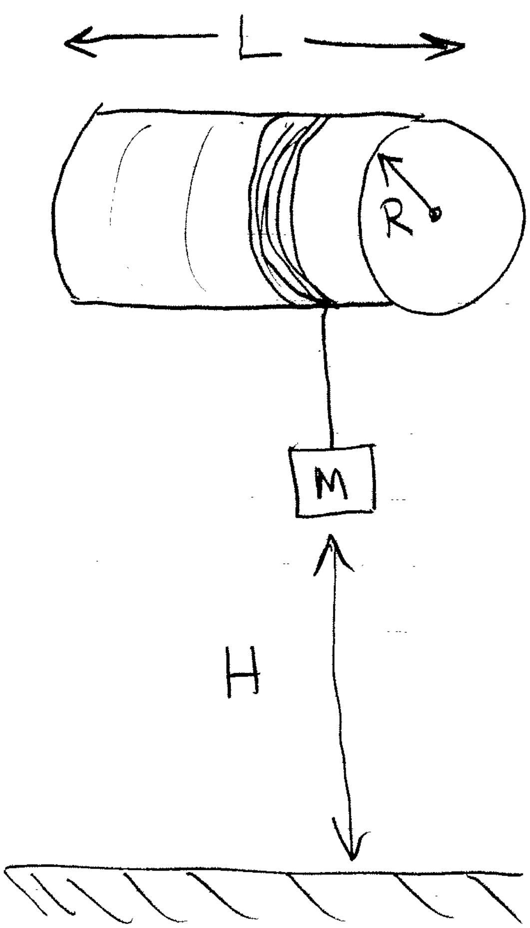
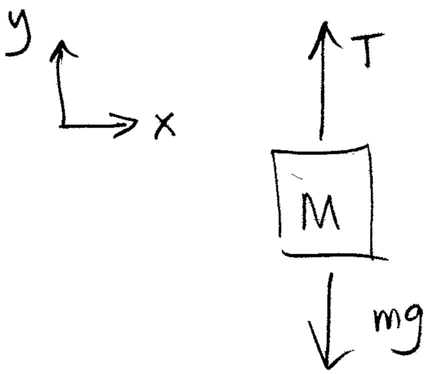


Joe has a barrel full of sand,
with length L
radius R
density ρ

The mass is
$$M = (\pi R^2 L)\rho$$

Joe wraps a rope around the barrel and attaches a bucket of mass m from the end of the rope. He suspends the barrel high above the ground, free to spin about its central axis. The bucket is motionless and a height H above the ground.

Joe releases the bucket. It begins to fall down, rotating the barrel as it drops.



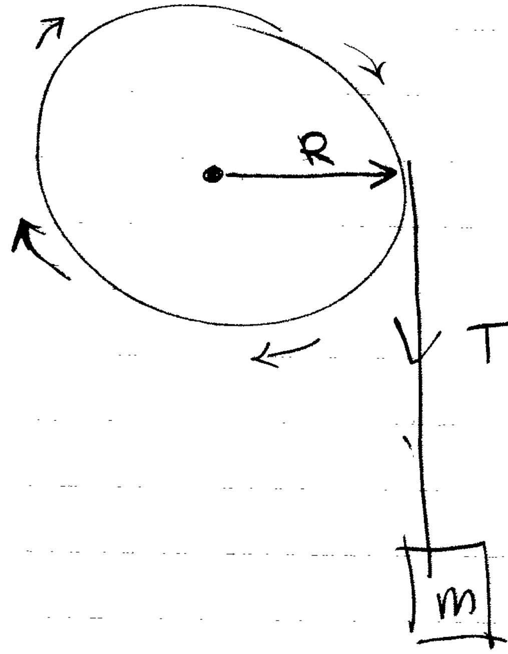
force	x	y
grav	0	$-mg$
rope	0	$+T$
total	$ma_x = 0$	$-ma_y$

negative means dropping

$$T - mg = -ma_y$$

$$\rightarrow T = m(g - a_y)$$ tension in rope.

end-on view of barrel



The rope pulls on the rim of the barrel, causing it to rotate.

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= \vec{R} \times \vec{T} \qquad \underline{\text{into}}\ \text{page}$$

$$|\tau| = RT \sin(90°)$$
$$= RT$$

The moment of inertia of the barrel around its central axis is

$$I = \tfrac{1}{2} M R^2$$
$$= \tfrac{1}{2}\left[(\pi R^2 L)\rho\right] R^2$$
$$= \tfrac{1}{2} \pi L \rho R^4$$

So, as the rope pulls, the barrel accelerates in angular speed:

$$\alpha = \frac{\tau}{I} = \frac{RT}{I} = \frac{R(mg - ma_y)}{\frac{1}{2}\pi L \rho R^4}$$

But if the rope doesn't slip,

$$\alpha = \frac{a_y}{R}$$

So

$$\frac{a_y}{R} = \frac{mg - ma_y}{\frac{1}{2}\pi L \rho R^3}$$

We can solve for a_y, the linear acceleration of the bucket.

$$a_y = \frac{mg}{\frac{1}{2}\pi L \rho R^2} - \left(\frac{m}{\frac{1}{2}\pi L \rho R^2}\right) a_y$$

$$a_y \left(1 + \frac{m}{\frac{1}{2}\pi L \rho R^2}\right) = \left(\frac{m}{\frac{1}{2}\pi L \rho R^2}\right) g$$

$$a_y = \frac{\frac{2m}{\pi L \rho R^2}}{\left(1 + \frac{2m}{\pi L \rho R^2}\right)} g$$

And then the time for bucket to reach the ground is

$$H = \frac{1}{2} a_y t^2$$

$$\rightarrow \boxed{t = \sqrt{\frac{2H}{a_y}}}$$

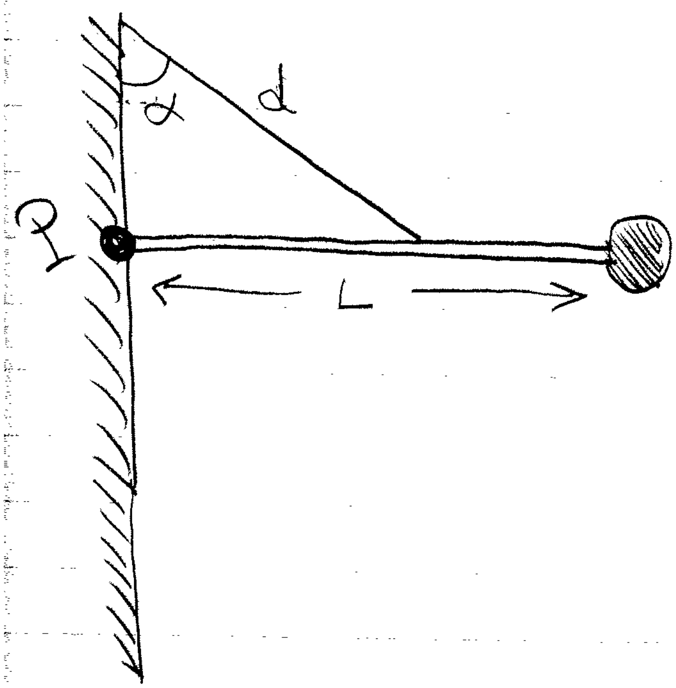
If $L = 2\,m$ $M = 3142\,kg$

$R = 0.5\,m$

$\rho = 2000\,kg/m^3$ $a_y = 0.00317\,g = 0.0311\,m/s^2$

$m = 5\,kg$

$H = 2.5\,m$ $t = 12.7\,sec$

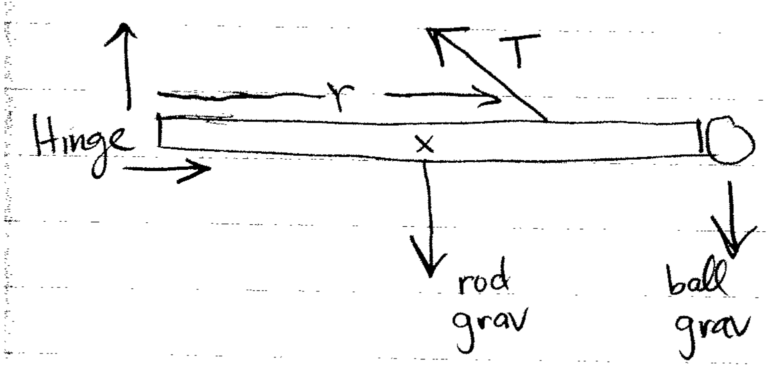


A flagpole consists of two parts:

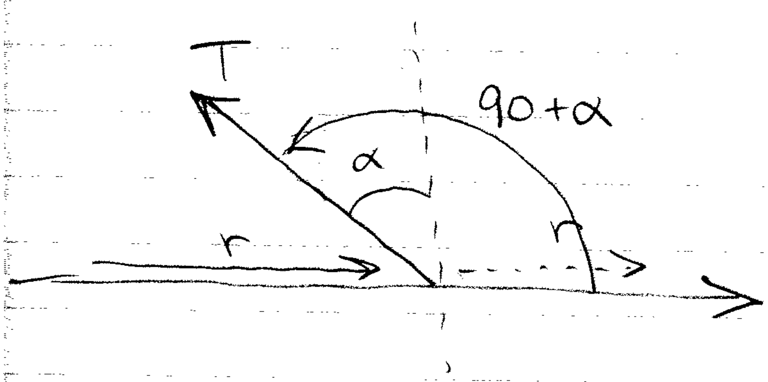
pole, length L, mass m
ball, mass M (compact)

The flagpole is connected to the wall at hinge point P. A wire of length d runs at angle α to support the flagpole.

When all is in equilibrium, what is the tension in the wire?



name	x	y	torque
rod grav	0	$-mg$	$-mg\left(\frac{L}{2}\right)$
ball grav	0	$-Mg$	$-MgL$
hinge	$+H_x$	$+H_y$	0
wire	$-T\sin\alpha$	$+T\cos\alpha$	$+Tr\sin(90+\alpha)$
	0	0	0



Note the lever arm of the wire is

$$r = d\sin\alpha$$

The angle between tension and radius from hinge to wire is

$$90+\alpha$$

$$\sin(90+\alpha) = \cos\alpha$$

$$\tau_{wire}\Big| = rT\sin(90+\alpha)$$

So, using the fact that sum of torques is zero,

$$-mg\left(\frac{L}{2}\right) - MgL + T(d\sin\alpha)\cos\alpha = 0$$

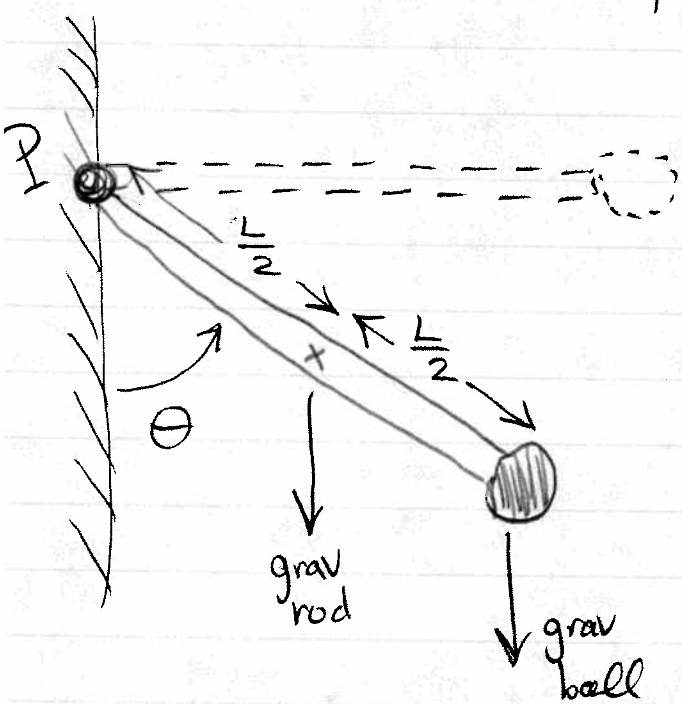
So

$$T\,d\sin\alpha\cos\alpha = gL\left(M + \tfrac{1}{2}m\right)$$

$$\boxed{T = \frac{gL\left(M + \frac{1}{2}m\right)}{d\sin\alpha\cos\alpha}}$$

The wire breaks! Now the pole begins to rotate around the hinge as it falls. Assume no friction in the hinge.

When the pole has swung downwards so that the remaining angle is θ, the torque on pole around hinge is



$$|\tau| = \left(\frac{L}{2}\right)mg\sin\theta$$

$$+ LMg\sin\theta$$

$$= gL\left(M + \tfrac{1}{2}m\right)\sin\theta$$

→

The moment of inertia around end of rod is

$$I = \frac{1}{3} m L^2 + M L^2$$

So the angular acceleration at this moment is

$$\alpha = \frac{\tau}{I} = \frac{gL\left(M + \frac{1}{2}m\right)}{\left(\frac{1}{3}m + M\right)L^2} \sin\theta$$

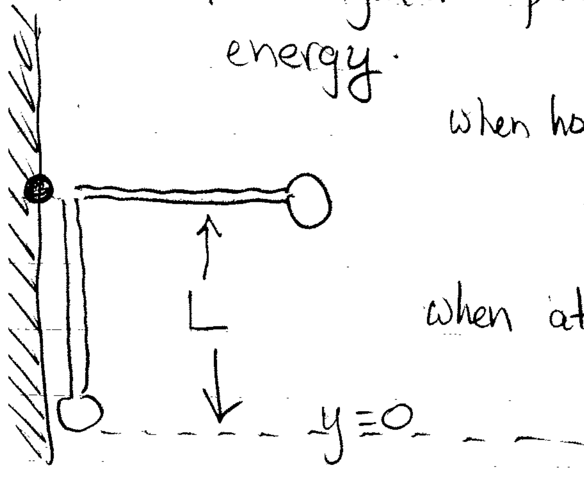
Is this simple harmonic motion?

$$\frac{d^2\theta}{dt^2} = (-) \frac{g\left(\frac{1}{2}m + M\right)}{\left(\frac{1}{3}m + M\right)L} \sin\theta$$

torque acts to decrease angle θ

No! We started at $\theta = \frac{\pi}{2}$ rad $= 90°$, so $\sin\theta \neq \theta$, so over the entire swing, not SHM.

At bottom of swing, just before flagpole strikes wall, its angular speed can be computed via conservation of energy.



when horizontal $E = KE + GPE$

$= 0 + Lmg + LMg$

when at bottom $E = KE + GPE$

$= \frac{1}{2} I \omega^2 + \left(\frac{L}{2} mg\right) + 0$

So

$$\frac{1}{2}I\omega^2 + \left(\frac{L}{2}\right)mg = Lmg + LMg$$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}Lmg + LMg$$

$$I\omega^2 = Lmg + 2LMg$$

$$I\omega^2 = gL(m+2M)$$

$$\omega^2 = \frac{gL(m+2M)}{I}$$

$$= \frac{gL(m+2M)}{\left(\frac{1}{3}m+M\right)L^2}$$

$$\rightarrow \quad \omega = \left[\left(\frac{m+2M}{\frac{1}{3}m+M}\right)\frac{g}{L}\right]^{1/2}$$

And the speed of the ball at tip of flagpole must be

$$V = L\omega$$

$$= L\left[\left(\frac{m+2M}{\frac{1}{3}m+M}\right)\frac{g}{L}\right]^{1/2}$$

$$\boxed{V = \left[\left(\frac{m+2M}{\frac{1}{3}m+M}\right)gL\right]^{1/2}}$$