



Potter's wheel has $R = 0.4\text{ m}$, $h = 0.05\text{ m}$, $\rho = 7780 \frac{\text{kg}}{\text{m}^3}$.

Thus

$$\text{mass } M = \pi R^2 h \rho = 196 \text{ kg}$$

$$\text{mom of Inertia } I = \frac{1}{2} M R^2 = 15.6 \text{ kg}\cdot\text{m}^2$$

Wheel is at rest. Potter then turns on motor, which applies a torque

$$\vec{\tau}(t) = 20 \text{ N}\cdot\text{m} (1 - e^{-t}) \hat{j} \quad t \text{ in seconds}$$

So, at time $t = 2 \text{ sec}$,

$$\begin{aligned} \vec{\tau} &= 20 \text{ N}\cdot\text{m} (1 - e^{-2}) \hat{j} \\ &= 17.29 \text{ N}\cdot\text{m} \hat{j} \\ \vec{\alpha} &= \frac{\vec{\tau}}{I} = \frac{17.29 \text{ N}\cdot\text{m} \hat{j}}{15.6 \text{ kg}\cdot\text{m}^2} = 1.11 \frac{\text{rad}}{\text{s}^2} \hat{j} \end{aligned}$$

To find the angular velocity at $t = 2 \text{ sec}$, $\omega(2)$, we need to integrate the angular acceleration from $t = 0$ to $t = 2$. We can't use the simple kinematic formulae because the acceleration is not constant!

$$\vec{\omega}(2) - \vec{\omega}(0) = \int_{t=0}^{t=2} \vec{\alpha}(t) dt = \int_{t=0}^{t=2} \frac{\vec{\tau}(t)}{I} dt$$

But $\omega(0) = 0$, so

$$\begin{aligned}\vec{\omega}(2) &= \int_{t=0}^{t=2} \frac{20 \text{ N}\cdot\text{m}}{15.6 \text{ kg}\cdot\text{m}^2} (1 - e^{-t}) dt \uparrow \\ &= 1.282 \frac{\text{rad}}{\text{s}^2} \left[\int_0^2 1 dt - \int_0^2 e^{-t} dt \right] \uparrow \\ &\approx 1.282 \frac{\text{rad}}{\text{s}^2} [(2-0)s - (-e^{-2} - (-e^0))s] \\ &= 1.282 \frac{\text{rad}}{\text{s}^2} [2s - (-0.1353 + 1)s] \uparrow \\ &= 1.282 \frac{\text{rad}}{\text{s}^2} [2s - 0.8647s] \uparrow \\ \boxed{\vec{\omega}(t=2s) = 1.46 \frac{\text{rad}}{\text{s}}} \end{aligned}$$

The instantaneous power delivered by the motor is

$$\text{instantaneous } \underline{P} = \vec{\tau}(t) \cdot \vec{\omega}$$

Here, $\vec{\tau}$ and $\vec{\omega}$ are in same direction, so

$$\underline{P}(t=2s) = \tau(t=2s) \cdot \omega(t=2s)$$

$$= 17.29 \text{ N}\cdot\text{m} \cdot 1.46 \frac{\text{rad}}{\text{s}}$$

$$\boxed{\underline{P} = 26.1 \text{ Watts}}$$