

There is an incoming wave

$$y_i(x,t) = A \sin(kx - \omega t)$$

When it hits a wall, it can create reflections, which could be

$$y_2(x,t) = C \sin(kx + \omega t)$$

$$y_4(x,t) = E \cos(kx + \omega t)$$

or any mixture of these \uparrow . What are the proper values of coeffs C and E ? Use the boundary at $x=0$, where the rope is tied to the wall $\Rightarrow y(x=0) = 0$

$$\text{Sum is } y_i(x,t) + y_2(x,t) + y_4(x,t) = 0$$

$$A \sin(kx - \omega t) + C \sin(kx + \omega t) + E \cos(kx + \omega t) = 0$$

Plug in $x=0$

$$A \sin(-\omega t) + C \sin(+\omega t) + E \cos(+\omega t) = 0$$

Here, note a $\sin(\omega t)$ term cannot cancel a $\cos(\omega t)$ term at all values of time t , so we must have separately

$$\text{sin terms: } A \sin(-\omega t) + C \sin(\omega t) = 0$$

$$\text{cos terms: } E \cos(\omega t) = 0$$

$$\Rightarrow \boxed{E = 0}$$

$$-A \sin(\omega t) + C \sin(\omega t) = 0$$

$$\Rightarrow \boxed{A = C}$$