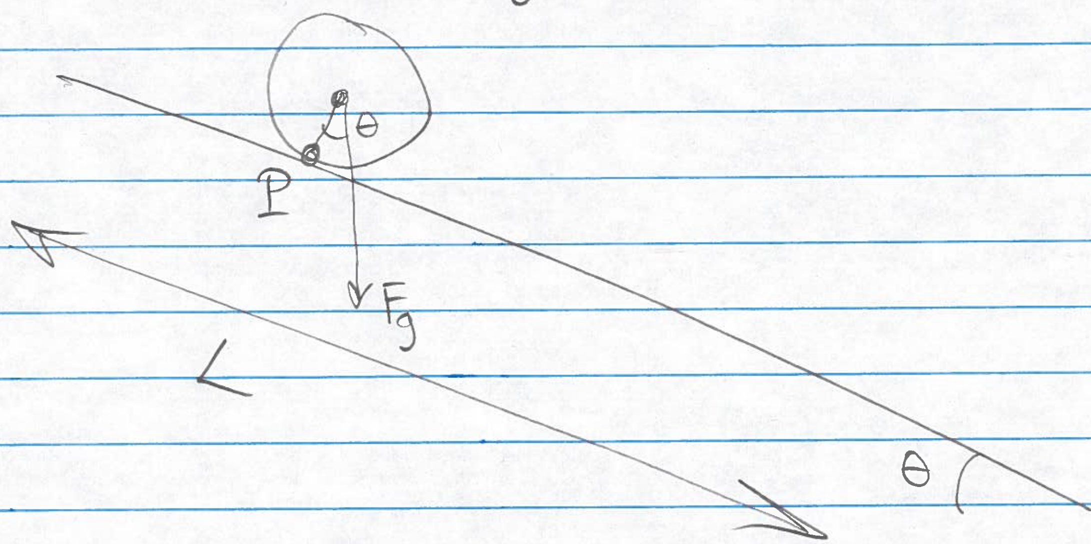
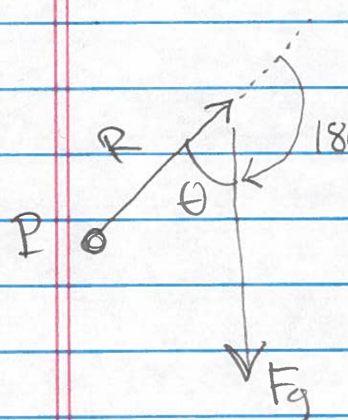
# W10B: Rolling motion

3/12/2026




Hollow ball rolls down ramp, angle $\theta$, length $L$. How long to reach the bottom?

Use torque around point $P$ where ball touches ramp.




$$\vec{\tau} = |r||F_g| \sin(\text{angle between})$$

$$= R \cdot Mg \cdot \sin(180° - \theta)$$

$$= RMg \sin\theta \quad \text{into page}$$

Now, moment of inertia around $P$ is

$$I = I_{com} + MR^2$$

$$= \frac{2}{3}MR^2 + MR^2$$

$$= \frac{5}{3}MR^2$$

So the angular acceleration is

$$\vec{\alpha} = \frac{\vec{\tau}}{I} = \frac{RMg\sin\theta}{\frac{5}{3}MR^2}$$

$$= \frac{3}{5}\frac{1}{R}g\sin\theta \quad \text{into page}$$

And that means the linear acceleration is

$$a = R\alpha = \frac{3}{5}g\sin\theta \quad \text{down ramp}$$

So if ramp has length L, the time to reach bottom is

$$(x_f - x_i) = V_i t + \frac{1}{2}at^2$$

$$L = 0 + \frac{1}{2}at^2$$

$$\rightarrow t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2L}{\frac{3}{5}g\sin\theta}}$$

$$\boxed{t = \sqrt{\frac{10}{3}\frac{L}{g\sin\theta}}}$$

do show this form, too → need it shortly

$$= \sqrt{\frac{10}{3}\frac{L^2}{gh}} \quad \text{since } \frac{h}{L} = \sin\theta$$

p3

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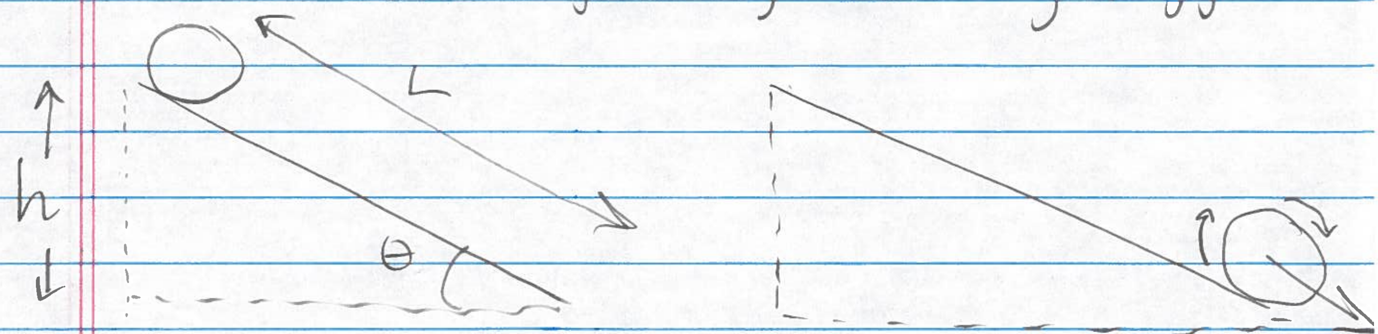
We can also analyze rolling motion using energy.




Ask students to lead class through the energy method

Before : $E_i = GPE + KE$

$= Mgh = Mg\,L\sin\theta$

After $E_f = GPE + KE$

$= 0 + \frac{1}{2}Mv_f^2 + \frac{1}{2}I_{com}\omega_f^2$

$= \frac{1}{2}Mv_f^2 + \frac{1}{2}\left[\frac{2}{3}MR^2\right]\omega_f^2$

But $R\omega_f = v_f$, so $R^2\omega_f^2 = v_f^2$

$\therefore\ E_f = \frac{1}{2}Mv_f^2 + \frac{1}{3}Mv_f^2 = \frac{5}{6}Mv_f^2$

Can solve for final speed $v_f$:

$E_i = E_f \Rightarrow Mgh = \frac{5}{6}Mv_f^2$

$v_f = \sqrt{\frac{6}{5}gh}$

to here

And now that we know the final speed, we can find the time to move down the ramp.

$$x_f - x_i = \frac{1}{2}(v_i + v_f)t$$

$$L = \frac{1}{2}(0 + v_f)t$$

$$\rightarrow t = \frac{2L}{v_f} = \frac{2L}{\sqrt{\frac{6}{5}gh}}$$

Instructor can do this part

Now a bit of algebra can make it similar to the value we derived earlier:

$$t = 2\sqrt{\frac{5}{6}} \cdot \frac{\sqrt{L^2}}{\sqrt{gh}}$$

$$= \sqrt{\frac{20}{5}} \cdot \sqrt{\frac{5}{6}} \sqrt{\frac{L^2}{gh}}$$

$$= \sqrt{\frac{10}{3}} \sqrt{\frac{L^2}{gh}} = \sqrt{\frac{10}{3} \frac{L^2}{gh}}$$

↑ Same as value derived by torque.

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Instructor's Cheat Sheet — not for students!

If an object with center of mass $I_{com}$

$$I_{com} = Q M R^2$$

rolls down a ramp of length L, height h, angle θ, then around contact point

$$I = (1+Q) M R^2$$

$$\alpha = \frac{R M g \sin\theta}{I} = \frac{1}{R} g \sin\theta \cdot \left(\frac{1}{1+Q}\right)$$

$$a = \left(\frac{1}{1+Q}\right) g \sin\theta = \left(\frac{1}{1+Q}\right) g \frac{h}{L}$$

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2(1+Q)L}{g \sin\theta}}$$

$$= \sqrt{\frac{2(1+Q)L^2}{gh}}$$

This ↗ will be handy for the "rolling race".

$$KE_{bottom} = \frac{1}{2}(1+Q) M v_f^2$$

$$v_f = \sqrt{2\left(\frac{1}{1+Q}\right) g h}$$

given $L = 100$ cm
$h = 2.4$ cm
$g = 980 \frac{cm}{s^2}$

Student Names: ____________________

<------- Use symbols only: M, R, g, L, h ------------------->

| Q | I+Q | Object | Moment of inertia | ~~Total KE at bottom~~ ang accel | Linear accel | Final velocity | Time to reach bottom | Time to reach bottom (sec) | Ranking |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{2}{5}$ | $\frac{7}{5}$ | Lacrosse Ball | $\frac{2}{5}MR^2$ | $\frac{5}{7}\frac{1}{R}g\frac{h}{L}$ | $\frac{5}{7}g\frac{h}{L}$ | $\sqrt{\frac{10}{7}gh}$ | $\sqrt{\frac{14}{5}\frac{L^2}{gh}}$ | 3.45 / 3.28 | 2 / 2 |
| $\frac{2}{3}$ | $\frac{5}{3}$ | Racquet Ball | $\frac{2}{3}MR^2$ | $\frac{3}{5}\frac{1}{R}g\frac{h}{L}$ | $\frac{3}{5}g\frac{h}{L}$ | $\sqrt{\frac{6}{5}gh}$ | $\sqrt{\frac{10}{3}\frac{L^2}{gh}}$ | 3.77 / 3.60 | 4 / 4 |
| $\frac{1}{2}$ | $\frac{3}{2}$ | Wooden Disk | $\frac{1}{2}MR^2$ | $\frac{2}{3}\frac{1}{R}g\frac{h}{L}$ | $\frac{2}{3}g\frac{h}{L}$ | $\sqrt{\frac{4}{3}gh}$ | $\sqrt{3\frac{L^2}{gh}}$ | 3.57 / 3.45 | 3 / 3 |
| 1 | 2 | Metal Hoop | $MR^2$ | $\frac{1}{2}\frac{1}{R}g\frac{h}{L}$ | $\frac{1}{2}g\frac{h}{L}$ | $\sqrt{gh}$ | $\sqrt{4\frac{L^2}{gh}}$ | 4.12 / 3.90 | 5 / 5 |
| 0 | 1 | Physics Cart | $\approx 0$ | 0 | $g\frac{h}{L}$ | $\sqrt{2gh}$ | $\sqrt{2\frac{L^2}{gh}}$ | 2.92 / 2.82 | 1 / 1 |

actual obs times

actual rank

racquet ball (hollow) 2.50, 2.47, 2.69

lacrosse ball (solid) 2.38, 2.47, 2.50

giant puck (solid)

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too steep! Need longer run times.

Change 4 blocks → 2 blocks

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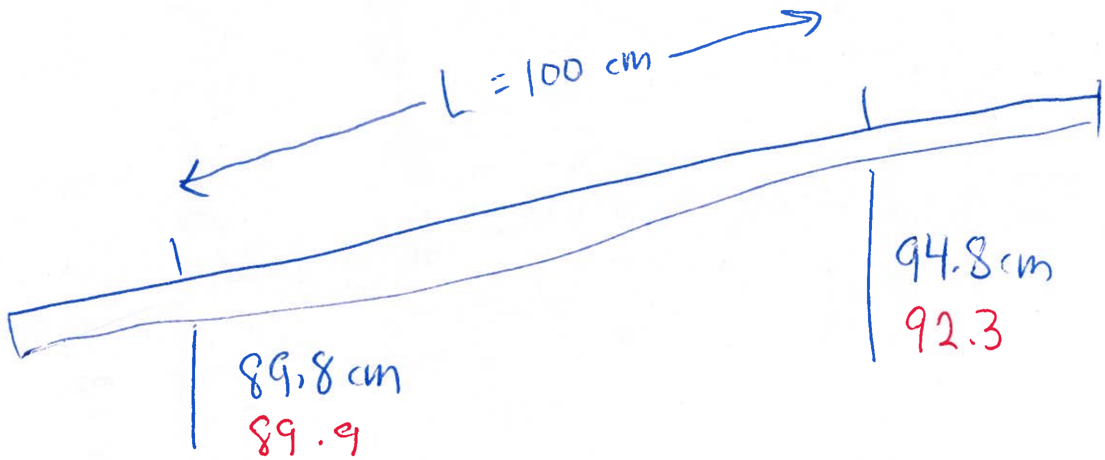
| | |
|---|---|
| * racquet ball (hollow) | 3.69, 3.57, 3.97, 3.53 |
| lacross ball (solid) | 3.44, 3.41, 3.28 |
| * giant puck (solid) | 3.44, 3.53?, 3.37 |
| * metal hoop (silver) | 3.90, 3.84, 3.97 |
| small metal hoop (rusty) | 3.69, 3.84, 4.03 (unreliable) |
| 50 g mass | 3.66, 3.63, 3.69 |
| * cue ball (solid) | 3.25, 3.28, 3.31 ← better |
| silver pend ball (solid) | 3.34, 3.38, 3.50 |

To move instructor's table, first unlatch rollers on table's legs.

Roll table out from podium a bit

Insert ~~4~~ 2 blocks under two legs each to ~~tt~~ tilt table.

Then relatch rollers to prevent table from moving




h = 5.0 cm 2.4

$\theta = \sin^{-1}\left(\frac{5.0\text{ cm}}{100.0\text{ cm}}\right) = 2.87^\circ$ $1.38^\circ$

Frictionless sliding $a = g\sin\theta =$ ~~287~~ 49.1 23.5 $\frac{\text{cm}}{s^2}$ $t = \sqrt{\frac{2L}{a}} = 2.02$ s 2.92

yellow car 2.66, 2.78, 2.50 (veers to right)

brown car 2.60, 2.60, 2.63 (runs straight) 5.57

* heavy cart 2.00, 2.00, 2.03 nice! 2.79, 2.82, 2.85