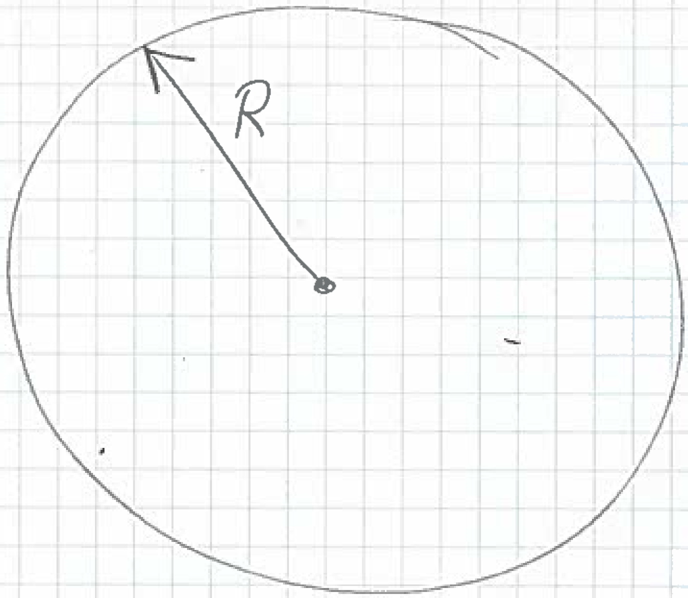

A giant sphere of solid gold has a mass

$$M = (2.06 \pm 0.21) \times 10^7 \text{ kg}$$

What is the radius of this sphere?

Look up density of gold, and find

$$\rho = 19{,}300 \pm 200 \ \frac{\text{kg}}{\text{m}^3}$$

The connection between mass, radius and density is

$$M = \frac{4}{3}\pi R^3 \rho$$

Re-arranging,

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

We can determine the radius of the sphere:

$$R = \left(\frac{3 \cdot 2.06 \times 10^7 \text{ kg}}{4\pi\,(19{,}300 \text{ kg/m}^3)}\right)^{1/3}$$

$$= 6.34 \text{ m}$$

But what is the uncertainty in radius, ΔR?

Recall

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

Since this looks complicated, let's find the uncertainty in two steps:

a) the uncertainty in the expression inside the parantheses – without the $1/3$ power

b) the uncertainty of the whole thing, including the $1/3$ power.

To simplify, we'll give the expression inside the parantheses a new name. How about Q?

The quantity inside the parantheses can be called

$$Q = \frac{3M}{4\pi\rho}$$

Division rule says

$$\frac{\Delta Q}{Q} = \frac{\Delta M}{M} + \frac{\Delta\rho}{\rho}$$

$$= \frac{0.21 \times 10^{7}\ \text{kg}}{2.06 \times 10^{7}\ \text{kg}} + \frac{200\ \frac{\text{kg}}{\text{m}^3}}{19{,}300\ \frac{\text{kg}}{\text{m}^3}}$$

$$= 0.102 + 0.010$$

So

$$\frac{\Delta Q}{Q} = 0.112 = 11.2\%$$

Now, the radius of the sphere is

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = (Q)^{1/3}$$

So the uncertainty in radius R can be found via

$$\frac{\Delta R}{R} = \frac{\Delta Q}{Q} \cdot \frac{1}{3}$$

$$= (0.112) \cdot \frac{1}{3} = 0.037$$

Only 3.7%

Now we can finally determine the uncertainty in the radius, ΔR:

$$\Delta R = R\,(0.037)$$

$$= (6.34\ m)(0.037) = 0.23\ m$$

Thus, the radius of the gold sphere is

$$\boxed{R = 6.34 \pm 0.23\ m \quad \text{or} \quad 6.3 \pm 0.2\ m}$$