We are adding together two travelling waves:

$$y_1(x,t) = A\sin\left(\frac{n\pi x}{L} - \omega_n t\right) \quad \text{goes} \rightarrow$$
$$y_2(x,t) = A\sin\left(\frac{n\pi x}{L} + \omega_n t\right) \quad \text{goes} \leftarrow$$

When they meet, the sum is the superposition

$$y_1 + y_2 = A\sin\left(\frac{n\pi x}{L} - \omega_n t\right) + A\sin\left(\frac{n\pi x}{L} + \omega_n t\right)$$

We can simplify with a trig idenitity

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin(\alpha)\cos(\beta)$$

Let $\alpha = \frac{n\pi x}{L}$ $\quad \beta = \omega_n t$

$$\Rightarrow \boxed{y_1 + y_2 = 2\sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t)}$$

This form is very useful for checking boundary conditions

Ex: $n=3$, $L = 18\,m$, as in class example.

So we can see that the sum of two travelling waves matches the position $y=0$ at both ends of string

At $x = 0\,m$, $\quad y_1 + y_2 = 2\sin\left(\frac{3\pi}{18}\cdot 0\right)\cos(\omega_3 t)$
$$= 0$$

At $x = 18\,m$, $\quad y_1 + y_2 = 2\sin\left(\frac{3\pi}{18}\cdot 18\right)\cos(\omega_3 t)$
$$= 2\sin(3\pi)\cos(\omega_3 t)$$
$$= 0$$