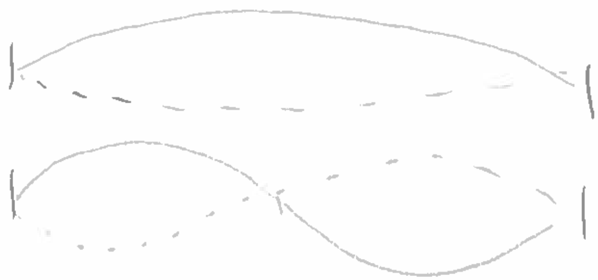
Waves and musical instruments

Many musical instruments are built around <u>waves travelling on strings</u>: piano, violin, guitar, harp, etc.



$\lambda = 2L$

$\lambda = L$

Given a string of
mass m
length L
$\rightarrow$ linear mass dens μ
tension T

If we pluck the string, we set up waves going back and forth between the fixed ends. The interference of those oppositely-directed waves creates standing waves, as shown above. But, crucially,

only certain wavelengths may exist

$\rightarrow$ only certain frequencies will be created

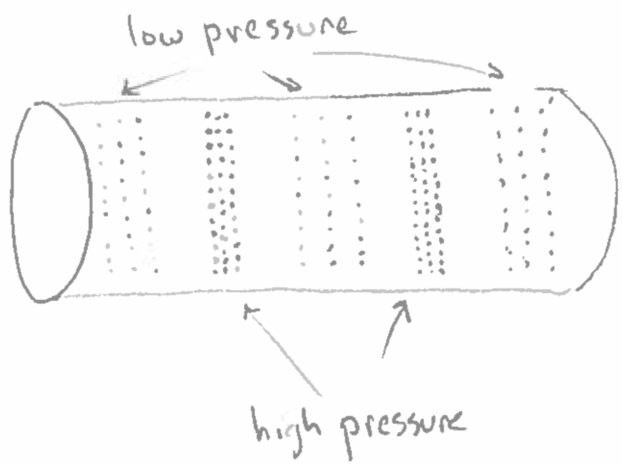
In the case of a string of length L, we have

$$\left.\begin{aligned} \lambda_N &= \frac{2L}{N} \\ v &= \sqrt{\frac{T}{\mu}} \end{aligned}\right\} \quad f_N = \frac{v}{\lambda_N} = \left[\frac{\sqrt{\frac{T}{\mu}}}{2L}\right] N$$

so if $f_1 = 100\ \text{Hz}$
$f_2 = 200\ \text{Hz}$
$f_3 = 300\ \text{Hz}$
$\vdots$

A real string, attached to a real frame, may yield some combination of these permitted frequencies.

But many instruments are designed around a different principle: oscillations of air inside a hollow tube. We call these <u>wind</u> instruments; flute, horn, pipe organ, etc.



A sound wave inside the tube means that the pressure of air varies from place to place, and as a function of time.

$$\text{Pressure}(x,t) = A\cos(kx - \omega t)$$

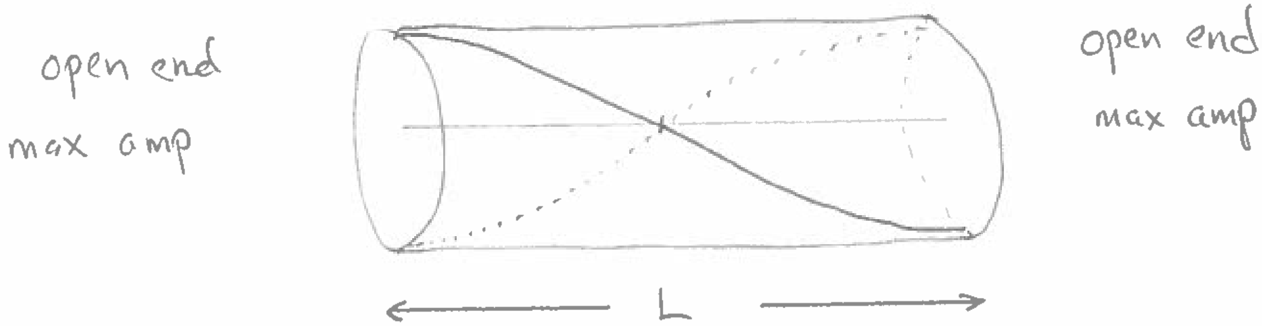
A wave travelling to the right, meeting a wave travelling to the left, will create a standing wave inside the tube:

$$\text{Pressure}(x,t) = A\cos(kx)\cos(\omega t)$$

Just as a standing wave on a string has rules – "Motion must be zero at each end" – a standing wave of air in a tube also has rules:

1) if one end of the tube is open, pressure must have a maximum amplitude
2) if one end of the tube is closed, pressure must have zero amplitude

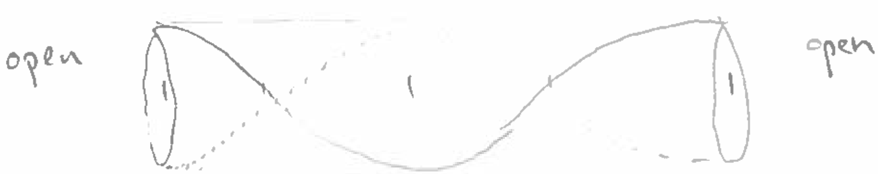
We can draw waves in a tube if we follow a convention: use vertical position to represent amplitude of pressure variations.



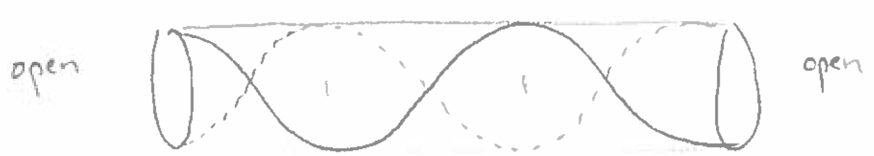
So, for example, a tube of length L open at both ends is shown above. A standing wave in the tube must have maximum amplitude at both ends; in other words, must have an anti-node at each end.

What is the wavelength of this wave?

$$\lambda = 2L$$



$$\lambda = L$$



$$\lambda = \frac{2L}{3}$$

Other wavelengths are possible, too: in fact, the sequence should look familiar: $\lambda_N = \frac{2L}{N}$

What are the frequencies of these waves?

Well, these are waves in air - disturbances in pressure that we perceive as sound. They travel at

$$V_s = \text{speed of sound in air} = 343 \tfrac{m}{s}$$

So the frequency for any given wavelength is

$$f = \frac{V_s}{\lambda}$$

$N =$	$\lambda =$	$f =$
1	$2L$	$V_s/2L$
2	L	V_s/L
3	$2L/3$	$3V_s/2L$
4	$L/2$	$2V_s/L$

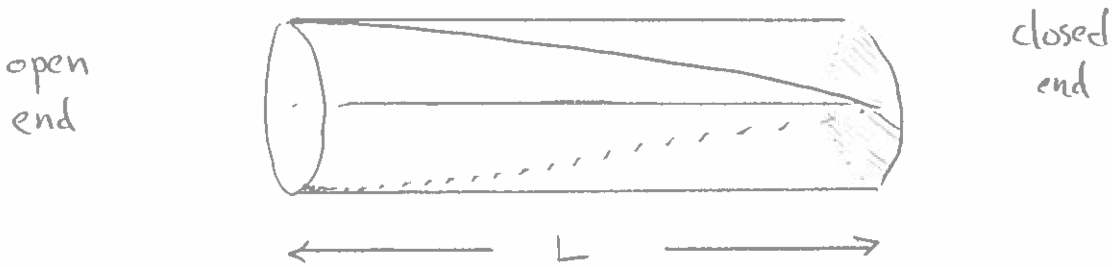
Suppose a piccolo has a tube of length $L = 30$ cm, open at both ends. What is the lowest frequency it can play?

$$f_1 = \frac{V_s}{2L} = \frac{343 \text{ m/s}}{2(0.30\text{ m})} = 572 \text{ Hz}$$

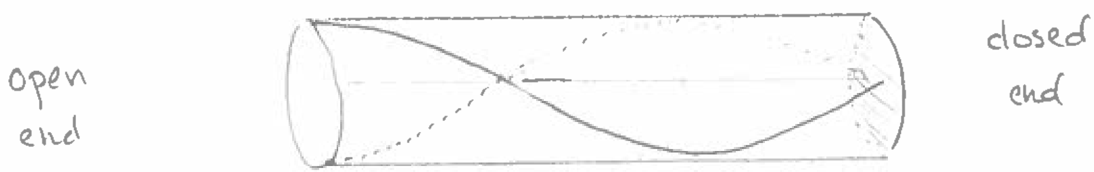
But tubes may not be open at both ends. Suppose a tube is closed at one end. Then a schematic of a standing wave inside it must show

max amplitude at open end

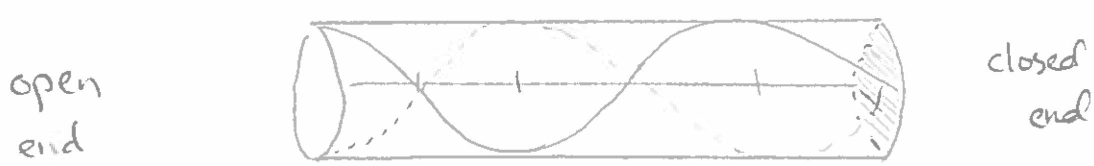
min amplitude at closed end



The wavelength of this standing wave is $4L$.



The next-longest standing wave has $\lambda = \frac{4}{3}L$



And the next has wavelength $\lambda = \frac{4L}{5}$

For tubes with one closed end, the permitted wavelengths have a new pattern:

$$\lambda_N = \frac{4L}{N}$$

Let's compare the frequencies which might be created by two similar wind instruments. Each has a tube of length $L = 3.43$ m long, but one is open at both ends, and the other only at one end.

N	open at both ends λ	open at both ends f	open at only one end λ	open at only one end f
1	6.86 m	50 Hz	13.72 m	25 Hz
2	3.43	100	4.57	75
3	2.29	150	2.74	125
4	1.72	200	1.96	175
	⋮	⋮	⋮	⋮
N		$50 \cdot N$		$25 \cdot (2N-1)$

Note the different pattern in the permitted frequencies: the tube open at both ends has all multiples of the fundamental, while the tube open at only one end has only odd multiples...

Could that affect the overall "feel" of the instrument? Yes!