



Two long pieces of string are joined at $x=0$. We are given

	String I	String II
ρ	1.5 g/cm^3	$1.5 \text{ g/cm}^3 = 1500 \text{ kg/m}^3$
radius r	7.8 mm	3.9 mm

So we can compute

Cross section area $A = 1.911 \times 10^{-4} \text{ m}^2$

$4.778 \times 10^{-5} \text{ m}^2$

linear density $\mu = 2.867 \times 10^{-1} \text{ kg/m}$

$7.168 \times 10^{-2} \text{ kg/m}$

Joe watches on point on the string and sees vertical motion with

total dist $1.54 \text{ cm} \rightarrow$ amplitude $A_I = \frac{1.54 \text{ cm}}{2} = 0.77 \text{ cm}$
 $= 7.7 \times 10^{-3} \text{ m}$

period $P = 3.0 \text{ s} \rightarrow$ ang freq $\omega = \frac{2\pi}{P} = 2.094 \frac{\text{rad}}{\text{s}}$



A photograph shows 4 peaks over a span of 5 m, so the wavelength must be

$$\lambda = \frac{5 \text{ m}}{3 \text{ waves}} = 1.66 \text{ m}$$

The impedance of the string in region I can be calculated via

$$\begin{aligned}\text{Wave speed } v_I &= f \lambda = \left(\frac{\omega}{2\pi}\right) \lambda = \left(\frac{2.094 \text{ rad/s}}{2\pi \text{ rad}}\right) (1.66 \text{ m}) \\ &= 0.556 \text{ m/s}\end{aligned}$$

which allows us to determine the tension

$$\begin{aligned}v_I &= \sqrt{\frac{T}{\mu_I}} \rightarrow T = \mu_I v_I^2 = (2.867 \times 10^{-1} \text{ kg/m}) (0.556 \text{ m/s})^2 \\ &= 0.08849 \text{ N}\end{aligned}$$

and then impedance

$$\begin{aligned}Z_I &= \sqrt{T \mu_I} = \sqrt{(0.08849 \text{ N}) (2.867 \times 10^{-1} \text{ kg/m})} \\ &= 0.159 \text{ kg/s}\end{aligned}$$

The wave travelling along string in region I carries power

$$\begin{aligned}P_I &= \frac{1}{2} (\mu_I v_I) \omega^2 A_I^2 \\ &= \frac{1}{2} (2.867 \times 10^{-1} \text{ kg/m}) (0.556 \text{ m/s}) (2.094 \text{ rad/s})^2 (7.7 \times 10^{-3} \text{ m})^2 \\ &= 2.07 \times 10^{-5} \text{ W}\end{aligned}$$

Now, in region II, the string has the same tension T , but different mass density. So its impedance is

$$\begin{aligned}Z_{II} &= \sqrt{T \mu_{II}} = \sqrt{(0.08849 \text{ N}) (7.168 \times 10^{-2} \text{ kg/m})} \\ &= 7.96 \times 10^{-2} \frac{\text{kg}}{\text{s}} = \frac{1}{2} Z_I\end{aligned}$$



We can compute the amplitude of the transmitted wave via

$$\begin{aligned}A_t &= \frac{2Z_I}{Z_I + Z_{II}} A_i \\&= \frac{2Z_I}{1.5Z_I} A_i = 1.33 A_i = 1.33 (7.7 \times 10^{-3} \text{ m}) \\&= 1.027 \times 10^{-2} \text{ m}\end{aligned}$$

and so derive the power carried into region II by this transmitted wave

$$P_{II} = \frac{1}{2} (\mu_{II} v_{II}) \omega^2 A_t^2$$

where

$$\begin{aligned}v_{II} &= \sqrt{\frac{T}{\mu_{II}}} = \sqrt{\frac{0.08849 \text{ N}}{7.168 \times 10^{-2} \frac{\text{kg}}{\text{m}}}} = 1.111 \frac{\text{m}}{\text{s}} \\&= 2v_I\end{aligned}$$

So

$$\begin{aligned}P_{II} &= \frac{1}{2} (7.168 \times 10^{-2} \frac{\text{kg}}{\text{m}}) (1.111 \frac{\text{m}}{\text{s}}) (2.094 \frac{\text{rad}}{\text{s}})^2 (1.027 \times 10^{-2} \text{ m})^2 \\&= 1.84 \times 10^{-5} \text{ W}\end{aligned}$$