Light waves from two lasers approach a pool of oil. The lasers have wavelengths in a vacuum

$$\lambda_1 = 400 \text{ nm} \quad \text{in Vacuum}$$
$$\lambda_2 = 600 \text{ nm} \quad \text{in Vacuum}$$

a) angular frequencies are

$$\omega_1 = 2\pi f_1 = 2\pi\left(\frac{c}{\lambda_1}\right) = 4.7091 \times 10^{15} \ \frac{\text{rad}}{\text{s}}$$
$$\omega_2 = 2\pi f_2 = 2\pi\left(\frac{c}{\lambda_2}\right) = 3.1394 \times 10^{15} \ \text{rad/s}$$

and these remain the same in the oil.

b) wavelengths in vacuum are

$$\lambda_1 = 400 \text{ nm} \quad \text{in vacuum}$$
$$\lambda_2 = 600 \text{ nm} \quad \text{in vacuum}$$

but these will change in oil.

The oil has index of refraction

$$n(\lambda) = \left(1 + \frac{0.1 \cdot 300 \text{ nm}}{\lambda}\right)$$

c) In the oil, the wavelengths become

$$\lambda_1' = \frac{\lambda_1}{n(\lambda_1)} = \frac{400 \text{ nm}}{1.075} = 372.1 \text{ nm}$$

$$\lambda_2' = \frac{\lambda_2}{n(\lambda_2)} = \frac{600 \text{ nm}}{1.050} = 571.4 \text{ nm}$$

d) and wave numbers

$$k_1' = \frac{2\pi}{\lambda_1'} = 1.6886 \times 10^7 \ \frac{\text{rad}}{\text{m}}$$

$$k_2' = \frac{2\pi}{\lambda_2'} = 1.0996 \times 10^7 \ \frac{\text{rad}}{\text{m}}$$

e) To find the group velocity as the waves travel together through the oil, we start with the sum of the waves:

$$\text{Sum} = A\sin(k_1'x - \omega_1 t) + A\sin(k_2'x - \omega_2 t)$$

Use trig ID

$$\sin(P) + \sin(Q) = 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

So

$$\text{Sum} = 2A\sin\left(\frac{(k_1'+k_2')x}{2} - \frac{(\omega_1+\omega_2)t}{2}\right)\cos\left(\frac{k_1'-k_2'}{2}x - \frac{\omega_1-\omega_2}{2}t\right)$$

use for V_{ph} use for V_{gr}

$$V_{gr} = \frac{\omega_1 - \omega_2}{k_1' - k_2'} = 2.66 \times 10^8 \ \frac{m}{s}$$

f) The phase velocity is

$$V_{ph} = \frac{\omega_1 + \omega_2}{k_1' + k_2'} = 1.13 \times 10^8 \ \frac{m}{s}$$

So $V_{gr} > V_{ph}$ in this instance, and as usual.