Spinning Wheel Uncertainty Calculations

Trial	Time from start	Time for 3 rev	Angular Speed	Ang Accel
1	0 s	1.68 ± 0.1 s	11.22 ± 0.67 $\frac{rad}{s}$	
				$-0.22\ \frac{rad}{s^2}$
2	10	2.10 ± 0.1	8.98 ± 0.43	
3	20	2.31 ± 0.1	8.16 ± 0.35	
				$-0.04\ \frac{rad}{s}$
4	30	2.43 ± 0.1	7.76 ± 0.32	

Angular speed $\omega = \dfrac{\text{angle displaced}}{\text{time}} = \dfrac{3 \times 2\pi \text{ rad}}{1.68 \text{ s}} = 11.22\ \dfrac{\text{rad}}{\text{s}}$

Uncert in ang speed $\dfrac{\Delta\omega}{\omega} = \dfrac{\Delta(\text{angle})}{\text{angle}} + \dfrac{\Delta(\text{time})}{\text{time}}$

$= \dfrac{0}{6\pi \text{ rad}} + \dfrac{0.1 \text{ s}}{1.68 \text{ s}} = 0.060$

$\rightarrow \Delta\omega = \omega(0.060) = 11.22\ \dfrac{\text{rad}}{\text{s}}(0.060) = 0.67\ \dfrac{\text{rad}}{\text{s}}$

Angular accel $\alpha = \dfrac{\text{change in ang speed}}{\text{change in time}} = \dfrac{(8.98 - 11.22) \text{ rad/s}}{(10-0) \text{ s}}$

$= -0.22\ \dfrac{\text{rad}}{\text{s}^2}$

But how to find uncertainty in angular acceleration?

Consider the first two trials.

$$\omega_1 = 11.22 \pm 0.67 \ \frac{rad}{s} \qquad t_1 = 0 \ s$$

$$\omega_2 = 8.98 \pm 0.48 \ \frac{rad}{s} \qquad t_2 = 10 \ s$$

$$\alpha = \frac{\text{change in angular speed}}{\text{change in time}}$$

a division of two measured quantities

So

$$\frac{\text{uncert in } \alpha}{\alpha} = \left| \frac{\text{uncert in } [\omega_2 - \omega_1]}{[\omega_2 - \omega_1]} \right| + \left| \frac{\text{uncert in } [t_2 - t_1]}{[t_2 - t_1]} \right|$$

$$= \left| \frac{0.48 \ \frac{rad}{s} + 0.67 \ \frac{rad}{s}}{(8.98 - 11.22) \ \frac{rad}{s}} \right| + \left| \frac{0 \ s}{10 \ s} \right|$$

$$= \left| \frac{1.15 \ \frac{rad}{s}}{-2.24 \ \frac{rad}{s}} \right| + 0$$

$$= 0.51$$

$$\rightarrow \text{uncert in } \alpha = \alpha \cdot 0.51 = -0.22 \ \frac{rad}{s^2} \cdot 0.51$$

$$= \pm 0.11 \ \frac{rad}{s^2}$$

$$\rightarrow \boxed{\alpha = -0.22 \pm 0.11 \ \frac{rad}{s^2}}$$

Likewise, for second calculated angular acceleration

$$\omega_1 = 8.16 \pm 0.35 \ \frac{rad}{s} \qquad t_1 = 20\,s$$

$$\omega_2 = 7.76 \pm 0.32 \ \frac{rad}{s} \qquad t_2 = 30\,s$$

So

$$\frac{\text{uncert in } \alpha}{\alpha} = \left| \frac{\text{uncert in } [\omega_2 - \omega_1]}{\omega_2 - \omega_1} \right| + \left| \frac{\text{uncert in } [t_2 - t_1]}{t_2 - t_1} \right|$$

$$= \left| \frac{(0.35 + 0.32)\ \frac{rad}{s}}{-0.40\ \frac{rad}{s}} \right| + \left| \frac{0\,s}{10\,s} \right|$$

$$= \left| \frac{0.67\ \frac{rad}{s}}{-0.40\ \frac{rad}{s}} \right| + 0$$

$$= 1.68$$

$$\rightarrow \text{uncert in } \alpha = \alpha \cdot 1.68 = -0.04\ \frac{rad}{s^2} \cdot 1.68$$

$$= \pm\ 0.07\ \frac{rad}{s^2}$$

$$\rightarrow \boxed{\alpha = -0.04 \pm 0.07\ \frac{rad}{s^2}}$$

Yes, this is consistent with "no acceleration"