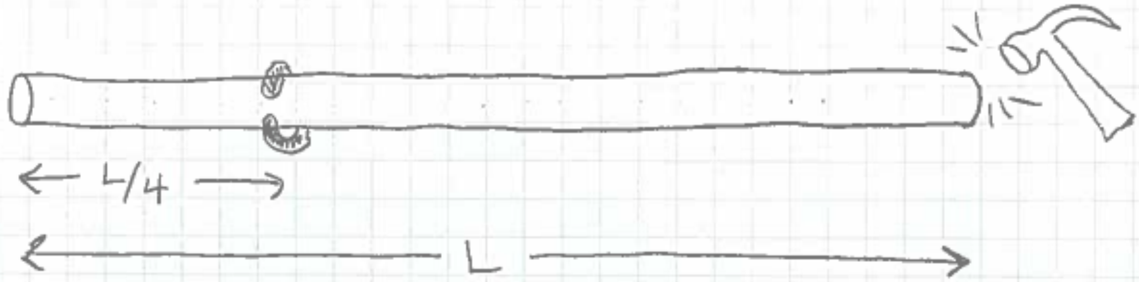


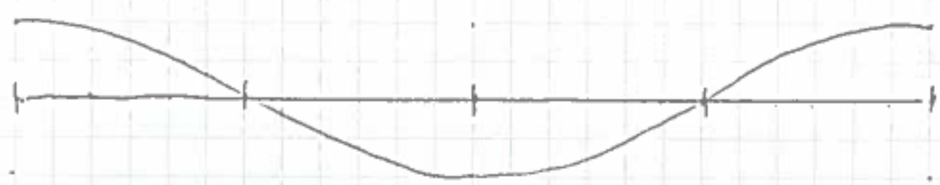
A long copper rod of length L is held firmly at a distance $\frac{L}{4}$ from the left end, then tapped on right end with a hammer. The rod rings with a faint, high pitch.

"Mon Dieu", exclaims Leopold, "5 octaves above middle C!"

How long is the rod?

First, we know that the rod's vibration

- has a node at $L/4$, where it is held
- has antinode at $x=0$, free left end
- has antinode at $x=L$, free right end



The longest wavelength satisfying these conditions is $\lambda = L$.

Next, we need to find the speed of longitudinal waves travelling through the rod. For long, thin objects, longitudinal waves move at

$$v = \sqrt{\frac{Y}{\rho}} \qquad \begin{aligned} Y &= \text{Young's modulus} \\ \rho &= \text{density} \end{aligned}$$

For copper

$$v = \sqrt{\frac{130 \times 10^9 \ N/m^2}{8960 \ kg/m^3}} = 3809 \ \frac{m}{s}$$

Now we can compute the frequency of the waves travelling through the rod

$$f\lambda = v$$

$$\rightarrow f = \frac{v}{\lambda} = \frac{3809 \text{ m/s}}{L}$$

Whoops! We don't know length L. But we do know the frequency:

middle C has	$f = 261.6$ Hz
one octave above " "	$f = 2 \cdot 261.6$ Hz
two " "	$f = 4 \cdot 261.6$ Hz
<u>five</u> octaves above	$f = 32 \cdot 261.6$ Hz $=$ 8371 Hz

So

$$L = \frac{v}{f} = \frac{3809 \text{ m/s}}{8371 \text{ Hz}} = \underline{\underline{0.455 \text{ m}}}$$