Globular Cluster Distances from RR Lyrae Stars ⋆

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Abstract. The most common methods to derive the distance to globular clusters using RR Lyrae variables are reviewed, with a special attention to those that have experienced significant improvement in the past few years. From the weighted average of these most recent determinations the absolute magnitude of the RR Lyrae stars at $[Fe/H] = -1.5$ is $M_V = 0.59 \pm 0.03$, corresponding to a distance modulus for the LMC $(m - M)_0 = 18.48 \pm 0.05$.

1 Introduction

Globular clusters (GC) have traditionally been considered as good tracers of the process that led to the formation of their host galaxy, whether this was a "monolithic" relatively rapid collapse of the primeval gas cloud, as described by [37], or a "hierarchical" capture of smaller fragments on a longer time baseline, as described by [74]. We refer the reader to [44] for a recent review on this issue.

Therefore, knowing the distance to GCs with high accuracy is important in several respects:

- Cluster distances, along with information on the dynamical, kinematic and chemical properties of the clusters, are essential to provide a complete description of the galaxy formation, early evolution and chemical enrichment history.
- Accurate distances are needed in order to derive the age of GCs from the stellar evolution theory, i.e. by comparing the absolute magnitude of the Main-Sequence Turn-off (MS-TO) region in the Color-Magnitude diagram with the corresponding luminosity of theoretical isochrones. The precise knowledge of absolute ages has important cosmological implications (e.g. the age of the Universe), whereas relative ages provide detailed information on the formation process of the host galaxy.
- The Luminosity Function (LF) of a GC system is one of the most powerful candles for extragalactic distance determinations, as it peaks at a rather bright luminosity ($M_V \sim -7.5$), and GCs are numerous in both spiral and elliptical galaxies. An accurate calibration is essential both for testing the assumption of universality of the LF and for deriving its absolute value,

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hence again the importance of disposing of accurate distances to local GC calibrators.

Distances to GCs can be obtained by several methods that use either the cluster as a whole (direct astrometry) or some "candle" stellar population belonging to the cluster, e.g. Horizontal Branch (HB) and RR Lyrae stars, Main Sequence stars, White Dwarfs, Eclipsing Binaries, the tip of the Red Giant Branch (RGB), the clump along the RGB. In the latter case the distance stems from the determination of the absolute magnitude of the selected "candle", which in turn may depend on a purely theoretical or a (semi)-empirical calibration.

Here we review the distance determinations to GCs using only RR Lyrae (and HB) stars. Distance determinations to GCs using also the other possible methods have been reviewed by [11].

2 RR Lyrae Stars as Standard Candles

RR Lyrae stars have been traditionally the most widely used objects for the purpose of distance determination (see [78] for a general review), because they are i) easy to identify thanks to their light variation; ii) luminous giant stars (although less bright than the Cepheids) hence detectable to relatively large distances; iii) typical of old stellar systems that do not contain Population I distance indicators (such as the classical Cepheids), and iv) much more numerous than the Population II Cepheids. But of course the property that qualifies them as *standard candles* is their mean brightness, which has been known to be "nearly constant" in any given globular cluster (within a narrow range of variation) since Bailey's work in early 1900.

However, accurate studies have shown a few decades later that the mean intrinsic brightness of the RR Lyrae stars is not strictly constant: first, it is a (approximately linear) function of metallicity, i.e. $M_V(RR) = \alpha[Fe/H]+\beta$ ([70]; [71]) with a variation of $\sim$0.25 mag over 1 dex variation in metallicity; second, even within the same cluster, i.e. at fixed metallicity, there is an intrinsic spread in the HB luminosity due to evolutionary effects, whose extent can vary from $\sim$0.1 to $\sim$0.5 mag as a function of metallicity ([72]); finally, it has recently been shown that the luminosity-metallicity relation is not strictly linear, because it depends also on the HB morphology and stellar population ([19]; [34]).

Notwithstanding these aspects that introduce a significant intrinsic variation in the absolute magnitude of the RR Lyrae variables, these stars remain excellent distance indicators once these effects are properly known and taken into account.

As a first approximation, and for the purpose of taking into account small corrections due to metallicity differences, when needed in comparing different results, we assume that the $M_V(RR)$-[Fe/H] relation is linear, with the parameters estimated by [28] as the average of several methods, i.e.

$$M_V(RR) = (0.23 \pm 0.04)[Fe/H] + (0.93 \pm 0.12) \tag{1}$$

[11] found the same slope and 0.92 for the zero-point.

We shall now consider the main methods of absolute magnitude determination for RR Lyrae stars, with special attention to those that have experienced a substantial improvement recently. The aim is to estimate the most accurate and reliable value for the β parameter in eq. (1), which can then be used to derive the distance to globular clusters and other old stellar systems of known metallicity. For this purpose also studies dealing with field RR Lyrae stars will be reviewed, on the verified assumption that globular cluster and field RR Lyrae stars share the same characteristics ([27]; [22]).

2.1 Statistical Parallaxes

This method works by balancing two measurements of the velocity ellipsoid of a given stellar sample, obtained from the stellar radial velocities and from the proper motions plus distances, via a simultaneous solution for a distance scale parameter. The underlying assumption is that the stellar sample can be adequately described by a model of stellar motions in the Galaxy.

[61] provided the most recent review of this method, and summarized the results previously obtained by various groups on field RR Lyraes using slightly different algorithms and assumptions but basically the same sample of stars and very similar input data. From the work of [45] on 147 RR Lyrae stars, which contains a very careful analysis and corrections for all relevant biases, the average magnitude is $M_V(RR) = 0.77 \pm 0.13$ at [Fe/H]=–1.6. A very interesting result is the analytic expression for the relative error in the distance scale parameter reported by [66] (and references therein). They show that for a group of stars with a given velocity dispersion and bulk motion, and with observational errors smaller than the velocity dispersion, the relative error in the distance scale parameter is proportional to $N^{-1/2}$ where N is the number of stars in the sample. For a halo stellar population such as the RR Lyraes, where observational errors in the radial and tangential velocities are typically 20-30 km/s and velocity dispersions are $\sim$ 100 km/s, a more effective way of improving the results is by increasing the number of stars in the sample rather than improving further the quality of the velocity determinations. [45] tried this way by defining the radial velocity ellipsoid using 716 metal-poor non-variable and 149 RR Lyrae stars, and matching it with the distance-dependent ellipsoid derived from the proper motions of the RR Lyraes alone. The result is $M_V(RR) = 0.80 \pm 0.11$ at [Fe/H]=–1.71. The accuracy of this hybrid solution, however, is hardly any better given the bigger chance of thick disk contaminants even at low metallicity. We remind that [4] find that the local fraction of metal-poor stars that might be associated with the Metal Weak Thick Disk (MWTD) is on the order of 30%-40% at abundances below [Fe/H]=–1.0, and a significant fraction of these may extend to metallicities below [Fe/H]=–1.6.

[33] applied this method to a sample of 262 local RR Lyrae variables. They separated "halo" from "thick disk" objects by metallicity ($[Fe/H] < -1.0$) and kinematic criteria, and assumed an initial distance scale $< M_V > (RR) = 1.01 + 0.15[Fe/H]$ (i.e. $M_V = 0.79$ at [Fe/H]=–1.5) to transform proper motions into space velocity components. They then determined $M_V(RR) = 0.76 \pm 0.12$ for the

"halo" population at [Fe/H]=–1.6. This result is in agreement with the previous ones from this method. However, the possibility of kinematic inhomogeneities within the "halo" sample is strongly reassessed by [10], who identify two different populations among the metal-poor subset in this sample of stars. These two spherical subsystems would have different dynamical characteristics and origins, the slowly rotating subsystem being associated to the Galactic thick disk, and the fast rotating (possibly with retrograde motion) subsystem belonging to the accreted outer halo.

It is not quite clear if and to what extent the kinematic selection criteria adopted to separate "halo" and "thick disk" populations introduce a bias in the subsequent kinematic analysis of this method, and the effects on the final result of the adopted distance scale and possible contamination from the MWTD stellar component. It is not impossible that the application of the Statistical Parallax method to the local RR Lyrae stars might need a more detailed and accurate modelling of the stellar motions in the Galaxy, as well as a much larger sample of stars to work on, in order to provide reliable and robust results.

For the purpose of the present review we can summarize the results of the Statistical Parallax method as $< M_V(RR) >= 0.78 \pm 0.12$ mag at [Fe/H]=–1.5.

2.2 Trigonometric Parallax for RR Lyr

Trigonometric parallaxes are the most straightforward method of distance determination, being based on geometrical quantities independent of reddening. Only with *Hipparcos* trigonometric parallaxes for a good number of HB and RR Lyrae stars have become available. However, they are not accurate enough for a reliable individual distance determination, except for the nearest star, RR Lyr, for which a relatively high precision estimate of π (4.38±0.59 mas) was derived by *Hipparcos* ([65]). A previous ground-based estimate (i.e. 3.0±1.9 mas) was reported in the Yale Parallax Catalog ([3]).

The new and very important result in this field is the determination of a more accurate parallax for RR Lyr using HST-FGS3 data ($\pi = 3.82 \pm 0.20$ mas) by [5]. This leads to a true distance modulus $\mu_0 = 7.09 \pm 0.11$ mag, or 7.06 ± 0.11 mag if one adopts instead the weighted average of all three parallax determinations $< \pi >= 3.87 \pm 0.19$ mas.

RR Lyr has $< V >= 7.76$ mag and [Fe/H]=–1.39 ([29]; [39]). Depending on whether one assumes $< A_V >= 0.07 \pm 0.03$ mag as the average absorption value from the reference stars surrounding RR Lyr, or $< A_V >= 0.11 \pm 0.10$ mag as the linearly interpolated local value from the same reference stars, one obtains $M_V = 0.61 \pm 0.11$ mag or $M_V = 0.57 \pm 0.15$ mag, respectively.

Following [5] we adopt $M_V = 0.61 \pm 0.11$ mag, which leads to $M_V(RR) = 0.58 \pm 0.13$ at [Fe/H]=–1.5. This final error takes into account also the cosmic scatter in luminosity due to the finite width of the instability strip, by adding in quadrature an adopted value for the cosmic dispersion of 0.07 mag. This effect, which is negligible when many stars are involved, should be taken into account when dealing with individual stars.

2.3 Trigonometric Parallaxes for HB Stars

Since, as we discuss in Sect. 2.5, RR Lyraes are HB stars, [46] adopted the approach of considering all field metal-poor HB stars with *Hipparcos* values of π in a magnitude limited sample, $V_0 < 9$. This selection criterium led to a sample of 22 stars, of which 10 were HB stars on the blue side of the instability strip, 3 were RR Lyrae stars, and 9 were red HB stars. Using the globular cluster M5 as a template to reproduce the shape of the HB, [46] estimated the correction in M_V to apply to each star in order to report it to the middle of the instability strip, and derived $< M_V >= 0.69 \pm 0.10$ mag at [Fe/H]=–1.41, or $< M_V >= 0.60 \pm 0.12$ mag at [Fe/H]=–1.51 excluding one red HB star suspected of belonging to the red giant population.

A reanalysis of this sample was performed by [66], who eliminated all red HB stars from the sample as a prudent way to ensure that no contamination from the Red Giant Branch (RGB) was present, applied a different weighting procedure by the observational errors, and considered the effect of intrinsic scatter in M_V in the estimate of the Malmquist bias. Their result was $< M_V >= 0.69 \pm 0.15$ at [Fe/H]=–1.62 (but [23] point out that this result may be questionable since the metallicity scale for blue HB stars is not well determined). Finally, [54] using all stars of this sample and taking into account the intrinsic scatter in the HB magnitudes when correcting for the Lutz-Kelker effect, derived $M_V = 0.62$ mag at [Fe/H]=–1.5.

A final reanalysis of this problem was performed by [23] who provided a revised value $< M_V >= 0.62 \pm 0.11$ at [Fe/H]=–1.5.

2.4 Baade-Wesselink (B-W)

This method derives the distance of a pulsating star by comparing the linear radius variation, that can be estimated from the radial velocity curve, with the angular radius variation, that can be estimated from the light curve.

It is common belief that the B-W results are "faint", based on the large amount of work done on field RR Lyrae stars during the past decade by several independent groups, and revised and summarized by [40]:

$$M_V(RR) = (0.20 \pm 0.04)[Fe/H] + (0.98 \pm 0.05) \tag{2}$$

hence $M_V(RR) = 0.68$ at [Fe/H]=–1.5.

This method was reapplied by [14] to RR Cet ([Fe/H]=–1.43, average value from [29] and [39]) with the following improvements with respect to the previous analyses:

- Use of various sets of model atmospheres, with and without overshooting treatment of convection, [α/Fe]=+0.4; some experimental models with no convection, that mimic the effects of a different treatment of convection e.g. the [16] approximation, were also tried.

- Use of the detailed variation of gravity with phase, rather than the mean value; the values of log$g$ at each phase step were calculated from the radius percentage variation (assuming $\Delta R/<R>\sim 15\%$) plus the acceleration component derived from the radial velocity curve.
- Use of new semi-empirical calibrations for bolometric corrections, based on the temperature scale for Population II giants defined from RGB and HB stars in several globular clusters using infrared colors ([64]).
- Use of various assumptions on the γ-velocity, and turbulent velocity = 2km/s and 4km/s over all or part of the pulsation cycle.
- Use of BVRIK photometric data.
- The matching of the linear and angular radius variations was performed on the phase interval $0.25 \leq \phi \leq 0.70$ to avoid shock-perturbed phases.

It was found that i) the use of K magnitudes and V–K colors provided the most reliable and stable results, and ii) all other options produced similar results within 0.03 mag, except the test case that used an unrealistically large amplitude of the γ-velocity curve.

The resulting mean magnitude for RR Cet is $M_V = 0.57 \pm 0.10$ mag, i.e. 0.55 ± 0.12 mag when reported to [Fe/H]=–1.5, and taking into account the cosmic dispersion (see Sect. 2.2).

2.5 Evolutionary models of Horizontal Branch stars

From the evolutionary point of view, RR Lyrae stars are low-mass stars in the stage of core helium burning located in a well defined part of the HB, i.e. the temperature range approximately 5900-7400 K, known as the "instability strip". Therefore theoretical models of HB stars within this temperature interval should in first approximation be able to describe the average properties of RR Lyrae variables, were they not be pulsating.

Theoretical models (hence the HB morphology and luminosity level) depend significantly on assumed input parameters. The strongest dependence besides [Fe/H] is on the helium abundance, but other parameters may have an effect, such as [CNO/Fe], peculiar surface abundances due to mixing during the RGB phase, diffusion or sedimentation, rotation, magnetic field strength, some other yet unknown factor that affects mass loss efficiency, or a combination of any of these, as well as theoretical assumptions such as the equation of state, the treatment of plasma neutrino energy loss, the correct treatment of conductive opacities in RGB stars, the 3-alpha reaction rate, etc. briefly on anything that can affect the ratio total mass vs core mass of the star.

For these reasons several research groups have been actively working on the construction of new HB models trying to include as much improved input physics as possible. Without entering into the details of the individual choices and assumptions, for which we refer the reader to the original papers, we report the results found by six independent groups, namely [35], [15], [25], [41] (based on the work by [79]), [34] (from outer halo globular clusters only), and [82].

We show in Fig. 1 how $M_V(HB)$ varies with [Fe/H], where $M_V(HB)$ is the mean absolute V magnitude of an HB star at logT_{eff}=3.85, that is taken to represent the equilibrium characteristics of an RR Lyrae star near the middle of the instability strip.

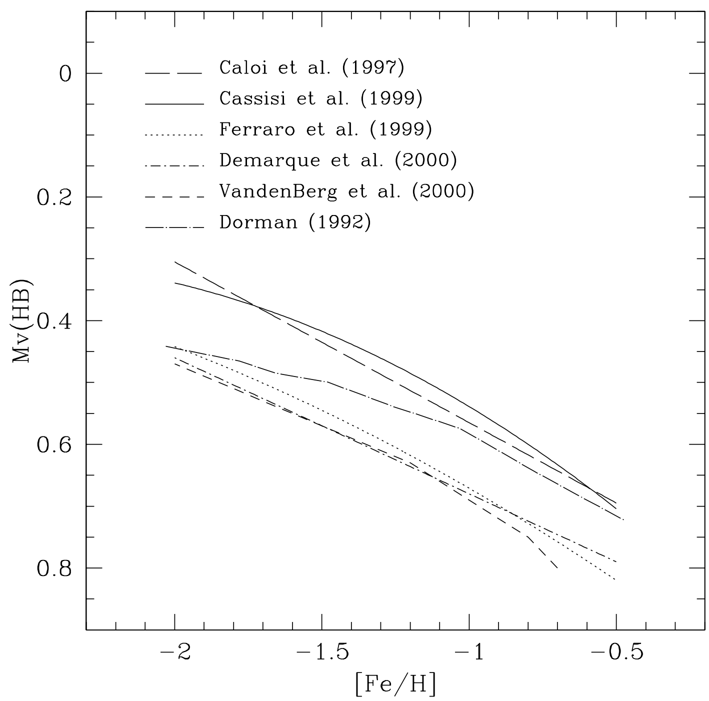

Fig. 1. M_V(HB) vs [Fe/H] from various sets of evolutionary models.

The original theoretical data usually refer to the Zero Age Horizontal Branch (ZAHB) rather than the average HB (or RR Lyrae) magnitude level. The two quantities $M_V(ZAHB)$ and $M_V(HB)$ are not identical. The stars in this evolutionary phase evolve rapidly away from the ZAHB: less than $\sim$ 10% of their total HB lifetime is spent on the ZAHB itself, and the remaining time is spent at 0.1-0.2 mag brighter luminosities ([41]). This aspect of stellar evolution has been discussed in several papers (see e.g. [17]; [21]; [24]).

Therefore the real ZAHB is intrinsically poorly populated, and when a comparison is made with the lower envelope of the observed HBs, which would represent the ZAHB, this is of difficult definition because of the uncertainties due to small sample statistics and photometric errors. So the comparison between HB theoretical models and observed HBs (including RR Lyrae stars) is made at the (brighter) magnitude level where the stars spend most of their HB lifetime. This is usually taken into account by correcting the theoretical $M_V(ZAHB)-[Fe/H]$ relation by a fixed offset (of the order of 0.08-0.10 mag), or by applying an empirical correction that is itself a function of [Fe/H], such as the one derived by [73]:

$$\Delta V(ZAHB - HB) = 0.05[Fe/H] + 0.16 \quad (3)$$

For the sake of simplicity we apply a fixed evolutionary correction of –0.08 mag to $M_V(ZAHB)$, which is very close to Sandage's correction near the middle of the metallicity range at [Fe/H]=–1.5.

We can derive a few conclusions from this comparison (see Fig. 1):

i) all models agree that the slope of the $M_V(HB) - [Fe/H]$ relation is not unique, i.e. this relation is *not universal* and is *not strictly linear*, as originally suggested by [26]. As a first approximation, however, all models can be roughly described by a linear relation with average slope ~ 0.23, excluding the oldest set of models ([35]) that are flatter.

ii) As far as the zero-point is concerned, there are two families of results differing by ~ 0.13 mag, i.e. [25] and [15] with $< M_V(HB) >$ =0.43 $\pm$ 0.12 at [Fe/H]=–1.5, and [41] ([79]), [34] and [82] with $< M_V(HB) >$ =0.56 $\pm$ 0.12. Again, [35] models differ as they fall exactly in between these two estimates.

2.6 Pulsation Models for RR Lyrae Stars

Visual Range New pulsation models have been calculated recently by [19], based on non-linear convective hydrodynamical models with updated opacities and the classical MLT treatment of convection ([6]). In combination with HB evolutionary models it is then possible to derive the Period-Luminosity-Metallicity relation for first overtone pulsators (RRc stars) at the blue edge of the instability strip, which in turn allows to estimate the luminosity $< M_V(RR) >$ of the RR Lyrae stars at the reference temperature logT_{eff}=3.85.

The behaviour of $M_V(RR)$ vs. [Fe/H] has been found to vary with the HB morphology and metallicity range, and could possibly be approximated by a quadratic relation. However, for the sake of simplicity this relation can be described by two linear relations that, for [α/Fe]=+0.3, are:

$$M_V(RR) = (0.17 \pm 0.04)[Fe/H] + (0.80 \pm 0.10) \; at \; [Fe/H] \; < \; -1.5 \quad (4)$$

and

$$M_V(RR) = (0.27 \pm 0.06)[Fe/H] + (1.01 \pm 0.12) \; at \; [Fe/H] \; > \; -1.5. \quad (5)$$

At the junction point [Fe/H]=–1.5 there is a discontinuity of 0.06 mag, the average value being $M_V(RR)$ =0.58$\pm$0.12.

The relations expressed in eq. (4) and (5), and comparison data points for a number of galactic globular clusters, are shown in Fig. 2. Compared with the theoretical HB models shown in Fig.1, these pulsation models are consistent with the family of results that produce the fainter magnitudes.

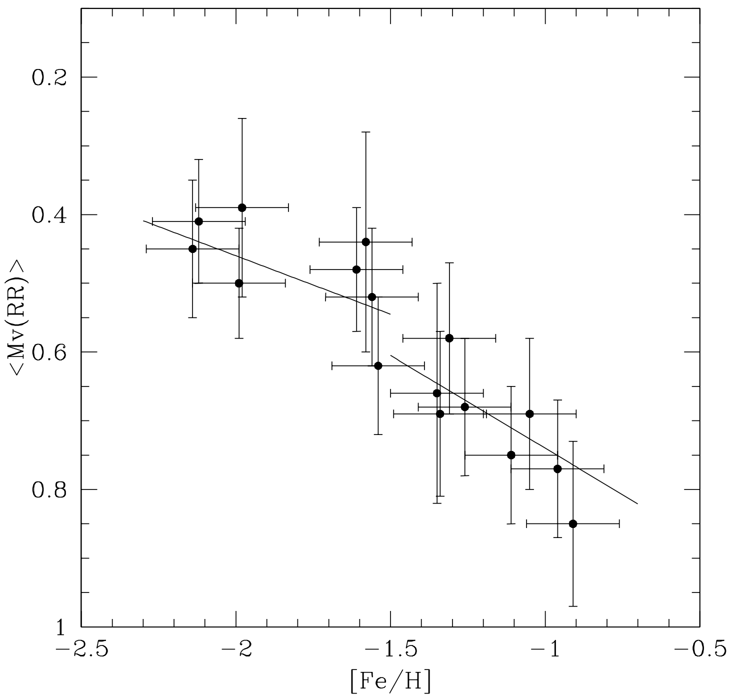

Fig. 2. $< M_V(RR) >$ vs [Fe/H] from the pulsation models by [19]. The dots represent galactic globular clusters, for comparison.

Infrared Range Infrared (K-band) observations of RR Lyrae stars had shown already several years ago that there exist a relatively tight relation between period P and mean K absolute magnitude $< M_K >$, with no (or little) dependence on metallicity ([63]; [62]; [49]). These empirical relations, however, had to be calibrated on some independent method of absolute magnitude determination, which was usually the Baade-Wesselink method in its various versions, hence different zero-points. Also the dependence on metallicity, admittedly small, was not assessed unambiguously. The values of M_K so derived could vary on a range of ~ 0.15 mag.

Based on the same set of updated non-linear convective pulsation models described above, [7] have defined a theoretical Period-Luminosity-Metallicity relation in the infrared K band (PL_KZ), which is much less sensitive to reddening and metallicity than the visual equivalent relation, and therefore is supposedly more accurate (total intrinsic dispersion σ_{Mk}=0.037 mag):

$$M_K = -2.071 logP + 0.167[Fe/H] - 0.766 \qquad (6)$$

Note that this relation has been derived for models with solar scaled metallicities; models with [α/Fe]=+0.3, that are well mimicked by models with $[M/H] =$

$[Fe/H] + 0.21$ according to the recipe by [69], would produce fainter M_K by $\sim$ 0.035 mag.

In principle, the non-linearity of the $logL(HB) - [Fe/H]$ relation and the change of slope at [Fe/H]$\sim$–1.5 could be taken into account by defining two separate linear relations for models with [Fe/H]<–1.5 and [Fe/H]>–1.5, respectively. In practice, the effect of non-linearity and the change of slope are negligibly small and the linear relation defined over the entire metallicity range that is relevant for RR Lyrae stars and globular clusters provides the same M_K values within the errors.

Very recently [9] have revised the above analysis and derived improved Period-(V–K)Color-Luminosity-Metallicity (PCL_KZ) relations for fundamental and first-overtone pulsators separately (see Bono, this conference, for more details). These new relations, also based on solar-scaled metallicity models, seem to give systematically brighter magnitudes by a few hundredths of a magnitude with respect to the PL_KZ relation expressed in eq. (6).

We now apply the PL_KZ relation in eq. (6) to a few test cases for which accurate data are available, i.e. the RR Lyrae stars in the globular clusters M3 and ω Cen, the 7 field RR Lyrae stars with $[Fe/H] = -1.5 \pm 0.15$ for which the Baade-Wesselink method was applied, and the field variable RR Lyr itself.

- **M3.**
  Seventeen non-Blazhko RRab variables and nine RRc variables in M3 have K photometry ([63]). Assuming [Fe/H]=–1.47 ([60]), and correcting the periods of the type-c variables to fundamental mode by the addition of a constant (0.127) to their $logP$ values, we obtain a K distance modulus $(m - M)_K = 15.03 \pm 0.05$, that can be considered as intrinsic modulus since the reddening of M3 is at most E(B–V)=0.01 mag ([36]), assuming $A_V = 3.1E(B-V)$ and $A_K = 0.11A_V$ ([20]). For these same stars $< V >= 15.64 \pm 0.05$ ([32]), hence $< M_V(RR) >= 0.58 \pm 0.08$.
- **ω Cen.**
  In addition to the K photometric data obtained by [63] on 30 RR Lyrae variables, new K photometry of 45 RR Lyrae variables has been recently obtained by [42]. There are no stars in common between these two sets of data, therefore the consistency of the photometric calibrations cannot be checked but *a posteriori*, by comparing the distance moduli derived from the two sets separately. Metal abundances for a number of RR Lyrae variables in ω Cen have been recently derived by [67], and have been used to derive M_K via eq. (6). We obtain $(m - M)_K = 13.69 \pm 0.09$ and 13.65 ± 0.13 from [63] and [42] data sets, respectively. The difference, well within the errors, could be ascribed to photometric calibration. Since [42] data are more recent and for a larger number of stars, we adopt their result that leads to $(m - M)_0 = 13.60 \pm 0.13$ assuming E(B–V)=0.13 and $A_K = 0.36E(B - V)$ ([20]). This result compares very well with the value of 13.65 ± 0.11 obtained by [80] using the eclipsing binary OGLE17 to derive the distance to ω Cen. If we now consider only the variables with [Fe/H]=–1.5±0.10, they have

$< V_0(RR) >= 14.14 \pm 0.11$ ([67]), hence $M_V(RR)$ =0.54±0.17 at [Fe/H]=–1.5.

- **Field RR Lyrae stars**
Approximately 30 field RR Lyrae stars have been analyzed with the Baade-Wesselink method and therefore have very accurate V and K light curves (see [40] for a review). From this sample we have selected 7 stars with $[Fe/H] = -1.5 \pm 0.15$, that are listed in Table 1. The V and K data are from [62] for all stars except UU Cet (data from [12]) and WY Ant (data from [77]). Using this average value of metallicity we have then derived the corresponding M_K values from eq. (6), the distance moduli and M_V values. Assuming a realistic error of ~ 0.1 mag on each individual estimate, the weighted average of these estimates is $< M_V >= 0.61 \pm 0.04$ mag. For comparison, the average Baade-Wesselink result on these same stars, as reported by [40], is 0.68 ± 0.15, whereas the application of the revised PCL_KZ relation leads [9] to estimate an average value of 0.54±0.03 mag.

Table 1. Field RR Lyrae stars with average [Fe/H]=–1.5±0.15 and good V and K data

Name	[Fe/H] (1)	[Fe/H] (2)	$< V_0 >$	$< K_0 >$	M_K	$(m-M)_0$	M_V
WY Ant	–1.48	–1.32	10.710	9.621	–0.518	10.139	0.571
RR Cet	–1.48	–1.38	9.625	8.524	–0.485	9.009	0.616
UU Cet	–1.28	–1.38	12.005	10.841	–0.565	11.406	0.599
RX Eri	–1.33	–1.63	9.529	8.358	–0.538	8.896	0.633
RR Leo	–1.60	–1.37	10.576	9.660	-0.302	9.962	0.614
TT Lyn	–1.56	–1.64	9.833	8.630	–0.553	9.183	0.650
TU UMa	–1.51	–1.38	9.764	8.656	–0.493	9.149	0.615

(1) From [39]
(2) From [29]

- **RR Lyr**
Eq. (6) can be applied to RR Lyr, for which $[Fe/H] = -1.39$±0.10 ([29];[39]), logP=–0.2466 ([52]), $< K >= 6.54 \pm 0.04$ mag ([38]), and $< V >= 7.76$ mag ([39]).
The result is $M_K = -0.487$, hence a distance modulus $(m-M)_0$=7.02 or 7.01 depending on the assumed absorption, i.e. $A_V = 0.07 \pm 0.03$ or 0.11 ± 0.10 (see Sect. 2.2). This in turn leads to $< M_V >= 0.67 \pm 0.11$ or 0.64 ± 0.11, respectively. By comparison, the results obtained by [5] using

a highly accurate estimate of the trigonometric parallax are $\sim$ 0.06 mag brighter (as described in more detail in Sect. 2.2).
This same analysis, performed by [8] in search of the "pulsation parallax" of RR Lyr, leads to $M_K = -0.541 \pm 0.062$ mag, whereas the application of the PCL_KZ relation leads to $M_K = -0.536 \pm 0.04$ mag ([9] and this conference). If we follow [5] choice of reddening ($A_V = 0.07$), then the value of M_V reported to [Fe/H]=–1.5 is 0.64±0.11 mag (or 0.59 from the PCL_KZ relation).

The weighted average of these 4 examples is $< M_V >= 0.61 \pm 0.03$ taking the results from the PL_KZ relation in eq. (6). We have seen that the revised and possibly improved PCL_KZ relation produces absolute magnitudes ~0.06 mag brighter; on the other hand all these values would be fainter by ~0.04 mag had non-solar-scaled metallicities (i.e. $[\alpha/Fe] = +0.3$) be taken into account.

Therefore we assume as average result of this section $< M_V >= 0.59 \pm 0.10$ at [Fe/H]=–1.5, where the error tries to account realistically for all the uncertainties still affecting this method.

Double-Mode Pulsators Another new result of pulsation models refers to double-mode RR Lyrae variables (RRd). From the pioneering work of [1] on stellar pulsation we know that the period of a fundamental (or first overtone) pulsator is related with its mass, luminosity and temperature via well known formulae, of which we report a recent redetermination by [18] that includes also some dependence on metallicity:

$$logP_0 = 11.242 + 0.841 logL - 0.679 logM - 3.410 logT_e + 0.007 logZ \quad (7)$$

and

$$logP_1 = 10.845 + 0.809 logL - 0.598 logM - 3.323 logT_e + 0.005 logZ \quad (8)$$

The double-mode pulsators, that pulsate simultaneously in the fundamental mode with period P_0 and in the first overtone with period P_1, allow to define a relation between stellar luminosity, temperature, periods and metallicity, where the dependence on mass is eliminated. Since periods and metallicities are observed quantities and temperatures can be derived from colors and adequate (empirical or theoretical) color-temperature calibrations, luminosities (hence distances) can be obtained.

Based on linear nonadiabatic pulsation models and various assumptions on opacities and detailed element abundances, [58] applied this method to the RRd stars in the Galactic globular clusters M15, M68 and IC4499. They did not provide direct values of M_V but only a comparison with the results of the Fourier-decomposition method, and estimated the distance modulus to the LMC as 18.45-18.55. The same data were later reanalyzed by [55] along with $\sim$ 180 RRd variables in the LMC from the MACHO database ([2]). No M_V values are given, but only distance moduli reported to the LMC. The weighted average of the four distance determinations to the LMC turns out to be $< (m - M)_o > (LMC) =$

18.50 ± 0.05. In this method the main source of error is due to ambiguity in the zero-point T_0 of the color-temperature transformation.

To derive the absolute V magnitude of RR Lyraes from the above results we need accurate observed V magnitudes of such stars in the LMC, with a good knowledge of their reddenings. The problem of the absolute and differential reddening across the LMC is a thorny problem that we cannot analyse here (see [31] and this conference for a detailed discussion); here we have assumed the values derived by the individual authors.

A few data sets can meet these requirements, in particular:

- [31] report the results of observations in two fields of the LMC bar, where 108 RR Lyrae stars were measured. The data in these two fields were corrected by their respective reddenings, i.e. 0.086 and 0.116. The mean magnitude of these RR Lyrae stars at average [Fe/H]=–1.5 is $< V_0 >= 19.06 \pm 0.06$. Spectroscopic metal abundances were also derived, and the slope of the luminosity-metallicity relation was found to be 0.214 ± 0.047, well consistent with the value of 0.23 used here.
- [83] presented and discussed the data for 160 RR Lyraes in 6 globular clusters (excluding NGC 1841 that may be significantly closer to us) at average [Fe/H]=–1.9. The data were corrected by the respective reddening for each individual cluster (ranging from 0.03 to 0.13 mag with average 0.07 mag). The mean magnitude of these 160 RR Lyrae stars is $< V_0 >= 18.98 \pm 0.06$.
- Other data for field RR Lyrae variables in the LMC are provided by the MACHO experiment ([2]): 680 stars, $< V_0 >= 19.14 \pm 0.10$ at [Fe/H]=–1.7, assumed reddening E(B–V)=0.10.
- The OGLE experiment ([81]): 6000 RR Lyrae stars, $< V_0 >= 18.91 \pm 0.10$ at [Fe/H]=–1.6, assumed reddening E(B–V)$\sim$0.143.

The error we associate to the MACHO and OGLE estimates is larger than the values quoted by the respective authors, but we believe it better represents the uncertainties due to photometric calibrations and reddening estimates still affecting these data sets. The large difference between these two results can only in part be accounted for by different values of the assumed reddening. Because of these uncertainties, we prefer not to use these results in the following considerations in spite of the very large number of involved stars.

A weighted average of the first two results only, after reporting them to [Fe/H]=–1.5, is $< V_0 >= 19.07 \pm 0.04$. Incidentally, we note that the average value of the last two results from the MACHO and OGLE data, that we have not considered because less accurate, is $< V_0 >= 19.06 \pm 0.07$ at [Fe/H]=–1.5, although the close agreement may be fortuitous.

If we then use the value estimated by [55] from RRd pulsators for the distance to the LMC, namely $< (m - M)_o > (LMC) = 18.50 \pm 0.05$ (see also A. Walker, this conference), then the average magnitude of the RR Lyrae stars is $< M_V >= 0.57 \pm 0.06$.

2.7 Fourier Parameters of Light Curves

During the past decade a series of studies were conducted, aimed at deriving *empirical* relations between the Fourier parameters of the light curves of RR Lyrae variables and their physical parameters. In particular, *RRc* variables were studied by [75] and [76], and *RRab* variables were studied by Kovács and collaborators in several papers (e.g. [50]; [56]; [57]; [59]).

This method is based on the assumption that period and shape of the light curves are correlated with the intrinsic physical parameters of the star. There is no known theoretical justification for this assumption, however well defined empirical correlations do indeed seem to exist, and the quoted studies have tried to define the combinations of Fourier parameters that best correlate with e.g. metallicity, intrinsic colors and absolute magnitude. The advantages of this method are potentially relevant, since its application only requires the use of accurate V light curves, that are now becoming available for large numbers of variables thanks to the many photometric surveys carried out in the past few years for different purposes. In particular we consider the relation

$$M_V(RR) = -1.876 logP - 1.158 A_1 + 0.821 A_3 + K \tag{9}$$

derived by [59] from 383 RRab variables in globular clusters. This formula fits the data with $\sigma = 0.04$ mag. The zero-point K, however, must be determined by some calibrator. The most recent and accurate estimate of K has been obtained by [53] using RR Lyr. This stars is affected by the Blazhko effect (a 41-d modulation of its amplitude whose amplitude in turn varies over a 4-year period). The Fourier parameters of this star correspond approximately to those of a normal RRab star only near maximum amplitude of the primary Blazhko cycle and minimum amplitude of the secondary cycle ([51]). By analysing data taken during one such epoch [53] finds that the Fourier coefficients A_1 and A_3 are respectively 0.31539 and 0.09768. Using M_V=0.61 for RR Lyr (see Sect. 2.2) he then finds K=0.43.

Based on this calibration, we apply the relation in eq. (9) to 55 normal RRab stars in M3 whose Fourier parameters have been recently determined from very accurate light curves ([13]). We find an average value $M_V = 0.615 \pm 0.003$, with an *rms* deviation for a single star of 0.02 mag. This very small *formal* error is purely statistical, and is due to the large number of stars involved in this estimate combined with a "tightening" effect by a factor ~ 2 of these M_V estimates with respect to the observed V values, whose intrinsic distribution has instead a $\sigma \sim$ 0.05 mag ([13]).

A further test can be done using the field variable RR Cet for which excellent light curves are available. For this star the Fourier coefficients A_1 and A_3 are respectively 0.31924 and 0.10760, and logP=–0.257, hence M_V=0.63 mag to which we can associate an *rms* error of 0.05 mag.

Both M3 and RR Cet have very similar metallicity, $[Fe/H] = -1.47$ and -1.43 respectively, and if we report the average of these two determinations to $[Fe/H] = -1.5$ we obtain $M_V = 0.61 \pm 0.05$ mag.

3 Summary and Conclusions

We have reviewed the methods of absolute magnitude determination for RR Lyrae variables, that can be used for distance determinations to globular clusters and all other stellar systems containing this type of stars.

We have adopted $M_V(RR)$ at [Fe/H]=–1.5 as the most convenient reference parameter (i.e. zero-point magnitude) for distance determination, assuming in first approximation that the dependence of $M_V(RR)$ on metallicity [Fe/H] is linear with a slope ~ 0.23.

We collect in the following Table 2 all the determinations of $M_V(RR)$ described in the previous sections. If we take the weighted average of these results, we obtain $<\mathbf{M}_V\mathbf{(RR)}> = \mathbf{0.59 \pm 0.03}$ mag (r.m.s. error of the mean) at **[Fe/H]=–1.5**.

Table 2. Summary of $M_V(RR)$ determinations at [Fe/H]=–1.5 from the methods described in the text.

Method	$M_V(RR)$ at [Fe/H]=–1.5	Reference
Statistical parallaxes	0.78±0.12	Sect. 2.1
Trigonometric parallaxes (RR Lyr)	0.58±0.13	Sect. 2.2
Trigonometric parallaxes (HB stars)	0.62±0.11	Sect. 2.3
Baade-Wesselink (RR Cet)	0.55±0.12	Sect. 2.4
HB stars: evolutionary models - bright	0.43±0.12	Sect. 2.5
HB stars: evolutionary models - faint	0.56±0.12	Sect. 2.5
Pulsation models (visual)	0.58±0.12	Sect. 2.6
Pulsation models (PL_KZ)	0.59±0.10	Sect. 2.6
Pulsation models (RRd)	0.57±0.06	Sect. 2.6
Fourier parameters	0.61±0.05	Sect. 2.7
Weighted average value	0.59±0.03	

The last two values of the list, from the double-mode pulsators and Fourier parameters, have smaller errors than the other results mainly because of the large number of stars considered by these two methods. If we do not wish to attach to them more weight than they probably deserve for intrinsic merits, and consider instead a typical error of ±0.10 mag for each of them, the previous average result and related error remain unchanged. Similarly, we may want to consider the results from the HB evolutionary models separately for the bright and faint groups: this would make a difference of at most 0.01 mag on the weighted average.

We note that the average value derived above is virtually identical to the value obtained by [28], only the error is now significantly smaller. Also [11] obtained a very similar average result (0.57 ± 0.04) by including the values from other distance determination methods, e.g. Eclipsing Binaries and Main-Sequence fitting to local Sub-Dwarfs. We might be tempted to conclude that we are approaching a robust result on this issue.

Using the value of $< V_0 >= 19.07 \pm 0.04$ at [Fe/H]=–1.5 estimated in Sect. 2.6 for the RR Lyrae variables in the LMC, this translates into a distance modulus to the LMC $(m-M)_0$ = 18.48±0.05. We refer to A. Walker and G. Clementini (this conference) for independent distance determinations to the LMC.

It may be interesting to compare the present result with two other $M_V(RR)$ or $M_V(HB)$ determinations that are important for different reasons:

i) The method of globular cluster Main-Sequence fitting to local Sub-Dwarfs (SD) is considered probably the most accurate and reliable method presently available, provided adequate precautions are taken in analyzing the data. The most recent results are given by [47], who have reanalyzed three clusters (47 Tuc, NGC6397 and NGC6752) using the most accurate data and assumptions, in particular high resolution (VLT-UVES) abundances and accurate photometry and reddening for MS and SD stars, all in the same scale and with the same treatment. The result obtained by [47] is $M_V(RR)$ = 0.61±0.07 mag at [Fe/H]=–1.5.

ii) Color-Magnitude diagrams have been derived for several globular clusters in M31 using *HST* data ([68]). From these an estimate of the mean HB magnitudes at the middle of the instability strip could be derived. These estimates are of course affected by significantly larger errors than any of those discussed in this review, however they are important because they allow to compare the same type of results in the Milky Way and in M31, in the framework of the similarities and differences between these two galaxies.
A preliminary analysis of 17 clusters shows that a slope ∼0.23 is adequate to describe the $V_0(HB) - [Fe/H]$ relation, and $< V_0(HB) >= 25.06 \pm 0.15$ at [Fe/H]=–1.5. The corresponding value of $M_V(HB)$ depends on the assumed distance to M31: if we assume the widely used value $(m-M)_0 = 24.43 \pm 0.06$ by [43], based on the Cepheid distance scale, then $M_V(HB)$ = 0.63±0.16 mag. An independent distance determination to the centroid of the M31 globular cluster system by [48], by fitting theoretical isochrones to the observed red giant branches of 14 globular clusters in M31, yields $(m-M)_0 = 24.47 \pm 0.07$, hence $M_V(HB)$ = 0.59±0.17 at [Fe/H]=–1.5.

It is reassuring to see that these results, in spite of the different intrinsic accuracy and statistical weight, agree with the average value estimated from the data listed in Table 2 within 1σ.

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