Luge at 2010 Olympics

1. Under ideal conditions,

$$\frac{1}{2} m v_f^2 = mg(H_i - H_f)$$

$$V_f = \sqrt{2g(H_i - H_f)} = \sqrt{2(9.8\,\tfrac{m}{s^2})(929\,m - 786\,m)}$$

$$= 52.9\ m/s$$

2. Work done by gravity on rider + sled is

$$W_g = \int \vec{F}_g \cdot d\vec{x} = |F_g|\,|\text{vertical drop}|$$

$$= (93\,kg)(9.8\,\tfrac{m}{s^2})(143\,m)$$

$$\cong 130 \times 10^3\ J$$

3. Actual top speed is $V_f = 40\ m/s$. So friction did a lot of negative work:

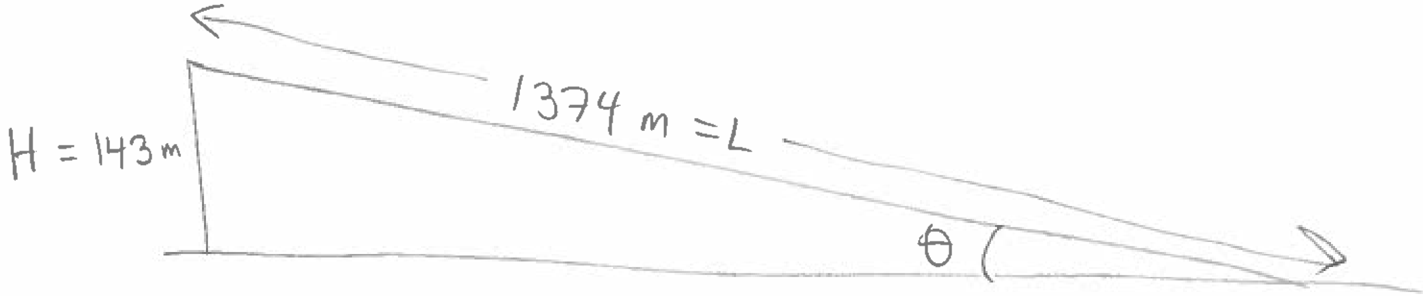
$$KE_i + GPE_i = KE_f + GPE_f - W_f$$

$$\longrightarrow W_f = (KE_f - KE_i) - (GPE_i - GPE_f)$$

$$= \left[\tfrac{1}{2}(93\,kg)(40\,\tfrac{m}{s})^2 - 0\right] - \left[(93\,kg)(9.8\,\tfrac{m}{s^2})(143\,m) - 0\right]$$

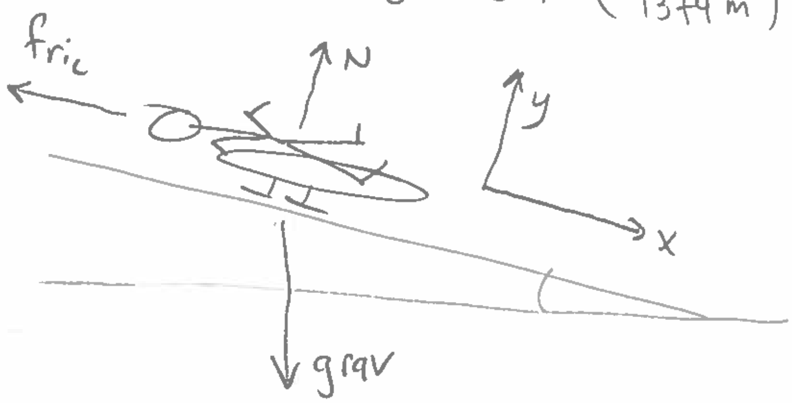
$$= 74.4 \times 10^3\ J - 130 \times 10^3\ J$$

$$= -55.9 \times 10^3\ J$$



Angle of track is, on average,

$$\theta = \sin^{-1}\left(\frac{143\text{ m}}{1374\text{ m}}\right) = 6.0^\circ$$



force	x	y
grav	$+mg\sin\theta$	$-mg\cos\theta$
Normal	0	$+F_N$
friction	$-\mu F_N$	0
total	ma_x	$ma_y = 0$

We can derive

$$F_N = mg\cos\theta$$

$$\rightarrow \quad \text{friction} = \mu mg\cos\theta$$

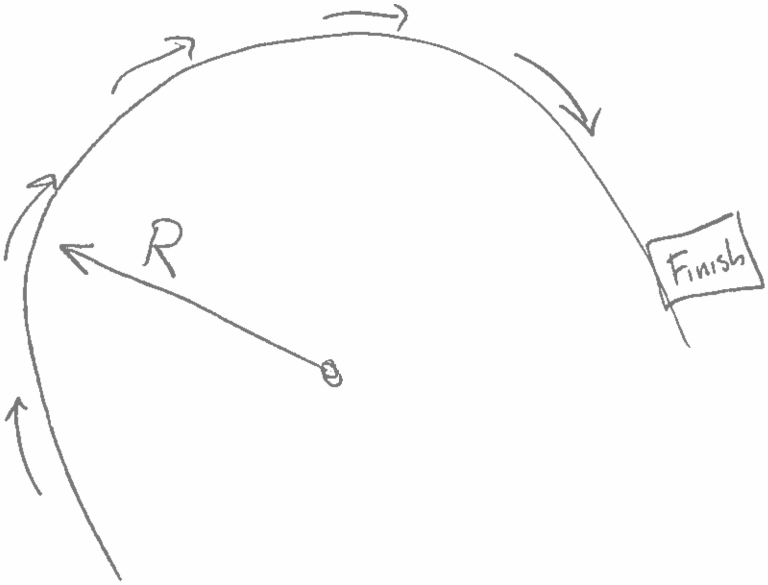
During slide down length $L = 1374$ m, the work done by friction is

$$W_f = \int \vec{F}_f \cdot dx \cong -|F_f|L$$

$$\cong -(\mu mg\cos\theta)L$$

But we know $W_f \cong -55.9 \times 10^3$ J

$$\rightarrow \quad \mu \cong \frac{-55.9\times10^3\text{ J}}{(93\text{ kg})(9.8\,\frac{\text{m}}{\text{s}^2})\cos 6.0^\circ\,(1374\text{m})} = \underline{0.045}$$



3 cm = 20 m

7 cm = 50 m

The final turn has radius of curvature $R \sim 50$ m.

If sled enters at $V = 40$ m/s,

$$a_c = \frac{V^2}{R} = \frac{(40\ \text{m/s})^2}{50\ \text{m}} = 32\ \text{m/s}^2$$

$$= 3.3\ \text{gees}$$

The angular speed of a rider in this turn will be

$$\vec{\omega} = \frac{V}{R} = \frac{40\ \text{m/s}}{50\ \text{m}} = 0.8\ \frac{\text{rad}}{\text{s}} \quad \underline{\text{into}}\ \text{page}$$

The torque exerted on a rider of mass 70 kg + sled 23 kg must be

$$\vec{\tau} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \vec{r} \times \vec{F} = 0$$

Since $\vec{\omega}$ = constant as rider goes around turn, $\frac{d\vec{\omega}}{dt} = 0$

and so $\vec{\tau} = 0$.

The force exerted on rider as he goes around turn comes from the walls of track, which push inward, toward center of circle. But then

$$\vec{r} \times \vec{F} = 0 = \vec{\tau}$$

because force is anti-parallel to radius vector.