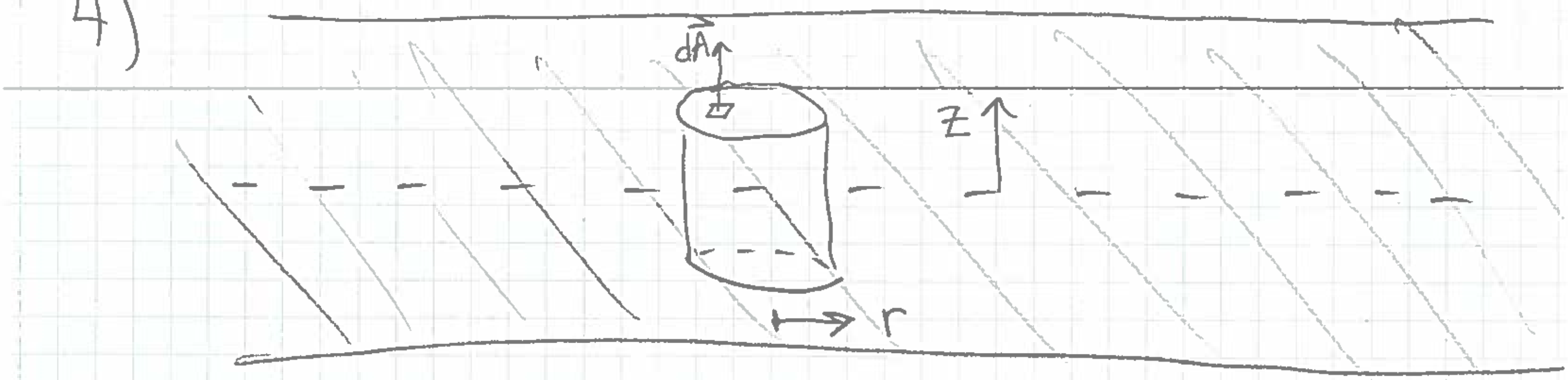
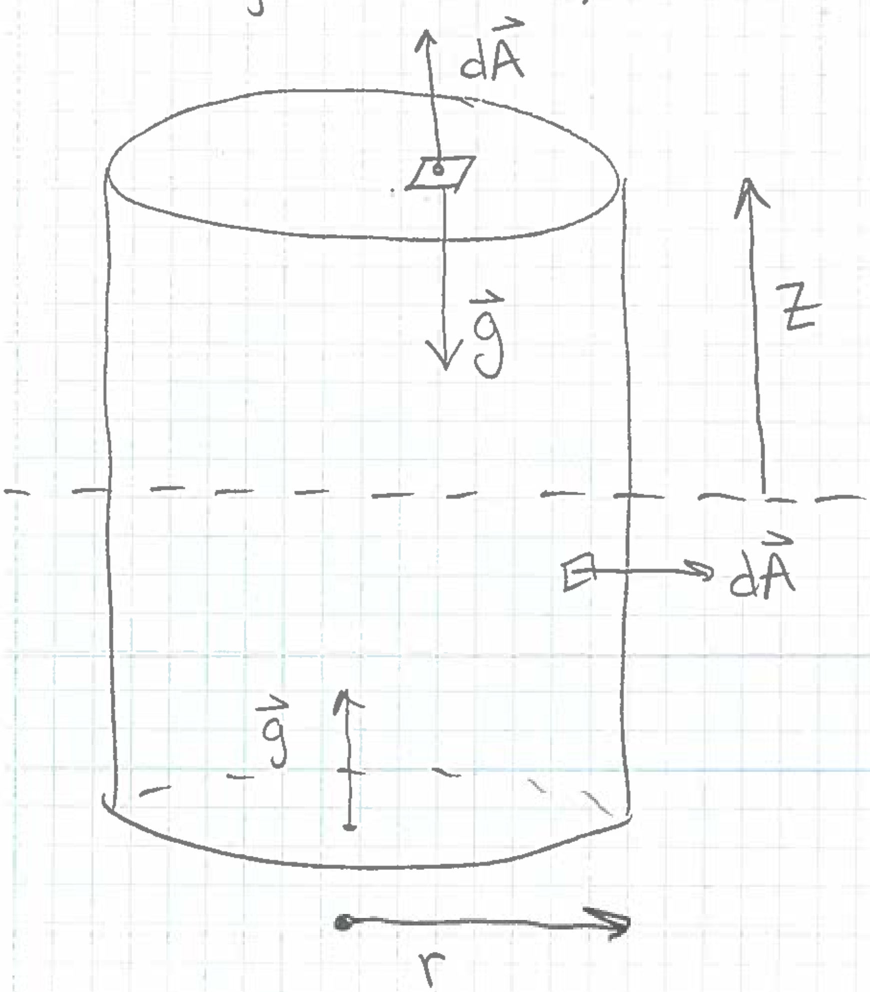
4)



Disk of Milky Way is infinite slab of stars with average mass density

$$\rho = 7 \times 10^{-20} \frac{kg}{m^3}$$

Sun sits inside disk, a height z above midplane. The gravitational force on it is <u>down</u>, back to midplane. How large is this force?



Use Gauss' Law for gravity

$$\oint \vec{g} \cdot d\vec{A} = -4\pi G M_{enc}$$

Integral over a cylinder simplifies:

on sides, $\vec{g} \perp d\vec{A}$

So $\vec{g} \cdot d\vec{A} = 0$

Only need to integrate over caps.

top: $\int \vec{g} \cdot d\vec{A} = |g||A| \cos 180^\circ$

$= -|g| \pi r^2$

bottom: $\int \vec{g} \cdot d\vec{A} = -|g| \pi r^2$

So $\oint \vec{g} \cdot d\vec{A} = -2|g|\pi r^2$

So now we can find strength of grav field $\vec{g}$ at height z above the midplane.

$$-2\pi r^2 |g| = -4\pi G M_{enc}$$

But

$$M_{enc} = \rho \, (\text{Volume of cylinder})$$
$$= \rho \left(\pi r^2 \cdot 2z \right)$$

So

$$-\cancel{2\pi r^2} |g| = -4\pi G \left(\cancel{2\pi r^2} \rho z \right)$$

$$\rightarrow |g| = \left(4\pi G \rho \right) z$$

And since $\vec{g}$ points down when $\vec{z}$ is above midpoint

$$\vec{g} = -\left(4\pi G \rho \right) \vec{z}$$

But

$$\vec{g} = \frac{d^2 \vec{z}}{dt^2} \qquad \text{gravitation acceleration}$$

So

$$\frac{d^2 \vec{z}}{dt^2} = -\left(4\pi G \rho \right) \vec{z}$$

This is simple harmonic motion with

$$\omega = \sqrt{4\pi G \rho}$$
$$= \sqrt{4\pi \left(6.67 \times 10^{-11}\right)\left(7 \times 10^{-20}\right)}$$
$$= 7.7 \times 10^{-15} \text{ rad/s}$$

From which we can compute the period of oscillation

$$\underline{P} = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{7.7 \times 10^{-15} \text{ rad/s}}$$

$$= 8.2 \times 10^{14} \text{ s}$$

$$\cong 26 \text{ million years}$$