Next, we need to calculate the integral

$$\int h\,dm = \int_{h=0}^{h=H} h\left(\pi R^2 \rho_0 \left[1+\frac{h}{H}\right]\left[1-\frac{h}{H}\right]^2\right) dh$$

$$= \rho_0 \pi R^2 \int h\left(1-\frac{h}{H}-\frac{h^2}{H^2}+\frac{h^3}{H^3}\right) dh$$

$$= \rho_0 \pi R^2 \int \left(h-\frac{h^2}{H}-\frac{h^3}{H^2}+\frac{h^4}{H^3}\right) dh$$

$$= \rho_0 \pi R^2 \left[\frac{1}{2}h^2 - \frac{1}{3}\frac{h^3}{H} - \frac{1}{4}\frac{h^4}{H^2} + \frac{1}{5}\frac{h^5}{H^3}\right]_{h=0}^{h=H}$$

$$= \rho_0 \pi R^2 \left[\frac{1}{2}H^2 - \frac{1}{3}H^2 - \frac{1}{4}H^2 + \frac{1}{5}H^2\right]$$

$$= \rho_0 \pi R^2 \left[\frac{7}{60}H^2\right]$$

So

$$\text{height of center of mass} = \frac{\int h\,dm}{\int dm} = \frac{\frac{7}{60}\rho_0 \pi R^2 H^2}{\frac{5}{12}\rho_0 \pi R^2 H}$$

$$= \boxed{\frac{7}{25}H}$$

vs. $\frac{1}{4}H = \frac{6}{24}H$ for a uniform cone.

So center of mass is a little higher, as it should be.