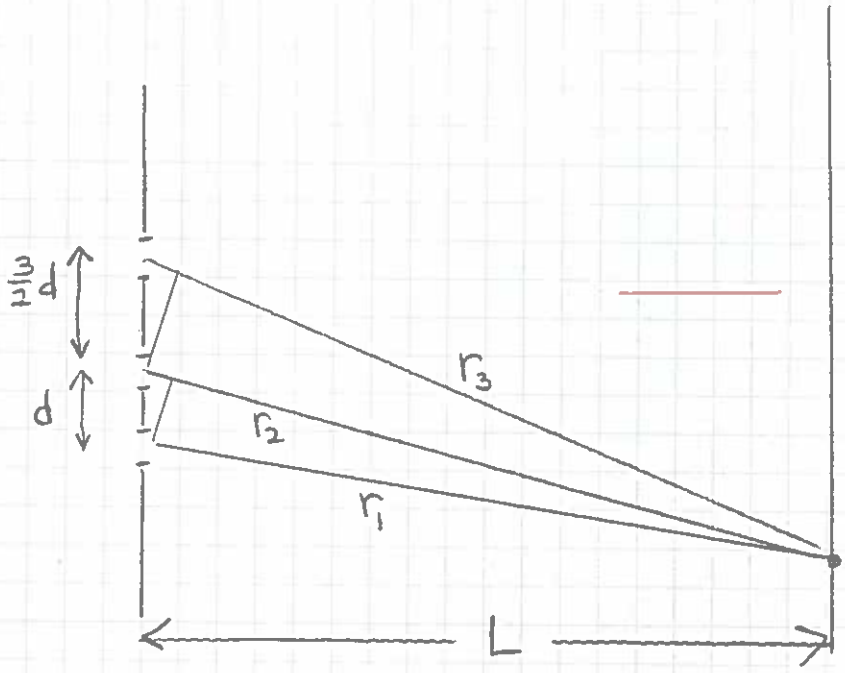


Joe sets up a 3-slit interference experiment, but makes a mistake punching the top slit in the wall: it is a distance $\frac{3}{2}d$ above the middle slit, not the proper distance d.

Joe shines light of wavelength λ through all 3 slits.

Are there any locations of complete constructive interference?

Consider point P, located at an angle θ away from the middle of the far wall. The waves reaching it are

$$y_1 = A\sin(kr_1 - \omega t)\cdot\frac{1}{r_1}$$
$$y_2 = A\sin(kr_2 - \omega t)\cdot\frac{1}{r_2}$$
$$y_3 = A\sin(kr_3 - \omega t)\cdot\frac{1}{r_3}$$

We can approximate

$$\frac{A}{r_1} \approx \frac{A}{r_2} \approx \frac{A}{r_3} = \frac{A}{r_{avg}}$$

and we can also replace the terms kr_1, kr_2, kr_3 inside the sine functions with phase offsets from the bottom slit's wave

$$\phi_1 = 0 \text{ rad}$$
$$\phi_2 = d\sin\theta\cdot\frac{2\pi}{\lambda} = \frac{2\pi d\sin\theta}{\lambda}$$
$$\phi_3 = \frac{5}{2}d\sin\theta\,\frac{2\pi}{\lambda} = \frac{5\pi d\sin\theta}{\lambda}$$

So we are adding

$$y_1 = \frac{A}{r_{avg}} \sin(0 - \omega t)$$

$$y_2 = \frac{A}{r_{avg}} \sin(\phi_2 - \omega t)$$

$$y_3 = \frac{A}{r_{avg}} \sin(\phi_3 - \omega t)$$

In order for y_1 and y_2 to interfere constructively

$$\phi_2 = 2\pi, 4\pi, 6\pi, \ldots, 2\pi \cdot N_2 \text{ rad}$$

In order for y_1 and y_3 to interfere constructively,

$$\phi_3 = 2\pi, 4\pi, 6\pi, \cdots, 2\pi \cdot N_3 \text{ rad}$$

and clearly

$N_3 > N_2$ because slit 3 is farther away.

So we need

$$\frac{2\pi d \sin\theta}{\lambda} = 2\pi N_2 \rightarrow N_2 = \frac{d\sin\theta}{\lambda}$$

$$\frac{5\pi d \sin\theta}{\lambda} = 2\pi N_3 \rightarrow N_3 = \frac{5}{2}\frac{d\sin\theta}{\lambda}$$

We can satisfy these conditions if (among other solutions)

$$N_2 = 2 \text{ and } N_3 = 5$$

Then

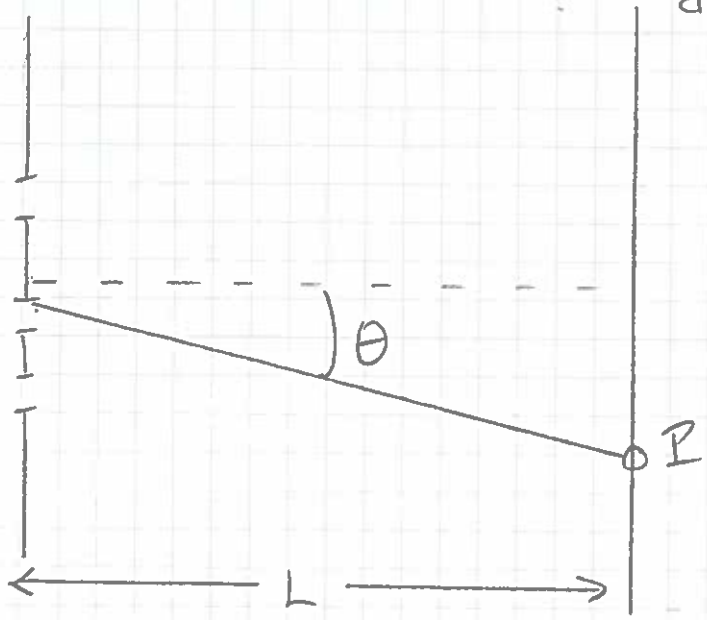
$d\sin\theta = 2\lambda$ ↑ slits 1 and 2 have path length $\Delta = 2\lambda$

$\frac{5}{2} d\sin\theta = 5\lambda$ ↑ slits 1 and 3 have path length $\Delta = 5\lambda$

→

And so constructive interference occurs if

$$\sin\theta = \frac{2\lambda}{d}$$



which is a larger angle than the usual

$$\sin\theta = \frac{\lambda}{d}$$