than this, perhaps considerably so.

By tabulating the residual differences between the absolute magnitudes of the individual dwarf stars and the values given by the formulæ just described, we find that the average difference, regardless of sign, for the stars of measured parallax is ±0·88m. for spectra A to F8, ±1·02m. for spectra G and G5, and ±1·15m. for K and M. For the stars in clusters, the average differences are ±0·70m. for spectra B0 to B9, ±0·66m. for A and A5, ±0·56m. for spectra F to F8, and ±0·80m. for G and G5.

These differences are larger for the stars of measured parallax than for the others (probably on account of the greater average uncertainty of the individual parallaxes and spectra in this case), but show no marked systematic variation with the class of spectrum. Their distribution follows very approximately the law of accidental errors, as is shown by Table VI., in which the observed numbers lying between certain limits are compared with those given by this law

TABLE VI.
Distribution of Differences from the Typical Absolute Magnitudes.

Stars with measured parallax			Stars in clusters		
Limits m. m.	Observed	Theory	Limits m. m.	Observed	Theory
±0·0 to ±0·8	65	61	±0·0 to ±0·5	59	58
±0·8 to ±1·6	41	44	±0·5 to ±1·0	42	42
±1·6 to ±2·4	21	23	±1·0 to ±1·5	21	24
±2·4 to ±3·2	10	9	±1·5 to ±2·0	10	8
±3·2 to ±4·0	3	3	±2·0 to ±2·5	4	4

The theoretical distribution for the stars in clusters corresponds to a probable error of ±0·61m., and that for the others to one of ±0·94m. Correction for the known influence of uncertainties of the parallaxes and spectra would reduce the latter to about ±0·75m. It appears, therefore, that the absolute magnitude of a dwarf star can be predicted with surprising accuracy from a mere knowledge of its spectrum. Half of all the dwarf stars are not more than twice as bright or as faint as the typical stars of their spectral classes. The corresponding uncertainty in the estimated parallax would be about one-third of its amount.

The parallaxes of the giant stars are so small, in comparison with the errors of even the best present methods of observation, that direct observations are not well adapted to determine to what degree they differ in brightness among themselves. An indirect method of determining this is, however, practicable, among those classes in which all the naked-eye stars are giants, by comparing the parallactic motions of those stars the proper-motions of which at right angles to the direction of the parallactic drift are large and small. A discussion by this method of the typical case of Class M (the details of which will be given elsewhere) shows that, if the distribution of the absolute magnitudes of these stars also follows the "law of errors," the probable error corresponding to it is approximately ±0·6m.—almost exactly the same as has already been found for the dwarf stars. The mean absolute magnitude of all the stars of this class which are visible to the naked eye is −0·5, and that of all the stars in a given region of space is +0·6. This method can scarcely be applied to the naked-eye stars of the other spectral classes (unless some way can be devised for weeding out the dwarf stars from among the giants); but it seems probable