Radio astronomers detect a pulse of radio waves from a distant object. They measure a small delay in time between arrival of the pulse at two wavelengths

$$\lambda_1 = 0.4\,m \qquad f_1 = \frac{c}{\lambda_1} \qquad \omega_1 = 2\pi f_1 = 4.709 \times 10^9 \ \frac{rad}{s}$$

$$\lambda_2 = 0.5\,m \qquad f_2 = \frac{c}{\lambda_2} \qquad \omega_2 = 2\pi f_2 = 3.767 \times 10^9 \ \frac{rad}{s}$$

$$\Delta t = t_2 - t_1 = 0.36\ s.$$

Now, assume distance to object is D. The waves don't travel at c through space, due to diffuse plasma of electron density

$$n_e = 0.03 \ \frac{electrons}{cm^3}$$

The speed of radio waves through plasma depends on their frequency - due to dispersion.

$$V_{pl} = c\sqrt{1 - \frac{9.4 \times 10^7 \ \frac{rad^2}{s^2}}{\omega^2}}$$

If we plug in frequencies ω_1, ω_2, we can compute the speed of each wave through plasma:

$$V_1 = c\left(1 - \frac{9.4 \times 10^7 \ rad^2/s^2}{\omega_1^2}\right)^{1/2} \cong c\left(1 - \frac{1}{2}\frac{9.4 \times 10^7}{(4.709 \times 10^9)^2}\right)$$

$$\cong c\left(1 - 2.11 \times 10^{-12}\right)$$

Likewise,

$$V_2 \cong c\left(1 - 3.31 \times 10^{-12}\right)$$

The time for each wave to travel distance D is

$$t_1 = \frac{D}{V_1} = \frac{D}{c\left(1-2.11\times10^{-12}\right)} \approx \frac{D}{c}\left(1+2.11\times10^{-12}\right)$$

$$t_2 = \frac{D}{V_2} = \frac{D}{c\left(1-3.31\times10^{-12}\right)} \approx \frac{D}{c}\left(1+3.31\times10^{-12}\right)$$

So time difference is

$$t_2 - t_1 = \frac{D}{c}\left[1+3.31\times10^{-12} - \left(1+2.11\times10^{-12}\right)\right]$$

$$t_2 - t_1 = \frac{D}{c}\left(1.19\times10^{-12}\right)$$

We can compute distance to source :

$$D = \frac{c\,(t_2-t_1)}{1.19\times10^{-12}} = \frac{c\,(0.36\,s)}{1.19\times10^{-12}}$$

$$= c\left(3.02\times10^{11}\,s\right) \cong c\,(9565\text{ yr})$$

$$\cong 9565 \text{ light years.}$$