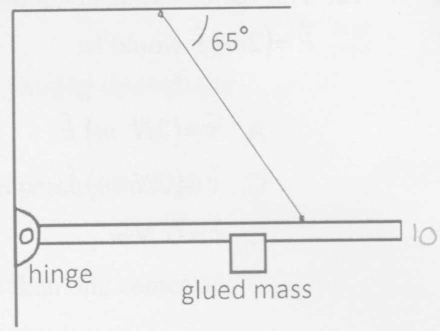
Multiple choice

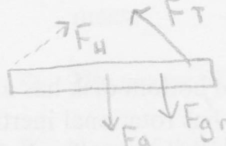
1. B
2. D
3. E
4. A
5. C
6. B
7. B
8. E
9. A
10. B
11. C
12. B
13. A

Show all work completely, legibly and in logical order, starting with basic concepts.

14. A long uniform horizontal rod of total length 10.0 m and total mass 150 kg it attached to a wall by a hinge (pivot). It is supported by a wire attached at 7.00 m from the hinge. The wire makes and angle of 65° with the ceiling as shown.



The maximum tension allowed in the wire before it will break is 2016 N. You want to glue a block of mass 60.0 kg on the rod as far away from the hinge as possible.



$\Sigma T = 0 \quad \Sigma F = 0$

a) **(14 points)** Calculate the maximum distance away from the hinge that you can glue the block.

$\Sigma T = F_T R \sin\theta - F_{gm} d - F_g R_2 = 0$

$= 2016(7)\sin 65 = 150(9.8)5 + 60(9.8)d$

$12789 = 7350 - 588d$

$5439.82 = 588d$

$\boxed{d = 9.25 m}$

b) **(6 points)** When the block is in this maximum position, calculate the net force acting on the rod by the hinge. Leave the answer in unit vector (Cartesian) components.

$\Sigma F_x = 0$

$F_{Hx} - F_{Tx} = 0$

$F_{HX} = F_T \cos\theta$

$F_{HX} = 2016\cos(65)$

$F_{Hx} = 852 N$

$\Sigma F_y = 0$

$F_{TY} + F_{HY} - F_g - F_{gm} = 0$

$2016 \sin 65 + F_{HY} = 150(9.8) + 60(9.8)$

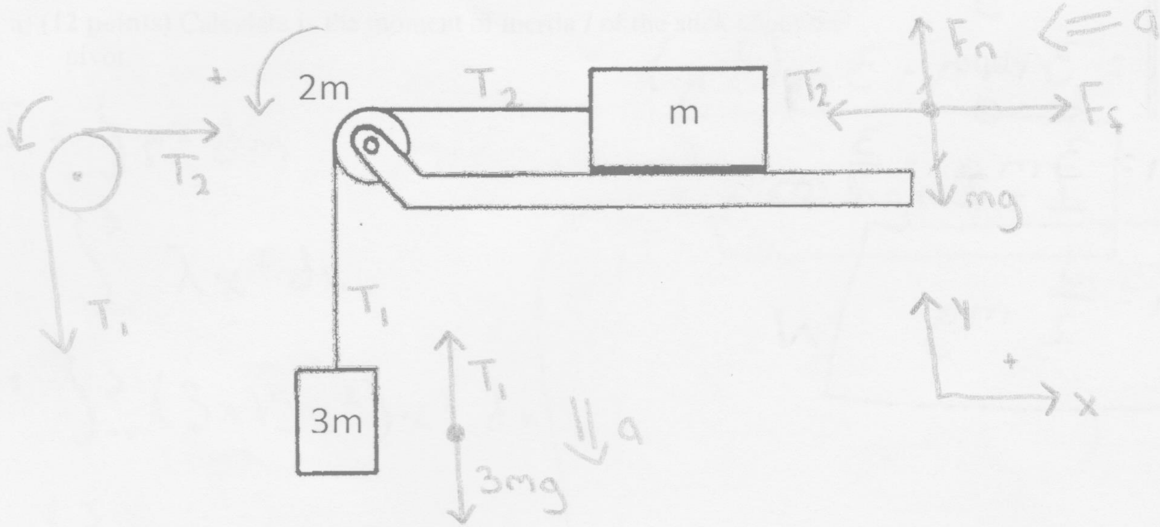
$1827 + F_{HY} = 1470 + 588$

$F_{HY} = 230 N$

$F_H = (852, 230) N$

15. A modified Atwood's machine consists of a block of mass m sliding on a rough table with kinetic friction coefficient 0.5. A hanging mass $3m$ pulls it by a massless string. The string passes over a pulley of mass $2m$. The pulley is a uniform disk of radius R. There is no slipping or sliding of the string over the pulley.

It is required to draw all relevant free body diagrams for the two masses and the pulley.



a) **(8 points)** Find the acceleration of the system when released from rest, as a multiple of the gravitational acceleration g.

$\sum T = I\alpha$

$T_1R - T_2R = I\alpha$

$T_1R - T_2R = \frac{1}{2}2mR^2\alpha$

$T_1 - T_2 = mR\alpha$

$\sum F_y = T_1 - 3mg = -3ma$

$T_1 = 3mg - 3ma$

$\sum F_x = F_f - T_2 = -ma$

$0.5mg - T_2 = -ma$

$T_2 = ma + \frac{1}{2}mg$

$3\cancel{m}g - 3\cancel{m}a - \cancel{m}a - \frac{1}{2}\cancel{m}g = \cancel{m}\frac{\cancel{R}a}{R}$

$3g - 3a - a - \frac{1}{2}g = a$

$3g - \frac{1}{2}g = 5a$

$2.5g = 5a$

$\boxed{a = \frac{1}{2}g}$

* a in m/s²

(Continued next page →)

b) **(6 points)** Find the tension on the vertical part of the string as a multiple of *mg*.

(Find T_1)

$T_1 = 3mg - 3ma$

$T_1 = 3mg - 3m(\frac{1}{2}g)$

$T_1 = 3mg - \frac{3}{2}mg$

$\boxed{T_1 = \frac{3}{2}mg}$ N

c) **(6 points)** Find the tension on the horizontal part of the string as a multiple of *mg*.

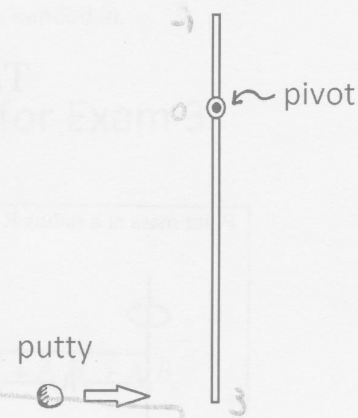
(Find T_2)

$T_2 = ma + \frac{1}{2}mg$

$T_2 = m(\frac{1}{2}g) + \frac{1}{2}mg$

$\boxed{T_2 = mg}$ N

16. A long thin rod of total length 4.00 m is pivoted 1.00 m from the top end. It is hung vertically and initially at rest as shown. The rod has a non-linear mass density given by $\boldsymbol{\lambda} = (\mathbf{3} + \mathbf{15}\boldsymbol{x}^2)$ kg/m, where $\boldsymbol{x} = \mathbf{0}$ is at the pivot. A small piece of putty of mass 0.200 kg is shot horizontally into the lower end of the rod, striking it with a speed of 20.0 m/s. The putty sticks to the rod.



a) **(12 points)** Calculate is the moment of inertia I of the stick about the pivot.

$$I = \int r^2\, dm$$
$$= \int_{-1}^{3} \lambda x^2\, dx$$
$$= \int_{-1}^{3} (3 + 15x^2)x^2\, dx$$
$$= \int_{-1}^{3} 3x^2 + 15x^4\, dx$$

$$\boxed{760 \text{ Kgm}^2 = I}$$

b) **(6 points)** Calculate the angular velocity ω of the stick the instant after the putty strikes and sticks to it.

$$\vec{L}_i = \vec{L}_f$$
$$(r \times p) + I\omega_i = (I_1 + I_2)\omega_f$$
$$(3 \cdot .2 \cdot 20) + 0 = (760 + 1.8)\omega_f$$
$$12 = 761.8\,\omega_f$$

$\boxed{\omega_f = 0.016 \text{ rad/sec out of page}}$

c) **(2 points)** If you were asked to find the maximum angle that the rod/putty combination would make with the vertical after the collision, explain <u>in words</u> how you would solve this problem. Do not actually solve. Maximum 3 sentences.

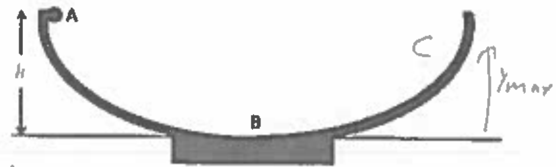
I would use conservation of energy. KE_{ROTATE} just after the collision would equal U_g at its max. Then trig can find θ max.

4)

$h = 0.15\,m$

$y_{max} = ?? \le h$

A uniform solid marble of mass $m = 0.020$ kg and diameter 0.010 m rolls without sliding down a symmetric steel bowl, starting from rest at point A, at the top of the left side. The top of each side is a distance $h = 0.15$ m above the bottom of the bowl. The left half of the bowl is rough enough to cause the marble to roll without slipping, but the right half is frictionless because it is coated with oil. Assume that there is no air resistance and no loss of energy due to kinetic friction in this problem.



(a) Use energy techniques to determine the translational speed v of the center of mass of the marble when it is a point B at the bottom of the bowl.

@ B   No nonconservative forces.

$K_t = \frac{1}{2} m v^2$

$K_R = \frac{1}{2} I \omega^2 \quad , \quad v = R\omega \quad I = \frac{2}{5} m R^2$

$= \frac{1}{2}\left(\frac{2}{5}\right) m R^2 \left(\frac{v}{R}\right)^2 = \frac{1}{5} m v^2$

$U = 0$

@ A

$K_t = 0 \quad , \quad K_R = 0$

$U = mgh$

$E_B = K_t + K_R + U = \frac{1}{2} m v^2 + \frac{1}{5} m v^2 + 0$

$E_A = K_t + K_R + U$
$= 0 + 0 + mgh$

conserve mechanical energy $\quad E_A = E_B$

$\cancel{m} g h = \cancel{m} v^2 \left(\frac{1}{2} + \frac{1}{5}\right)$

$\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

$v = \sqrt{\frac{10}{7} g h} = 1.45\,m/s$ ANS

(b) The transition from a rough to a smooth surface just after point B changes the marble's motion. It's rotational motion and translational motion are no longer constrained to change together. However both motions will still continue on into the right side of the bowl where there is no friction to stop either one. Use energy techniques to determine the maximum height y_{max} that the marble reaches on the right side of the bowl.

@ C   Rotational motion does not change → no external torque.

$K_t = \frac{1}{2} m v_c^2 \quad$ but $v_c = 0$ @ y_{max}

$K_R = \frac{1}{2} I \omega^2 \quad$ ω is same as at B; doesn't change on oil covered side since no friction.

$U = m g y_{max}$

Conserve Mechanical energy:

$\underline{E_B = E_C}$

$E_C = K_R + U$

$\frac{1}{2} m v^2 + \frac{1}{2} I\omega^2 = m g y_{max} + \frac{1}{2} I \omega^2$

$\frac{1}{2} \cancel{m} v^2 = \cancel{m} g y_{max} \Rightarrow y_{max} = \frac{v^2}{2g} = \frac{\frac{10}{7} g h}{2g} = \frac{5}{7} h = \frac{5}{7}(0.15\,m)$

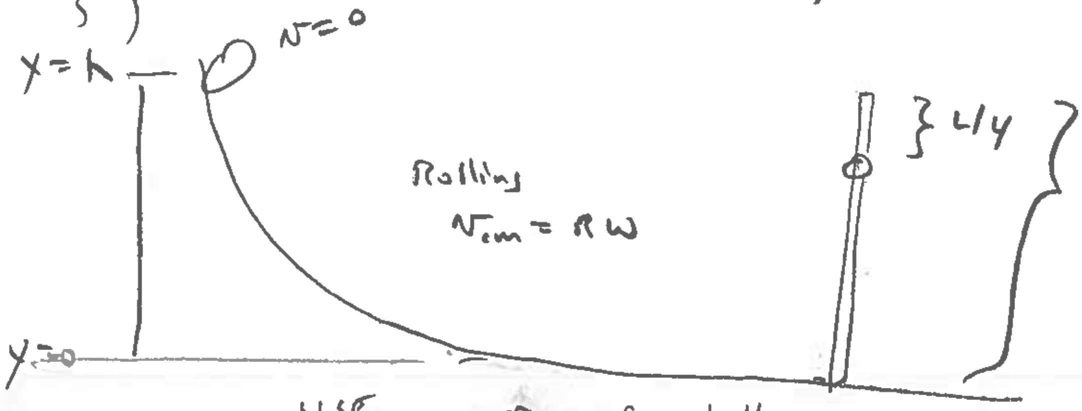
$= 0.107\,m$

ANS

4-1

5)

Hollow sphere $I = \frac{2}{3} m R^2$



Rolling

$v_{cm} = R\omega$

USE $v = v_{cm}$ for ball

$K_i = 0$

$U_i = mgh$

$$K_f = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{2}\left(\frac{2}{3} m R^2\right)\left(\frac{v}{R}\right)^2$$

$$= m v^2 \left(\frac{1}{2} + \frac{1}{3}\right)$$

$$= \frac{5}{6} m v^2$$

$$U_f = 0.$$

Rod: $I_{axis} = I_{cm} + m\left(\frac{L}{4}\right)^2$

$$= \frac{1}{12} m L^2 + \frac{1}{16} m L^2$$

$$I_{axis} = \frac{7}{48} m L^2$$

$$0 = \Delta K + \Delta U$$

$$= \left(\frac{5}{6} \cancel{m} v^2 - 0\right) + \left(0 - \cancel{m} g h\right)$$

$$\boxed{v = \sqrt{\frac{6}{5} g h}}$$

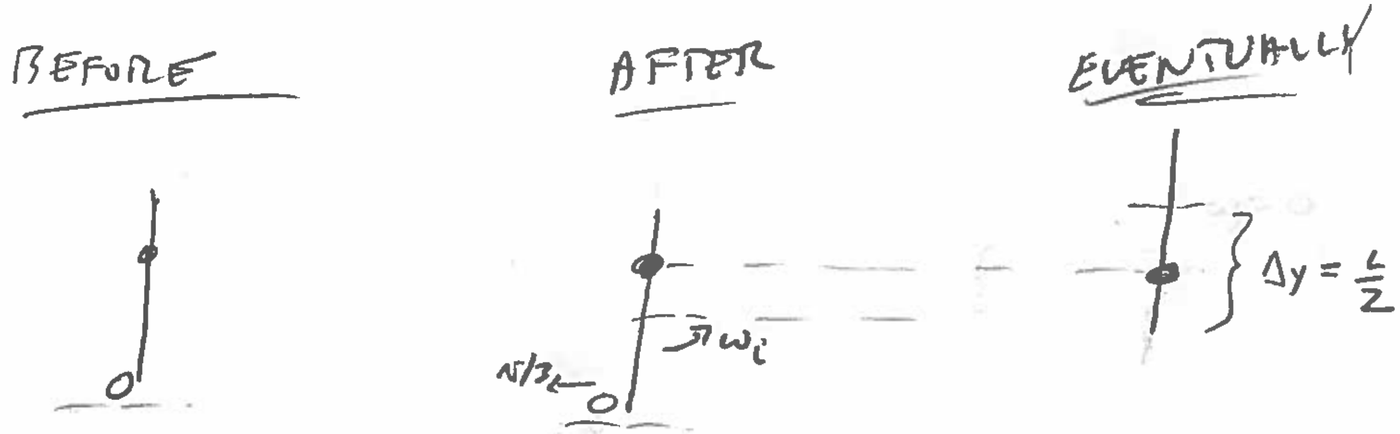
Check units

$$\sqrt{m/s^2 \, m} = m/s \checkmark$$

ENERGY CONSERVATION ON RAMP GIVES INITIAL CONDITION FOR COLLISION OF BALL & ROD

$v_{ball} = v$ $\omega_{Rod} = 0$

so $\omega_{ball} = v/R$



PLAN

i) USE CONSERVATION OF ENERGY TO FIND ω_i for Rod immediately after crash

ii) USE CONSERVATION OF ANGULAR MOMENTUM TO RELATE ω_i & $v/3$ TO v

USE $L = I\omega$ FOR BALL SINCE IT IS A RIGID BODY

AND $v_{cm} = R\omega$ since it rolls

– can't use $\vec{L} = \vec{r} \times \vec{p}$ (not a point particle)

i) No Translation

$$\frac{1}{2} I \cancel{\omega_f^2}^{\,0} + mg\,\Delta y = \frac{1}{2} I \omega_i^2$$

$$\cancel{m} g (L/2) = \frac{1}{\cancel{2}}\left(\frac{7}{48} \cancel{m} L^2\right) \omega_i^2$$

$$\omega_i = \sqrt{\frac{48}{7}\frac{g}{L}}$$

check units

$$\sqrt{\frac{m/s^2}{m}} = 1/s$$

$\therefore$ rad/s ✓

5-2

ii) $\Sigma L_i = \Sigma L_f$

from energy techniques — rod — negative (rebound) — from energy techniques

$$\underbrace{I_{Ball}\,\omega_{Ball} + 0}_{\text{Before}} = \underbrace{I_{ball}\,\omega_{ball} + I_{rod}\,\omega_i}_{\text{After}}$$

$$\left(\frac{2}{3}mR^2\right)\left(\frac{v}{R}\right) = \left(\frac{2}{3}mR^2\right)\left(-\frac{v}{3R}\right) + \left(\frac{7}{48}mL^2\right)\left(\frac{48}{7}\frac{g}{L}\right)^{1/2}$$

$$\left(\frac{2}{3}mR^2\right)\left(\frac{1}{R}\right)\underbrace{\left(1+\frac{1}{3}\right)}_{4/3} v = \sqrt{\frac{7}{48}}\, m L^{3/2} g^{1/2}$$

$$\frac{8}{9} vR = \sqrt{\frac{7}{48}\, g L^3}$$

from before:

$v^2 = \frac{6}{5} g h$

$$\left(\frac{8}{9}R\right)^2 v^2 = \frac{7}{48} g L^3$$

$$\left(\frac{5}{6}\right)\left(\frac{81}{64}\right)\frac{1}{R^2}\,\cancel{\frac{64}{81}}\,\cancel{R^2}\,\frac{6}{5}\cancel{g}\,h = \frac{7}{48}\cancel{g}\,L^3\,\frac{1}{R^2}\left(\frac{81}{64}\right)\left(\frac{5}{6}\right)$$

$$\therefore \boxed{h = \frac{315}{2048}\frac{L^3}{R^2}}$$

Ans

OR

$$h = 0.154\,\frac{L^3}{R^2}$$

check units

$m = \frac{m^3}{m^2} = m$ ✓