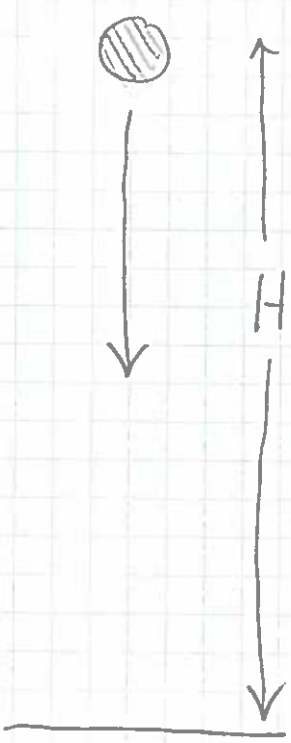
Bob drops a ball. It falls a distance H in a time $t = 0.87 \pm 0.03$ seconds.

How far is H? What is uncertainty in H?



First, we need the equation for H in terms of t:

$$H = \frac{1}{2} g t^2$$

↳ has uncertainty

↳ no uncertainty

↳ no uncertainty

$\frac{1}{2} = \frac{1}{2} \pm 0$ no uncertainty

$g = 9.8 \frac{m}{s^2} \pm 0 \frac{m}{s^2}$ no uncertainty

$t = 0.87\,s \pm 0.03\,s$ yes, uncertainty exists

The rule for multiplying factors says

$$\frac{\Delta(H)}{H} = \frac{\Delta(\frac{1}{2})}{\frac{1}{2}} + \frac{\Delta(g)}{g} + \frac{\Delta(t)}{t}$$

$$= \frac{0}{\frac{1}{2}} + \frac{0\ m/s^2}{9.8\ m/s^2} + \frac{0.03\,s}{0.87\,s}$$

$$\frac{\Delta H}{H} = 0 + 0 + 0.034$$

So we can compute the uncertainty in the height H of the ball's displacement:

$$\Delta H = H(0.034)$$

But what is the value H? We compute its value ignoring uncertainties:

$$H = \frac{1}{2} g t^2 = \frac{1}{2}\left(9.8 \frac{m}{s^2}\right)(0.87 s)^2$$

$$= 4.57 \text{ m}$$

Now we can compute uncertainty

$$\Delta H = (4.57 \text{ m})(0.034)$$

$$= 0.16 \text{ m}$$

$\rightarrow$ $\boxed{H = 4.57 \pm 0.16 \text{ m}}$