UP1 Equation Sheet for Exams

Linear Motion:

Vector addition: If $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$

then $\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$

and $|\vec{C}| = \sqrt{{C_x}^2 + {C_y}^2}$ and $\theta = \tan^{-}1\left(\frac{C_y}{C_x}\right)$

1D/2D Kinematics $v_x = \frac{dx}{dt}$, $a_x = \frac{dv_x}{dt} \longrightarrow x = \int v\ dt$, $v = \int a\ dt$

For Constant Acceleration:

$v_{xf} = v_{xi} + a_x\ t$; $x_f = x_i + v_i\ t + \frac{1}{2}a_x\ t^2$; $v_{xf}^2 = v_{xi}^2 + 2\ a_x\ (x_f - x_i)$; $x_f = x_i + \frac{1}{2}(v_{ix} + v_{fx})\ t$

For Constant Velocity: $\Delta x = v_x\ t$ **Centripetal Acceleration:** $a_{cent} = \frac{v^2}{r}$

Newton's 2nd Law: $\Sigma\vec{F} = m\vec{a}$ **Friction:** $f_k = \mu_k n$; $f_s \leq \mu_s n$

Dot Product: $\vec{A}\cdot\vec{B} = |\vec{A}|\ |\vec{B}|\ \cos\theta$, **or** $\vec{A}\cdot\vec{B} = (A_xB_x + A_yB_y + A_zB_z)$

Work by Constant Force: $W = \vec{F}\ \cdot\ \Delta\vec{r}$

General Equation for Work: $W = \int_{r_i}^{r_f} \vec{F}\ \cdot\ d\vec{r} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$

Kinetic Energy: $K = \frac{1}{2}mv^2$ **Work-KE:** $W_{net} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Conservation of Energy: $K_i + U_{g,i} + U_{el,i} = K_f + U_{g,f} + U_{el,f} + \Delta U_{int}$

$U_{el} = \frac{1}{2}kx^2$ $U_g = mgh$ $\Delta U_{int} = |\textbf{work done by friction}| = |\mu_k n d|$

Power: $P = \frac{dW}{dt} = \frac{\Delta\ \text{energy}}{\Delta\ \text{time}}$

Force-Potential Energy Relationship: $F_x = -\frac{dU}{dx}$ *and* $\Delta U = -W_{ab} = -\int_a^b F_x\ dx$

Linear Momentum: $\vec{p} = m\vec{v}$ **Impulse = Change in Momentum:** $\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$

For Time-dependent Force: $\int \vec{F}\ dt = \Delta\vec{p}$ **For Average Force:** $\vec{F}_{ave}\Delta t = \Delta\vec{p}$

CoLM (Cons. of Linear Momentum): $\hat{x}:\ \Sigma\vec{p}_{x,i} = \Sigma\vec{p}_{x,f}$ $\hat{y}:\ \Sigma\vec{p}_{y,i} = \Sigma\vec{p}_{y,f}$

Center of Mass: Discrete Particles or Uniform-Density Objects: $\vec{r}_{cm} = \frac{\Sigma m_i r_i}{\Sigma m_i}$

Non-Uniform-Density Objects: $\vec{r}_{cm} = \frac{\int \vec{r}\ dm}{\int dm} \longrightarrow x_{cm} = \frac{\int x\ dm}{\int dm}$, **and** $y_{cm} = \frac{\int y\ dm}{\int dm}$

1D: $dm = \lambda dr$; **2D:** $dm = \sigma dA$; **3D:** $dm = \rho dV$

Rotational Motion:

Rotational Kinematics: $\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt} \quad \longrightarrow \quad \theta = \int \omega \ dt, \quad \omega = \int \alpha \ dt$

For Constant Acceleration: $\omega_f = \omega_i + \alpha \ t; \quad \theta_f = \theta_i + \omega_i \ t + \frac{1}{2}\alpha \ t^2;$

$\omega_f{}^2 = \omega_i{}^2 + 2 \ \alpha \ (\theta_f - \theta_i); \quad \theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f) \ t$

Relation to Linear Quantities $v_t = r\omega$ **and** $a_t = r\alpha$

Moment of Inertia (MOI) $I = \Sigma m_i r_i^2 = \int r^2 dm$ **Total Mass:** $M_{tot} = \Sigma m_i = \int dm$

1D: $dm = \lambda dr$; **2D:** $dm = \sigma dA$; **3D:** $dm = \rho dV$

Parallel Axis Theorem: $I_{||} = I_{cm} + Md^2$

Torque: $\vec{\tau} = \vec{r} \text{ x } \vec{F}$ **N2L For Rotation:** $\Sigma\vec{\tau} = I\vec{\alpha}$

Cross Product: $|\vec{A} \text{ x } \vec{B}| = |\vec{A}| \ |\vec{B}| \ \sin\theta$**, or**

$$|\vec{A} \text{ x } \vec{B}| = (A_y \ B_z - A_z \ B_y)\hat{\imath} \ + \ (A_z \ B_x - A_x \ B_z)\hat{\jmath} \ + \ (A_x \ B_y - A_y \ B_x)\hat{k}$$

Rolling Motion: $v_{cm} = R\omega$

CoE with Rotation:: If pivoted, $K_{tot} = \frac{1}{2}I\omega^2$ **only, If not pivoted,** $K_{tot} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{cm}^2$

Angular Momentum: $\vec{L} = I\vec{\omega}$ **or** $\vec{L} = \vec{r} \text{ x } \vec{p}$

CoAM (Cons. of Ang. Momentum): $\Sigma\vec{L}_i = \Sigma\vec{L}_f$

Oscillations and Waves:

General Equation of Motion for SHM: $\frac{d^2x}{dt^2} = -\omega^2 x$

Position as function of time: $x(t) = A\cos(\omega t + \phi)$ **Time info:** $T = \frac{1}{f}$**, and** $\omega = \frac{2\pi}{T} = 2\pi f$

Max velocity and acceleration: $v_{max} = \omega A$**, and** $a_{max} = \omega^2 A$

Period of Mass on Spring: $T = 2\pi\sqrt{\frac{m}{k}}$ $\left(\omega = \sqrt{\frac{k}{m}}\right)$

Period of Pendulum at Small Angles: $T = 2\pi\sqrt{\frac{I}{m_{tot}gd}}$ $\left(\omega = \sqrt{\frac{m_{tot}gd}{I}}\right)$

Traveling Sinusoidal Wave: $y(x,t) = A\cos(kx \pm \omega t + \phi)$

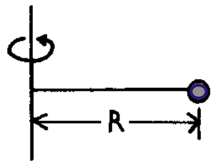
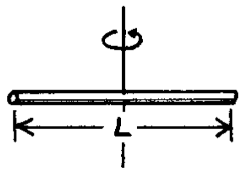
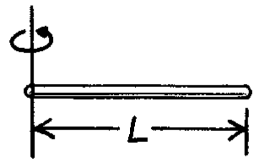
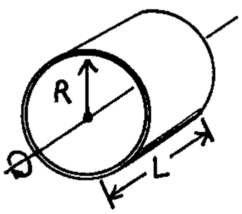
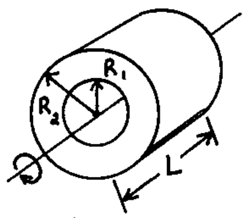
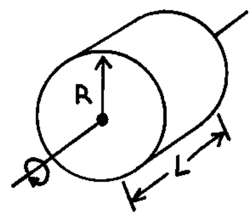
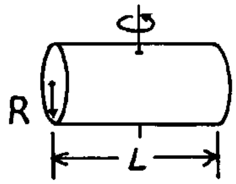
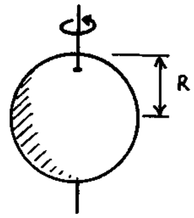
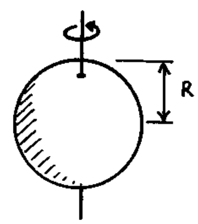
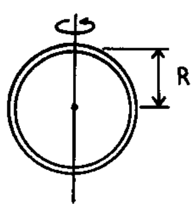
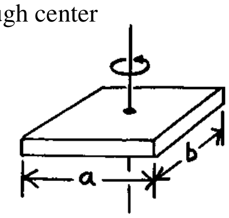
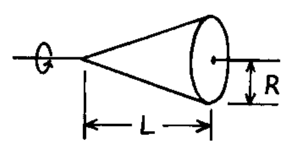
Wave number and Wavelength Relationship: $k = \frac{2\pi}{\lambda}$

Speed of Traveling Wave: $v = \lambda f = \frac{\omega}{k}$

Speed of Transverse Waves on String: $v_{string} = \sqrt{\frac{\text{tension}}{\text{linear mass density}}}$

Table of Selected Moments of Inertia

For uniform-density objects only

Point mass at a radius R  $I = MR^2$	Thin rod about axis through center perpendicular to length  $I = \frac{1}{12}ML^2$	Thin rod about axis through end perpendicular to length  $I = \frac{1}{3}ML^2$
Thin-walled cylinder about central axis  $I = MR^2$	Thick-walled cylinder about central axis  $I = \frac{1}{2}M(R_1^2 + R_2^2)$	Solid cylinder about central axis  $I = \frac{1}{2}MR^2$
Solid cylinder about central diameter  $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$	Solid sphere about center  $I = \frac{2}{5}MR^2$	Thin hollow sphere about center  $I = \frac{2}{3}MR^2$
Thin ring about diameter  $I = \frac{1}{2}MR^2$	Slab about perpendicular axis through center  $I = \frac{1}{12}M(a^2 + b^2)$	Cone about central axis  $I = \frac{3}{10}MR^2$

Note: All formulas shown assume objects of uniform mass density.