

Add up all slices to calculate mass of cone:

$$M = \int dm = \int_{h=0}^{h=H} \rho \pi R^2 \left(1 - \frac{h}{H}\right)^2 dh$$

$$= \rho \pi R^2 \int_{h=0}^{h=H} \left(1 - \frac{h}{H}\right)^2 dh$$

$$= \rho \pi R^2 \int_{h=0}^{h=H} \left(1 - 2\frac{h}{H} + \frac{h^2}{H^2}\right) dh$$

$$= \rho \pi R^2 \left[h - \frac{h^2}{H} + \frac{1}{3} \frac{h^3}{H^2} \right]_{h=0}^{h=H}$$

$$\underline{M = \frac{1}{3} \rho \pi R^2 H} \quad \text{total mass of cone.}$$

To calculate center of mass, we need to calculate

$$\text{height of center of mass} = \frac{\int h dm}{\text{total mass}}$$

$$= \frac{\int \rho \pi R^2 \left(1 - \frac{h}{H}\right)^2 h dh}{\frac{1}{3} \rho \pi R^2 H}$$