Now, suppose the cone is denser at the top than the bottom, so that

$$\rho(h) = \rho_0\left(1 + \frac{h}{H}\right)$$

In other words,

at bottom $\quad h = 0 \quad \rho = \rho_0$

at top $\quad h = H \quad \rho = 2\rho_0$

First, we need to calculate the mass of this cone.

$$M = \int dm = \int_{h=0}^{h=H} \rho_0\left(1+\frac{h}{H}\right)\pi R^2\left(1-\frac{h}{H}\right)^2 dh$$

$$= \rho_0 \pi R^2 \int_{h=0}^{h=H}\left(1+\frac{h}{H}\right)\left(1-\frac{2h}{H}+\frac{h^2}{H^2}\right) dh$$

$$= \rho_0 \pi R^2 \int_{h=0}^{h=H}\left(1-\frac{h}{H}-\frac{h^2}{H^2}+\frac{h^3}{H^3}\right) dh$$

$$= \rho_0 \pi R^2 \left[ h - \frac{1}{2}\frac{h^2}{H} - \frac{1}{3}\frac{h^3}{H^2} + \frac{1}{4}\frac{h^4}{H^3} \right|_{h=0}^{h=H}$$

$$= \rho_0 \pi R^2 \left[ H - \frac{1}{2}H - \frac{1}{3}H + \frac{1}{4}H \right]$$

$$= \underline{\frac{5}{12}\rho_0 \pi R^2 H}$$

which _is_ larger than mass of uniform cone.