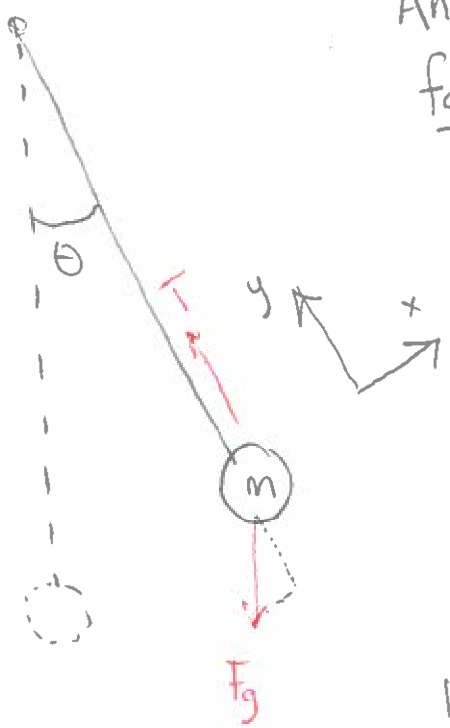
Analyzing a pendulum using forces.

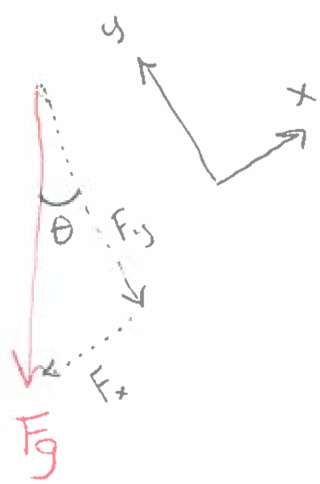


Tilt coord axis:

force	x	y
string	0	$+T$
grav	$-mg\sin\theta$	$-mg\cos\theta$
total	$-ma_x$	$ma_y = 0$

We see the tension in string must be

$$T = mg\cos\theta$$

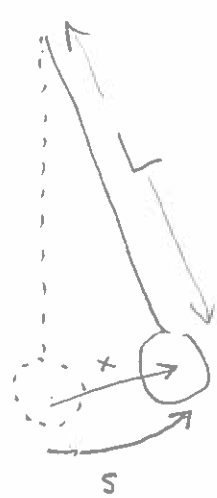


But we also see the force in x-dir is

$$F_x = -mg\sin\theta$$

So

$$a_x = -g\sin\theta$$



If the angle θ is small, then the linear displacement x from lowest point is almost the same as the arc length s.

$$x \approx s = L\theta \qquad (\theta \text{ in rad})$$

$$\rightarrow \frac{x}{L} \approx \theta$$

But for small angles, we know

$$\sin\theta \approx \theta \qquad \theta \text{ in rad}$$

So

$$a_x = -g\sin\theta \approx -g\theta \approx -g\frac{x}{L}$$

In other words, a small displacement from lowest point leads to a small acceleration back; larger displacement means larger acceleration back.

Hey!

$$a_x = \frac{d^2x}{dt^2}$$

So

$$\frac{d^2x}{dt^2} = -\left(\frac{g}{L}\right)x$$

This looks like ... the equation of simple harmonic motion!

$$x(t) = A\cos(\omega t + \phi)$$

depends on initial displacement

$$\omega = \sqrt{\frac{g}{L}}$$

depends on when we let go

So Period $P = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

So for a simple pendulum

$$P = 2\pi\sqrt{\frac{L}{g}}$$

I guess

$$P^2 = 4\pi^2 \frac{L}{g}$$

Can you use this information to determine "g" from your measurements?

$$P^2 = \left(\frac{4\pi^2}{g}\right) L$$

slope in graph

$$\rightarrow \quad g = \left(\frac{4\pi^2}{\text{slope}}\right)$$

check:

slope units $\frac{s^2}{m}$

so $\frac{1}{\text{slope}}$ $\frac{m}{s^2}$ ✓

Compute g; add error bars based on your data.