Jane creates a spring-mass system with

$$k = 156 \text{ N/m}$$
$$m = 21.8 \text{ kg}$$
$$b = 1.13 \text{ N/m/s}$$

She allows the system to come to rest at equilibrium.

Now she pulls the mass down with $F = 20$ N, a constant force.

static displacement $\boxed{x = \frac{F}{k} = \frac{20 \text{ N}}{156 \text{ N/m}} = 0.1282 \text{ m}}$

Next, she decides to make a dynamic test, applying a sinusoidally varying force with

$$F_{max} = 20 \text{ N} \quad \text{(same as before)}$$
$$\omega = 1.87254 \text{ rad/s}$$

The natural frequency of this system is

$$\omega_0 = \sqrt{\frac{k}{m}} = 2.6751 \text{ rad/s}$$

So, one can compute the amplitude of the driven system after it has settled into a steady-state behavior:

$$\text{dynamic amplitude } A = \frac{F_{max}/m}{\sqrt{\left[\omega_0^2 - \omega^2\right]^2 + \frac{b^2\omega^2}{m^2}}}$$

Plugging in the numbers, we find $\rightarrow$

$$A = \frac{20\ N / 21.8\ kg}{\sqrt{\left[\left(2.6751\ \frac{rad}{s}\right)^2 - \left(1.87254\ \frac{rad}{s}\right)^2\right]^2 + \frac{\left(1.13\ \frac{N}{m/s}\right)^2\left(1.87\ \frac{rad}{s}\right)^2}{(21.8\ kg)^2}}}$$

dynamic displacement $\boxed{A = 0.251\ m}$ Thus $\boxed{\frac{A}{X} = 1.960}$

The phase offset between the driving force and motion of the mass in response is

$$\phi = \tan^{-1}\left(\frac{\frac{b\omega}{m}}{\omega_0^2 - \omega^2}\right) = \tan^{-1}\left(\frac{0.09706}{3.6498}\right)$$

$$\boxed{\phi = 0.02659\ rad}$$

Jane is not satisfied. She sets the motor so the driving frequency ω is equal to the natural frequency ω_0. Now

dynamic displacement $$A = \frac{F_{max}/m}{\sqrt{\frac{b^2\omega_0^2}{m^2}}} = \frac{F_{max}/m}{\frac{b\omega_0}{m}}$$

$$\boxed{A = 6.616\ m}$$

Now the dynamic displacement is <u>much</u> larger than the static displacement:

$$\boxed{\frac{A}{X} = \frac{6.616\ m}{0.1282\ m} = 51.6}$$

Now the phase offset between the driving force and motion of the mass is

$$\phi = \tan\left(\frac{b\omega/m}{\omega_0^2 - \omega^2}\right)$$

This function is undefined when $\omega = \omega_0$, but as one makes $\omega \rightarrow \omega_0$, it is clear that the phase angle approaches

$$\boxed{\phi \rightarrow \pi/2 \text{ rad}}$$

The "Q" factor for this system is

$$Q \equiv \frac{\omega_0 m}{b} = \frac{(2.675 \frac{\text{rad}}{\text{s}})(21.8 \text{ kg})}{1.13 \text{ N/m/s}}$$

$$\boxed{Q = 51.6}$$

And note that this is the same as the ratio of dynamic to static displacement — when we force the system at its natural frequency.