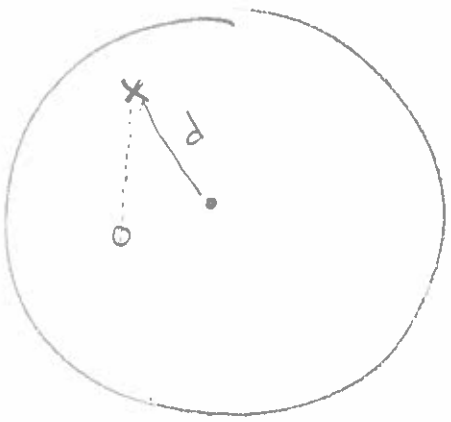
Analyzing a pendulum via torques



An object of mass M and moment of inertia

I_{com} around its center of mass

is suspended from a pivot located distance d from center of mass.

We call this a "physical pendulum".

a) What is moment of inertia around pivot?

$$I = I_{com} + Md^2$$

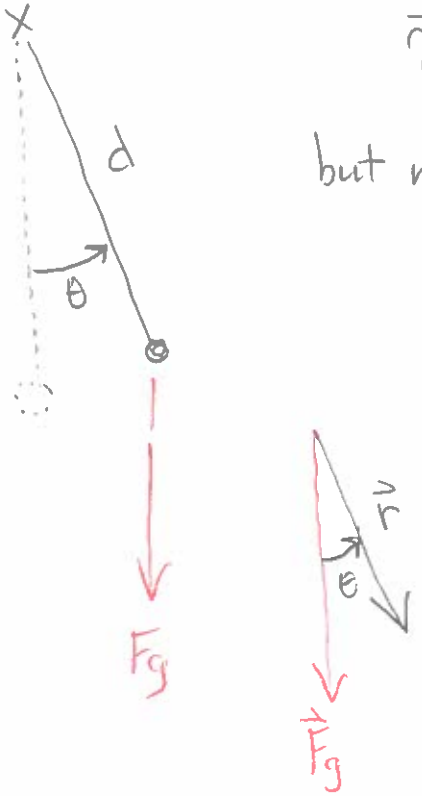
b) If object is swung by angle θ away from vertical, what is torque around pivot?

$$\vec{\tau} = \vec{r} \times \vec{F} = |d||Mg| \sin\theta \quad \underline{\text{into}}$$

but note angle displaced is

$$\vec{\theta} = \theta \quad \underline{\text{out}}$$

So $\vec{\tau}$ is opposite in dir to $\vec{\theta}$



c) what is angular acceleration of object when we release it?

$$\vec{\alpha} = \frac{\vec{\tau}}{I} = \frac{dMg\sin\theta}{I_{com} + Md^2}$$ into

But if angle θ is small,

$$\sin\theta \approx \theta$$

$$\rightarrow \vec{\alpha} \cong -\left(\frac{dMg}{I_{com} + Md^2}\right)\vec{\theta}$$

remember
$\vec{\alpha}$ goes in
$\vec{\theta}$ goes out

But $\alpha = \frac{d^2\theta}{dt^2}$

$$\rightarrow \frac{d^2\theta}{dt^2} = -\left(\frac{dMg}{I_{com} + Md^2}\right)\theta$$

$$\rightarrow \theta(t) = A\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{dMg}{I_{com} + Md^2}}$$

$$\rightarrow \text{Period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_{com} + Md^2}{dMg}}$$