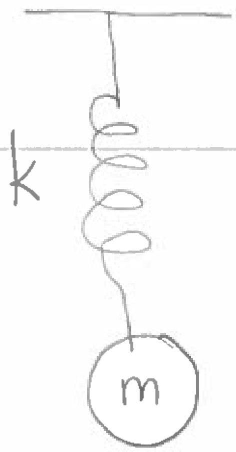
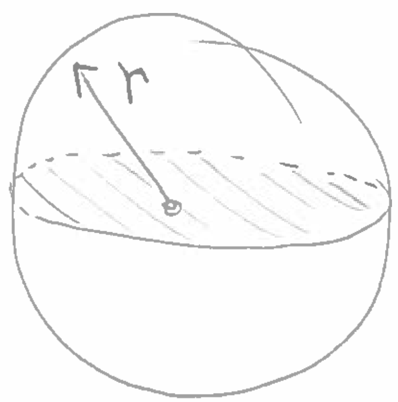


Spring of force constant $k = 39$ N/m holds a spherical weight of

mass $m = 4.95$ kg

density $\rho_1 = 2700$ kg/m³

The radius of the sphere r_1 can be calculated from



$$m = \frac{4}{3}\pi r_1^3 \rho_1$$

$$\rightarrow r_1 = \left(\frac{3m}{4\pi\rho_1}\right)^{1/3}$$

Now, the air resistance acting on the sphere depends on its cross-section area

$$A_1 = \pi r_1^2$$

After many trials, Fred measures air resistance coefficient to be

$$b_1 = 0.088 \ \frac{\text{kg}}{\text{s}}$$

The time constant of the sphere's motion is thus

$$\boxed{\tau_1 = \frac{2m}{b_1}} = 112.5 \text{ s}$$

And the position of the sphere can be written

$$y(t) = Ae^{-t/\tau}$$

if we ignore the initial conditions.

Fred watches the sphere as it moves.

When $y(t_{start}) = 0.140 \text{ m}$, he starts a timer

$y(t_{stop}) = 0.014 \text{ m}$, he stops the timer

How much time has passed? Divide these equations

$$\frac{y(t_{start})}{y(t_{stop})} = \frac{0.140 \text{ m}}{0.014 \text{ m}} = \frac{A e^{-t_{start}/\tau_1}}{A e^{-t_{stop}/\tau_1}}$$

$$10 = \frac{e^{-t_{start}/\tau_1}}{e^{-t_{stop}/\tau_1}} = e^{-[t_{start} - t_{stop}]/\tau_1}$$

$$\ln(10) = -[t_{start} - t_{stop}]/\tau_1$$

$$\rightarrow [t_{start} - t_{stop}] = -\tau_1 \ln(10)$$

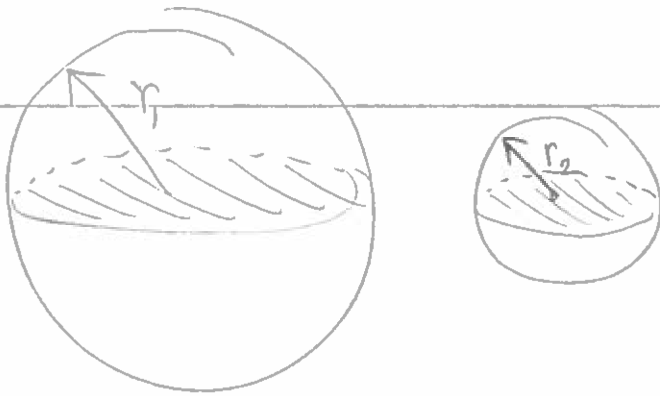
Of course, if we reverse the order of t_{start} and t_{stop}, we'll get a positive time difference — makes sense.

$$\boxed{[t_{stop} - t_{start}] = \tau_1 \ln(10)} = (112.5 \text{ s})(2.303)$$

$$= 259 \text{ s}$$

"Too quick a decay!" exclaims Fred. He wants to increase the duration of the system's motion. But how?

"Aha! I'll decrease air resistance!"



Fred builds a new sphere ~~with same mass m~~, but higher density $\rho_2 = 8100 \frac{kg}{m^3}$

Now

$$r_2 = \left(\frac{3m}{4\pi \rho_2} \right)^{1/3}$$

The smaller sphere has smaller cross-section area

$$A_2 = \pi r_2^2$$

The ratio are areas is

$$\frac{A_1}{A_2} = \frac{\cancel{\pi} r_1^2}{\cancel{\pi} r_2^2} = \frac{\left\{ \left[\frac{3m}{4\pi\rho_1} \right]^{1/3} \right\}^2}{\left\{ \left[\frac{3m}{4\pi\rho_2} \right]^{1/3} \right\}^2} = \frac{\rho_2^{2/3}}{\rho_1^{2/3}}$$

Since air resistance is linearly dependent on area, this means

$$\frac{b_1}{b_2} = \left(\frac{\rho_2}{\rho_1} \right)^{2/3} \rightarrow b_2 = b_1 \left(\frac{\rho_1}{\rho_2} \right)^{2/3}$$

Thus

$$b_2 = b_1 (0.481) = 0.042 \frac{kg}{s}$$

And

$$\tau_2 = \frac{2m}{b_2} = 234 \, s$$

So now $\boxed{[t_{stop} - t_{start}] = \tau_2 \ln(10)} = 539 \, s$