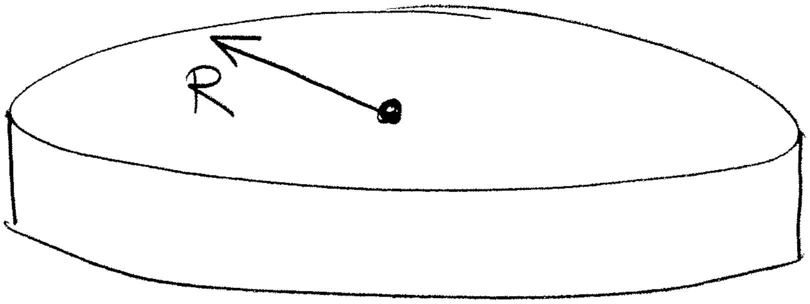
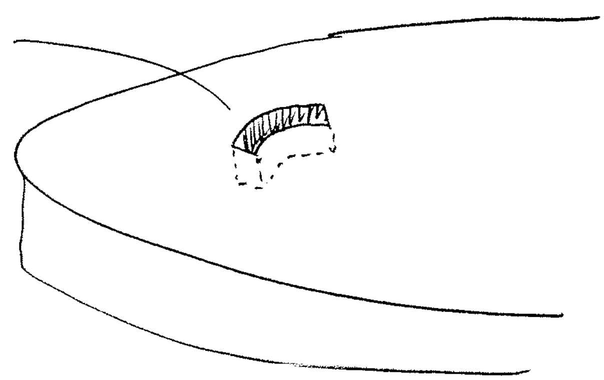


A solid disk has uniform density. We can express it in terms of a "surface density" — how much mass does a little chunk of area A have?

this little piece has an area on the top surface
$A = 0.3\ m^2$

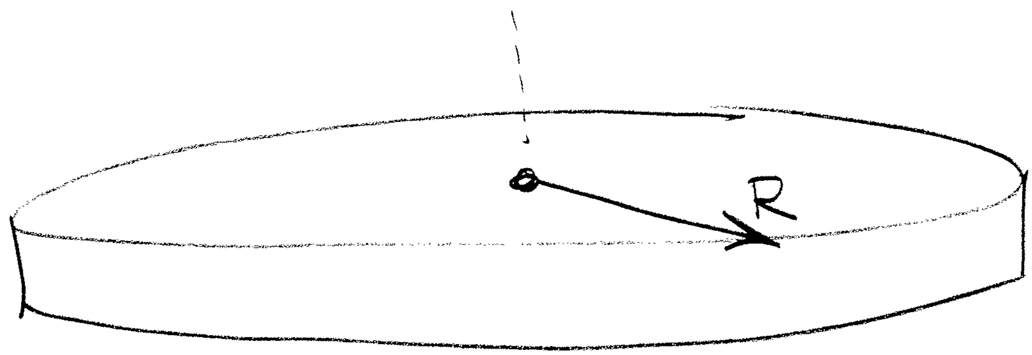


The mass of the disk underneath this little piece is
$m = 2.1\ kg$

So the "surface density" of this disk is

$$\sigma = \frac{mass}{area} = \frac{2.1\ kg}{0.3\ m^2} = 7\ \frac{kg}{m^2}$$

Physicists often use "surface density" $\frac{kg}{m^2}$ to describe thin, flat objects, just as they use "linear density" kg/m to describe long, thin objects.



If we are given a solid disk with

radius $R = 2\,m$

surface density $\sigma = 7\ kg/m^2$

there are two ways to determine the moment of inertia.

1) quick and easy – formula

2) longer – break into pieces

Method ①. Total area of disk $A = \pi R^2$

$= \pi (2m)^2$

$= 12.6\ m^2$

So

total mass of disk $M = \sigma \cdot A$

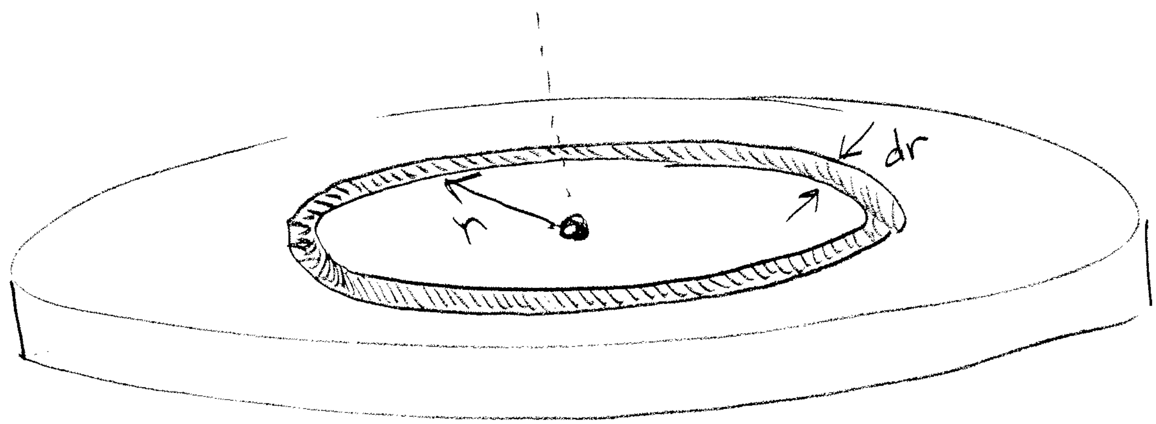
$= \left(7\ \frac{kg}{m^2}\right) \cdot \left(12.6\ m^2\right)$

$= 88.0\ kg$

Then

$$I = \frac{1}{2} M R^2 = \frac{1}{2}(88.0\ kg)(2m)^2 = 176\ kg \cdot m^2$$

Method 2 : break disks into <u>rings</u>, add up the mass of all the rings.



Look at one thin ring.

radius r
width dr

$$\text{area of ring} = (\text{length})(\text{width})$$
$$= (2\pi r)(dr)$$
$$= 2\pi r\, dr$$

$$\text{mass of ring} = (\text{area})(\text{surface density})$$
$$= (2\pi r\, dr)\,\sigma$$
$$= (2\pi r\, dr)\left(7\,\frac{kg}{m^2}\right)$$

The entire disk is just a sum of rings — so if I add up the mass of all rings, I should get the mass of the disk.

$$\text{total disk mass} = \int_{r=0}^{r=2m} (2\pi r\, dr)\left(7\,\frac{kg}{m^2}\right)$$

$$= \left(7\,\frac{kg}{m^2}\right) \int_{r=0}^{r=2m} 2\pi r\, dr$$

$$= \left(7\,\frac{kg}{m^2}\right) \left[\pi r^2 \right|_{0m}^{2m}$$

$$= \left(7\,\frac{kg}{m^2}\right) \left(12.6\ m^2 - 0\ m^2\right)$$

yes, this agrees with previous mass!

$$= 88\ kg$$

In the same way, I can add up the moment of inertia of all the rings to find the total moment of inertia of the disk.

one ring has mass $(2\pi r\, dr)\left(7\,\frac{kg}{m^2}\right)$

distance from center r

moment of inertia $(\text{mass})(\text{distance})^2$

So one little ring has moment of inertia

$$dI = \left(2\pi r \, dr\right)\left(7 \frac{kg}{m^2}\right)\left(r^2\right)$$

$$= \left(7 \frac{kg}{m^2}\right) 2\pi r^3 \, dr$$

$$= \left(14\pi \frac{kg}{m^2}\right) r^3 \, dr$$

The moment of inertia of all rings added together

$$I = \int_{r=0}^{r=2m} \left(14\pi \frac{kg}{m^2}\right) r^3 \, dr$$

$$= \left(14\pi \frac{kg}{m^2}\right) \int_0^{2m} r^3 \, dr$$

$$= \left(14\pi \frac{kg}{m^2}\right) \left| \frac{1}{4} r^4 \right|_0^{2m}$$

$$= \left(14\pi \frac{kg}{m^2}\right)\left(4 m^4 - 0 m^4\right)$$

$$= 176 \ kg \cdot m^2$$

Yes! This agrees with the quick-and-easy method.